ESSENTIAL ELEMENTS
AND ALTERNATE ACHIEVEMENT DESCRIPTORS FOR

Mathematics

Wisconsin Department of Public Instruction

ESSENTIAL ELEMENTS

Mathematics

Wisconsin Department of Public Instruction
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Madison, Wisconsin

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SECTION I

Wisconsin's Approach to Academic Standards

Foreword

In June 2010, Wisconsin adopted the Common Core State Standards in English Language Arts and Mathematics. These K-12 academic standards are aligned with college and work expectations, include rigorous content and application, and are internationally benchmarked. Additionally, the Common Core State Standards emphasize literacy in all of the disciplines. For all students to be career and college ready, including students with significant cognitive disabilities, educators should include both the content and the reading and writing skills that students need to demonstrate learning in the other disciplinary areas.

All students, including students with significant cognitive disabilities, deserve and have a right to a quality educational experience. This right includes, to the maximum extent possible, the opportunity to be involved in and meet the same challenging expectations that have been established for all students. Wisconsin educators collaborated with educators from 12 other states to create alternate achievement standards aligned to the Common Core State Standards. These alternate achievement standards are called the *Wisconsin Common Core Essential Elements (CCEEs) in English Language Arts and Mathematics*. The CCEEs satisfy the requirement of the U.S. Department of Education that Wisconsin have alternate achievement standards for its students with significant cognitive disabilities that are clearly linked to grade-level academic content standards, promote access to the general curriculum and reflect professional judgment of the highest expectation possible.

This document is a guide for parents, educators, school personnel, and other community members to support their work in teaching students with significant cognitive disabilities the academic skills necessary to succeed in life after graduation.

Tony Evers, PhD
State Superintendent

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Purpose of the Document

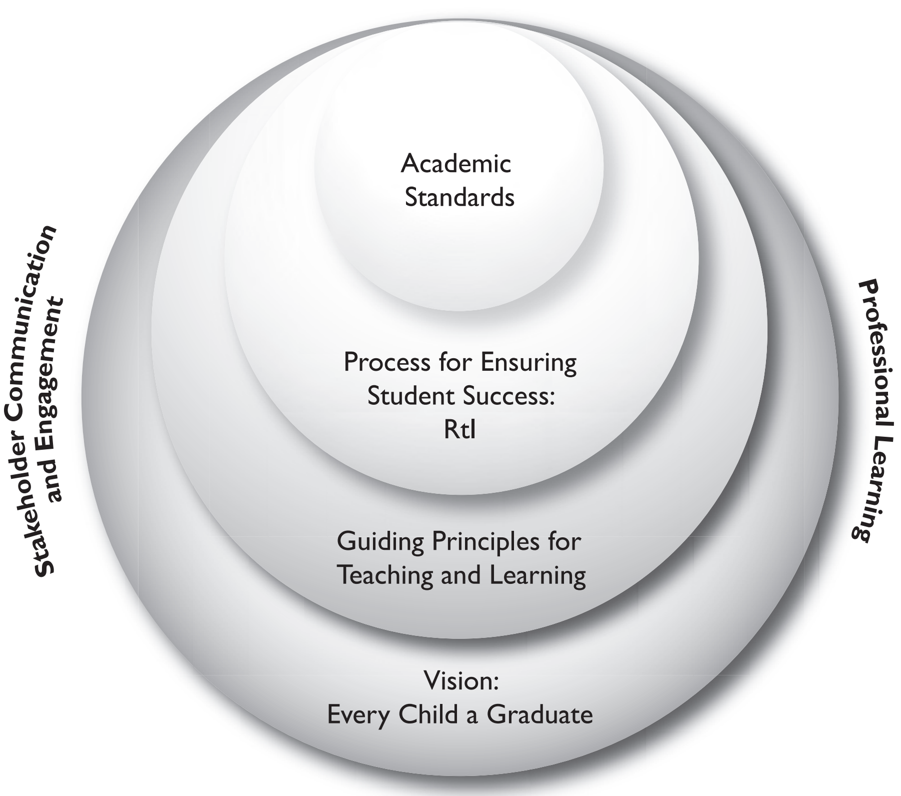
Sections I, 2 and 4 of this document were developed by Wisconsin educators to provide the vision and principles that support Wisconsin's Approach to Academic Standards. These principles, although initially developed for the CCSS, can be applied to the CCEEs and instructional practices of educators of students with significant cognitive disabilities.

To assist Wisconsin education stakeholders in understanding and implementing the **Common Core State Standards (CCSS)**, Wisconsin Department of Public Instruction (DPI) has developed guidance to be used along with the CCSS. These materials are intended to provide further direction and should not be viewed as administrative rule. This publication provides a vision for student success, guiding principles for teaching and learning, and locates the standards within a multi-level system of support where high quality instruction, balanced assessment, and collaboration function together for student learning. Information on the design and content of the CCSS is included, as is a guide to assist with facilitating local conversations about these internationally-benchmarked standards and how they impact instruction.

Aligning for Student Success

To build and sustain schools that support every student in achieving success, educators must work together with families, community members, and business partners to connect the most promising practices in the most meaningful contexts. Major statewide initiatives focus on high school graduation, Response to Intervention (RtI), and the *Common Core State Standards for English Language Arts, Disciplinary Literacy, and Mathematics*. While these are often viewed as separate efforts or initiatives, each of them is connected to a larger vision of every child graduating college and career ready. The graphic below illustrates how these initiatives function together for a common purpose. Here, the vision and set of guiding principles form the foundation for building a supportive process for teaching and learning rigorous and relevant content. The following sections articulate this integrated approach to increasing student success in Wisconsin schools and communities.

Relationship Between Vision, Principles, Process, Content



A Vision: Every Child a Graduate

In Wisconsin, we are committed to ensuring every child is a graduate who has successfully completed a rigorous, meaningful, 21st century education that will prepare him or her for careers, college and citizenship. Though our public education system continues to earn nation-leading graduation rates, a fact we can be proud of, one in ten students drop out of school, achievement gaps are too large, and overall achievement could be even higher. This vision for every child a graduate guides our beliefs and approaches to education in Wisconsin.

Guided By Principles

All educational initiatives are guided and impacted by important and often unstated attitudes or principles for teaching and learning. *The Guiding Principles for Teaching and Learning* emerge from research and provide the touchstone for practices that truly affect the vision of every child a graduate prepared for college and career. When made transparent, these principles inform what happens in the classroom, the implementation and evaluation of programs, and most important, remind us of our own beliefs and expectations for students.

Ensuring a Process for Student Success

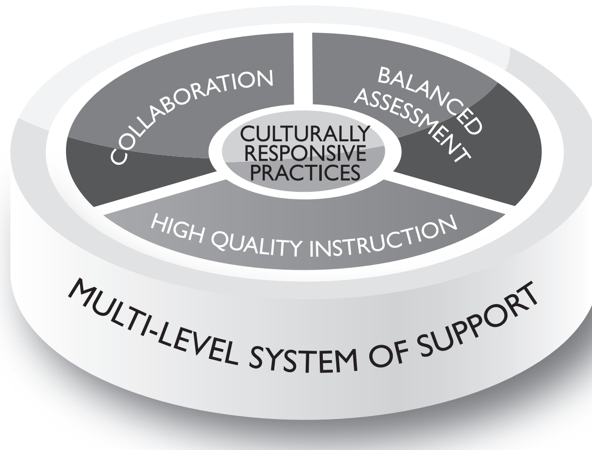
To ensure that every child in Wisconsin graduates prepared for college and career, schools need to provide high quality instruction, balanced assessment and collaboration reflective of culturally responsive practices. The Wisconsin Response to Intervention (RtI) framework helps to organize the components of a system designed to support student learning. Below, the three essential elements of high quality instruction, balanced assessment and collaboration interact within a multi-level system of support to ensure each student receives what he or she needs to access higher levels of academic and behavioral success.

At the school or district level, programs, initiatives and practices related to high quality instruction, balanced assessment and collaboration can be more powerful when organized or braided to function systemically to support all students. The focus must be on a comprehensive approach to student learning.

Connecting to Content: The Common Core State Standards

Within this vision for increased student success, rigorous, internationally-benchmarked academic standards provide the content for high quality curriculum and instruction, and for a balanced assessment system aligned to those standards. With the adoption of the CCSS, Wisconsin has the tools to build world-class curriculum, instruction and assessments for greater student learning. The CCSS articulate what we teach so that educators can focus on how instruction can best meet the needs of each student. When implemented within a multi-level system of support, the CCSS can help to ensure that every child will graduate prepared for college, work and a meaningful life.

Wisconsin's Vision for RtI



"Educators must work together with families, community members, and business partners to connect the most promising practices in the most meaningful contexts."

Guiding Principles for Teaching and Learning

These guiding principles are the underpinnings of effective teaching and learning for every Wisconsin teacher and every Wisconsin student. They are larger than any one initiative, process or set of standards. Rather, they are the lens we look through as we identify teaching and learning standards, design assessments and determine what good instruction looks like. These principles recognize that every student has the right to learn and are built upon three essential elements: high quality instruction, balanced assessment, and collaboration. They are meant to align with academic excellence, rigorous instruction, and college and career readiness for every Wisconsin student. For additional research, resources and probing questions to support professional learning on the six principles, please see the Wisconsin Research and Resources section of this document.

Every student has the right to learn.

It is our collective responsibility as an education community to make certain each child receives a high-quality, challenging education designed to maximize potential, an education that reflects and stretches his or her abilities and interests. This belief in the right of every child to learn forms the basis of equitable teaching and learning. The five principles that follow cannot exist without this commitment guiding our work.

Instruction must be rigorous and relevant.

To understand the world in which we live, there are certain things we all must learn. Each school subject is made up of a core of essential knowledge that is deep, rich, and vital. Every student, regardless of age or ability, must be taught this essential knowledge. What students learn is fundamentally connected to how they learn, and successful instruction blends the content of a discipline with processes of an engaging learning environment that changes to meet the dynamic needs of all students.

Purposeful assessment drives instruction and affects learning.

Assessment is an integral part of teaching and learning. Purposeful assessment practices help teachers and students understand where they have been, where they are, and where they might go next. No one assessment can provide sufficient information to plan teaching and learning. Using different types of assessments as part of instruction results in useful information about student understanding and progress. Educators should use this information to guide their own practice and in partnership with students and their families to reflect on learning and set future goals.

Learning is a collaborative responsibility.

Teaching and learning are both collaborative processes. Collaboration benefits teaching and learning when it occurs on several levels: when students, teachers, family members, and the community collectively prioritize education and engage in activities that support local schools, educators, and students; when educators collaborate with their colleagues to support innovative classroom practices and set high expectations for themselves and their students; and when students are given opportunities to work together toward academic goals in ways that enhance learning.

Students bring strengths and experiences to learning.

Every student learns. Although no two students come to school with the same culture, learning strengths, background knowledge, or experiences, and no two students learn in exactly the same way, every student's unique personal history enriches classrooms, schools, and the community. This diversity is our greatest education asset.

Responsive environments engage learners.

Meaningful learning happens in environments where creativity, awareness, inquiry, and critical thinking are part of instruction. Responsive learning environments adapt to the individual needs of each student and encourage learning by promoting collaboration rather than isolation of learners. Learning environments, whether classrooms, schools, or other systems, should be structured to promote engaged teaching and learning.

Reaching Every Student;
Reaching Every Discipline

Reaching Every Student

The CCSS set high, clear and consistent expectations for all students. In order to ensure that all students can meet and exceed those expectations, Wisconsin educators provide flexible and fluid support based on student need. Each student brings a complex system of strengths and experiences to learning. One student may have gifts and talents in mathematics and need additional support to reach grade-level standards in reading. A student may be learning English as a second language while remaining identified for gifted services in science. The following statements provide guidance for how to ensure that the CCSS provide the foundation for learning for every student in Wisconsin, regardless of their unique learning needs.

Application of Common Core State Standards for English Language Learners

The National Governors Association Center for Best Practices and the Council of Chief State School Officers strongly believe that all students should be held to the same high expectations outlined in the Common Core State Standards. This includes students who are English language learners (ELLs). However, these students may require additional time, appropriate instructional support, and aligned assessments as they acquire both English language proficiency and content area knowledge.

ELLs are a heterogeneous group with differences in ethnic background, first language, socioeconomic status, quality of prior schooling, and levels of English language proficiency. Effectively educating these students requires pre-assessing each student instructionally, adjusting instruction accordingly, and closely monitoring student progress. For example, ELLs who are literate in a first language that shares cognates with English can apply first-language vocabulary knowledge when reading in English; likewise ELLs with high levels of schooling can often bring to bear conceptual knowledge developed in their first language when reading in English. However, ELLs with limited or interrupted schooling will need to acquire background knowledge prerequisite to educational tasks at hand. Additionally, the development of native-like proficiency in English takes many years and may not be achieved by all ELLs especially if they start schooling in the US in the later grades. Teachers should recognize that it is possible to achieve the standards for reading and literature, writing and research, language development and speaking and listening without manifesting native-like control of conventions and vocabulary.

English Language Arts

The Common Core State Standards for English Language Arts (ELA) articulate rigorous grade-level expectations in the areas of reading, writing, speaking, listening to prepare all students to be college and career ready, including English language learners. Second-language learners also will benefit from instruction about how to negotiate situations outside of those settings so they are able to participate on equal footing with native speakers in all aspects of social, economic, and civic endeavors.

ELLs bring with them many resources that enhance their education and can serve as resources for schools and society. Many ELLs have first language and literacy knowledge and skills that boost their acquisition of language and literacy in a second language; additionally, they bring an array of talents and cultural practices and perspectives that enrich our schools and society. Teachers must build on this enormous reservoir of talent and provide those students who need it with additional time and appropriate instructional support. This includes language proficiency standards that teachers can use in conjunction with the ELA standards to assist ELLs in becoming proficient and literate in English. To help ELLs meet high academic standards in language arts it is essential that they have access to:

- Teachers and personnel at the school and district levels who are well prepared and qualified to support ELLs while taking advantage of the many strengths and skills they bring to the classroom;
- Literacy-rich school environments where students are immersed in a variety of language experiences;
- Instruction that develops foundational skills in English and enables ELLs to participate fully in grade-level coursework;

- Coursework that prepares ELLs for postsecondary education or the workplace, yet is made comprehensible for students learning content in a second language (through specific pedagogical techniques and additional resources);
- Opportunities for classroom discourse and interaction that are well-designed to enable ELLs to develop communicative strengths in language arts;
- Ongoing assessment and feedback to guide learning; and
- Speakers of English who know the language well enough to provide ELLs with models and support.

Application to Students with Disabilities

The Common Core State Standards articulate rigorous grade-level expectations in the areas of mathematics and English language arts. These standards identify the knowledge and skills students need in order to be successful in college and careers.

Students with disabilities, students eligible under the Individuals with Disabilities Education Act (IDEA), must be challenged to excel within the general curriculum and be prepared for success in their post-school lives, including college and/or careers. These common standards provide an historic opportunity to improve access to rigorous academic content standards for students with disabilities. The continued development of understanding about research-based instructional practices and a focus on their effective implementation will help improve access to mathematics and English language arts (ELA) standards for all students, including those with disabilities. Students with disabilities are a heterogeneous group with one common characteristic: the presence of disabling conditions that significantly hinder their abilities to benefit from general education (IDEA 34 CFR §300.39, 2004). Therefore, how these high standards are taught and assessed is of the utmost importance in reaching this diverse group of students.

In order for students with disabilities to meet high academic standards and to fully demonstrate their conceptual and procedural knowledge and skills in mathematics, reading, writing, speaking and listening (English language arts), their instruction must incorporate supports and accommodations, including:

- Supports and related services designed to meet the unique needs of these students and to enable their access to the general education curriculum (IDEA 34 CFR §300.34, 2004).
- An Individualized Education Program (IEP)[1] which includes annual goals aligned with and chosen to facilitate their attainment of grade-level academic standards.
- Teachers and specialized instructional support personnel who are prepared and qualified to deliver high-quality, evidence-based, individualized instruction and support services.

Promoting a culture of high expectations for all students is a fundamental goal of the Common Core State Standards. In order to participate with success in the general curriculum, students with disabilities, as appropriate, may be provided additional supports and services, such as:

- Instructional supports for learning, based on the principles of Universal Design for Learning (UDL),[2] which foster student engagement by presenting information in multiple ways and allowing for diverse avenues of action and expression.
- Instructional accommodations (Thompson, Morse, Sharpe & Hall, 2005), changes in materials or procedures, which do not change the standards but allow students to learn within the framework of the Common Core.
- Assistive technology devices and services to ensure access to the general education curriculum and the Common Core State Standards.

Some students with the most significant cognitive disabilities will require substantial supports and accommodations to have meaningful access to certain standards in both instruction and assessment, based on their communication and academic needs. These supports and accommodations should ensure that students receive access to multiple means of learning and opportunities to demonstrate knowledge, but retain the rigor and high expectations of the Common Core State Standards.

Implications for the Common Core State Standards for Students with Gifts and Talents

The CCSS provide a roadmap for what students need to learn by benchmarking expectations across grade levels. They include rigorous content and application of knowledge through higher-order skills. As such, they can serve as a foundation for a robust core curriculum, however, students with gifts and talents may need additional challenges or curricular options. In order to recognize what adaptations need to be made or what interventions need to be employed, we must understand who these students are.

According to the National Association for Gifted Children (2011), "Giftedness, intelligence, and talent are fluid concepts and may look different in different contexts and cultures" (para. 1). This means that there are students that demonstrate high performance or have the potential to do so in academics, creativity, leadership, and/or the visual and performing arts. Despite this diversity there are common characteristics that are important to note.

Students with gifts and talents:

- Learn at a fast pace.
- Are stimulated by depth and complexity of content.
- Make connections.

These traits have implications for how the Common Core State Standards are used. They reveal that as curriculum is designed and instruction, is planned there must be:

- Differentiation based on student readiness, interest, and learning style:
  - Pre-assessing in order to know where a student stands in relation to the content that will be taught (readiness), then teach those standards that the student has not mastered and enrich, compact, and/or accelerate when standards have been mastered. This might mean using standards that are beyond the grade level of the student.
  - Knowledge of our students so we are familiar with their strengths, background knowledge, experiences, interests, and learning styles.
  - Flexible grouping to provide opportunities for students to interact with peers that have similar abilities, similar interests, and similar learning styles (homogenous grouping), as well as different abilities, different interests, and different learning styles (heterogeneous grouping).
- Differentiation of content, process, and product.
  - Use of a variety of materials (differentiating content) to provide challenge. Students may be studying the same concept using different text and resources.
  - Variety of tasks (differentiating process). For example in a science lesson about the relationship between temperature and rate of melting, some students may use computer-enhanced thermometers to record and graph temperature so they can concentrate on detecting patterns while other students may graph temperature at one-minute intervals, then examine the graph for patterns.
  - Variety of ways to demonstrate their learning (differentiating product). These choices can provide opportunities for students with varying abilities, interests, and learning styles to show what they have discovered.
- Adjustment to the level, depth, and pace of curriculum.
  - Compact the curriculum to intensify the pace.
  - Vary questioning and use creative and critical thinking strategies to provide depth.
  - Use standards beyond the grade level of the students. Since the CCSS provide a K-12 learning progression, this is easily done.
  - Accelerate subject areas or whole grades when appropriate.
- Match the intensity of the intervention with the student's needs. This means that we must be prepared to adapt the core curriculum and plan for a continuum of services to meet the needs of all students, including those with gifts and talents.

References

Individuals with Disabilities Education Act (IDEA), 34 CFR §300.34 (a). (2004).

Individuals with Disabilities Education Act (IDEA), 34 CFR §300.39 (b)(3). (2004).

National Association for Gifted Children (2010). Redefining Giftedness for a New Century Shifting the Paradigm. Retrieved from http://www.nagc.org/index.aspx?id=6404.

National Association for Gifted Children (2011). What is giftedness? Retrieved from http://nagc.org/index.aspx?id=574.

Sousa, D.A. (200). How the gifted brain learns. Thousand Oaks, CA: Corwin Press.

Thompson, Sandra J., Amanda B. Morse, Michael Sharpe, and Sharon Hall. "Accommodations Manual: How to Select, Administer and Evaluate Use of Accommodations and Assessment for Students with Disabilities," 2nd Edition. Council for Chief State School Officers, 2005 http://www.ccsso.org/content/pdfs/AccommodationsManual.pdf . (Accessed January, 29, 2010).

Reaching Every Discipline
Wisconsin's Approach to Disciplinary Literacy

Background

In Wisconsin, we hold the vision that every child must graduate ready for post-secondary education and the workforce. To achieve this vision, students must develop the skills to think, read, communicate, and perform in many academic contexts. If students must develop these specific skills, every educator must then consider how students learn to read, write, think, speak and listen in their discipline.

The kinds of reading, writing, thinking, speaking and listening required in a marketing course are quite different when compared with the same processes applied in an agriculture, art or history course. For example, a student may have successfully learned the vocabulary and content needed to score an A on a freshman biology test, but finds he still struggles to understand relevant articles from *Popular Science Magazine*, or use his science vocabulary to post respected responses on an environmental blog he reads at home. This student knows biology content, but lacks the disciplinary literacy to think, read, write, and speak with others in this field. Without this ability, his content knowledge is limited only to the classroom, and cannot extend to the real world around him.

In Wisconsin, disciplinary literacy is defined as the confluence of content knowledge, experiences, and skills merged with the ability to read, write, listen, speak, think critically and perform in a way that is meaningful within the context of a given field.

Teaching for disciplinary literacy ensures that students develop the skills to use the deep content knowledge they learn in school in ways that are relevant to each of them, and to the world around them.

In 2009, *The State Superintendent's Adolescent Literacy Plan* offered recommendations for how to begin professional conversations about disciplinary literacy in Wisconsin. The plan recommended Wisconsin write standards for literacy that were specific to each discipline, and emphasized the need to accompany these literacy standards with discipline-specific professional learning.

Wisconsin's Approach to Disciplinary Literacy

In 2010, the Council of Chief State School Officers (CCSSO) responded to this need for standards by publishing Common Core State Standards for Literacy in History/Social Studies, Science and Technical Subjects in grades 6-12. These standards were adopted by State Superintendent Tony Evers in June 2010. Wisconsin applauds this bold move to begin a national conversation on disciplinary literacy, and recognizes the need to broaden this effort to include all disciplines, and every educator in every grade level.

The ability to read, write, think, speak, and listen, in different ways and for different purposes begins early and becomes increasingly important as students pursue specialized fields of study in high school and beyond. These abilities are as important in mathematics, engineering and art courses as they are in science, social studies and English.

To further solidify Wisconsin's expanded approach to disciplinary literacy, a statewide leadership team comprised of K-16 educators from diverse subject areas was convened. A set of foundations, was established and directs Wisconsin's approach to disciplinary literacy.

This document begins the conversation about literacy in all subjects. It will come to life when presented to teachers and they are able to showcase their subjects' connection to literacy in all subjects which will bring the literacy standards to life for their community of learners.

Wisconsin Foundations for Disciplinary Literacy

To guide understanding and professional learning, a set of foundational statements, developed in concert with *Wisconsin's Guiding Principles for Teaching and Learning*, directs Wisconsin's approach to disciplinary literacy.

- Academic learning begins in early childhood and develops across all disciplines.
- Content knowledge is strengthened when educators integrate discipline-specific literacy into teaching and learning.
- The literacy skills of reading, writing, listening, speaking and critical thinking improve when content-rich learning experiences motivate and engage students.
- Students demonstrate their content knowledge through reading, writing, listening, and speaking as part of a content literate community.

Wisconsin's Common Core Standards for Literacy in All Subjects

With the Wisconsin Foundations for Disciplinary Literacy, Wisconsin expands the Common Core State Standards for Literacy in History/Social Studies, Science and Technical Subjects, to include every educator in every discipline and at every level. The Common Core Standards for English Language Arts include the Literacy Standards in History/Social Studies, Science and Technical Subjects as well as other relevant standards materials, resources, and research that support discipline-specific conversations across all content areas and grade levels.

The Common Core State Standards for Literacy in all Subjects is included as part of every set of Wisconsin standards as each discipline is reviewed in accordance with the process for Wisconsin standards revision http://www.dpi.wi.gov/standards. This document includes relevant resources and research that may be helpful in advancing school and district conversations, and can also be downloaded at www.dpi.wi.gov/standards or purchased as a stand-alone document through www.dpi.wi.gov/publications.

SECTION 2

Wisconsin's Approach to Mathematics

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Wisconsin Foundations for Mathematics

Wisconsin's Guiding Principles for Teaching and Learning provide important guidance for the mathematics classroom. Within the discipline of mathematics, each of the six principles has specific implications for equity, pedagogy, instruction, and assessment. Mathematics educators should consider how the six guiding principles influence their teaching.

The following foundations provide direction for the teaching and learning of mathematics in Wisconsin.

Every student must have access to and engage in meaningful, challenging, and rigorous mathematics.

Equity in mathematics education requires recognition that the standards must be kept consistent while being flexible in instructional approach and methods of assessment to accommodate the strengths and weaknesses of all students. In order to optimize student learning, the high bar that is set for all should not be moved for some students; instead, the delivery system must be varied to allow access for all. Schools and classrooms need to be organized to convey the message that all students can learn mathematics and should be expected to achieve. Effective mathematics classroom practice involves assessing students' prior knowledge, designing tasks that allow flexibility of approach, and orchestrating classroom discussions that allow every student to successfully access and learn important mathematics.

Mathematics should be experienced as coherent, connected, intrinsically interesting, and relevant.

The PK-12 curriculum should integrate and sequence important mathematical ideas so that students can make sense of mathematics and develop a thorough understanding of concepts. The curriculum should build from grade to grade and topic to topic so that students have experiences that are coherent. The connections of mathematical ideas in a well-designed curriculum allow students to see mathematics as important in its own right, as well as a useful subject that has relevant applications to the real world and to other disciplines.

Problem solving, understanding, reasoning, and sense-making are at the heart of mathematics teaching and learning and are central to mathematical proficiency.

Using problem solving as a vehicle for teaching mathematics not only develops knowledge and skills, but also helps students understand and make sense of mathematics. By infusing reasoning and sense-making in daily mathematics instruction, students are able to see how new concepts connect with existing knowledge and they are able to solidify their understanding. Students who are mathematically proficient see that mathematics makes sense and show a willingness to persevere. They possess both understanding of mathematical concepts and fluency with procedural skills.

Effective mathematics classroom practices include the use of collaboration, discourse, and reflection to engage students in the study of important mathematics.

Collaboration and classroom discourse can significantly deepen student understanding of mathematical concepts. In addition to teacher-student dialogue, peer collaboration and individual reflection must also be emphasized. Representing, thinking, discussing, agreeing, and disagreeing are central to what students learn about mathematics. Posing questions and tasks that elicit, engage, and challenge students' thinking, as well as asking students to clarify their thinking and justify solutions and solution paths should be evident in all mathematics classrooms.

When today's students become adults, they will face new demands for mathematical proficiency that school mathematics should attempt to anticipate. Moreover, mathematics is a realm no longer restricted to a select few. All young Americans must learn to think mathematically, and they must think mathematically to learn.

(Adding It Up, National Research Council, 2001).

Standards for Mathematical Practice

The Standards for Mathematical Practice are central to the teaching and learning of mathematics. These practices describe the behaviors and habits of mind that are exhibited by students who are mathematically proficient. Mathematical understanding is the intersection of these practices and mathematics content. It is critical that the *Standards for Mathematical Practice* are embedded in daily mathematics instruction.

The graphic below shows the central focus on the *Standards for Mathematical Practice* within the familiar content areas of mathematics. Some of the behaviors and dispositions exhibited by students who are mathematically proficient are elaborated in the Characteristics of Mathematically Proficient Students.

Standards for Mathematical Content

The *Standards for Mathematical Content* describe the sequence of important mathematics content that students learn. They are a combination of procedures and understandings. These content standards are organized around domains and clusters which are specified by grade level, kindergarten through grade 8, and by conceptual category at high school. The domains at all levels are based on research-based learning progressions detailing what is known about students' mathematical knowledge, skill, and understanding. The progressions build from grade to grade and topic to topic, providing K-12 focus and coherence. Other important cross-grade themes that should be noted and investigated are concepts such as the role of units and unitizing, the properties of operations across arithmetic and algebra, operations and the problems they solve, transformational geometry, reasoning and sense-making, and modeling of and with mathematics.

The **narratives at each K-8 grade level** specify 2-4 key areas that are identified as the primary focus of instruction. These are referred to as **critical areas**. At the high school level, the narratives describe the **focus** for each conceptual category, as well as the connections to other categories and domains.

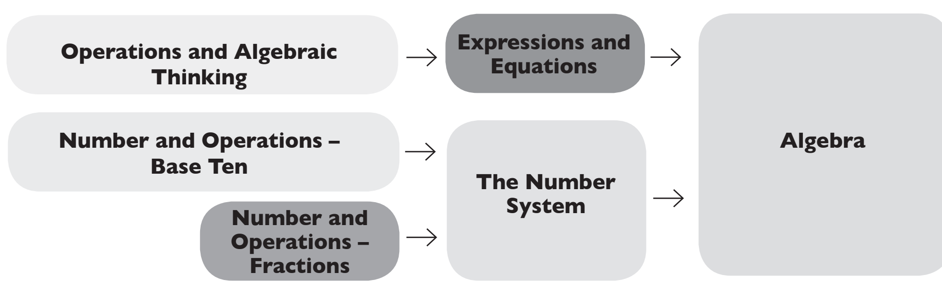
Learning mathematics with understanding is a focus of the CCSSM. Many of the *Standards for Mathematical Content* begin with the verb "understand" and are crucial for mathematical proficiency. It is generally agreed that students understand a concept in mathematics if they can use mathematical reasoning with a variety of representations and connections to explain the concept to someone else or apply the concept to another situation. This is how 'understand' should be interpreted when implementing the CCSSM.

> *One hallmark of mathematical understanding is the ability to justify, in a way appropriate to the student's mathematical maturity, why a particular mathematical statement is true or where a mathematical rule comes from... Mathematical understanding and procedural skill are equally important, and both are assessable using mathematical tasks of sufficient richness (CCSSM p. 4).*

While the *Standards for Mathematical Practice* should be addressed with all of the
Standards for Mathematical Content, the content standards that begin with the verb
"understand" are a natural intersection between the two.

K-12 Coherence and Convergence

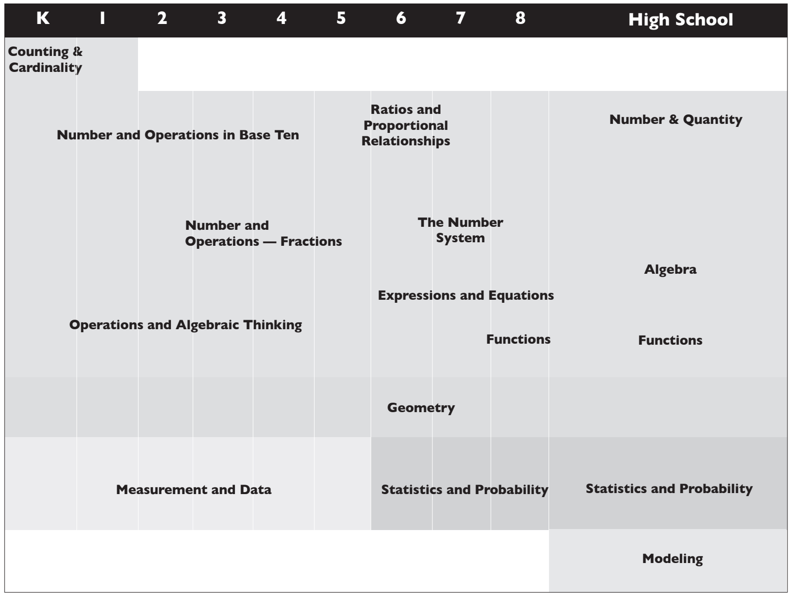
The *Standards for Mathematical Content* are built upon **coherence**, one of the design principles of the CCSSM. The intentional progression and sequencing of topics lays the foundation for the mathematics that is developed from kindergarten through high school. The diagram below depicts how domains at the elementary and middle school levels converge toward algebra at the high school. It is important that educators are knowledgeable about these progressions so that students learn mathematics with understanding and so that new content can build on prior learning



Focus and Organization of the Standards for Mathematical Content

The mathematics content of the CCSSM builds across grades and provides important underpinnings for the mathematics to be learned at subsequent levels. The coherence of the CCSSM lies in those connections, both within and across grade levels and topics. The graphic below illustrates the second design principle of the CCSSM – **focus**.

At the early elementary grades, the focus is largely on the areas of number and operations in base ten and algebraic thinking. This expands to a focus on fractions later in elementary school. The K-5 mathematics content provides the groundwork for the study of ratios, proportional reasoning, the number system, expressions and equations, and functions at the middle school level. By providing a focused mathematics experience in elementary and middle school, a strong foundation is developed for the content to be learned at the high school level.



Mathematical Proficiency

Mathematical proficiency is necessary for every student; therefore, understanding concepts and being fluent with procedural skills are both important. This means that educators must intentionally engage students at all levels so they are readily able to understand important concepts, use skills effectively, and apply mathematics to make sense of their changing world.

Adding it Up (National Research Council, 2001), a major research report that informed the development of the Common Core State Standards for Mathematics, emphasizes the five strands of mathematical proficiency: conceptual understanding, procedural fluency, adaptive reasoning, strategic competence, and productive disposition. These strands are not sequential, but intertwined and form the basis for the *Standards for Mathematical Content* and the *Standards for Mathematical Practice*. Together, these two sets of mathematics standards define what students should understand and be able to do in their study of K-12 mathematics.

Standards for Mathematical Practice	Characteristics of Mathematically Proficient Students*
Make sense of problems and persevere in solving them.	**Mathematically proficient students can:** Explain the meaning of a problem and restate it in their words. Analyze given information to develop possible strategies for solving the problem. Identify and execute appropriate strategies to solve the problem. Evaluate progress toward the solution and make revisions if necessary. Explain the connections among various representations of a problem or concept. Check for accuracy and reasonableness of work, strategy and solution. Understand and connect strategies used by others to solve problems.
Reason abstractly and quantitatively.	**Mathematically proficient students can:** Translate given information to create a mathematical representation for a concept. Manipulate the mathematical representation by showing the process considering the meaning of the quantities involved. Recognize the relationships between numbers/quantities within the process to evaluate a problem. Review the process for reasonableness within the original context.
Construct viable arguments and critique the reasoning of others.	**Mathematically proficient students can:** Use observations and prior knowledge (stated assumptions, definitions, and previous established results) to make conjectures and construct arguments. Compare and contrast logical arguments and identify which one makes the most sense. Justify (orally and in written form) the approach used, including how it fits in the context from which the data arose. Listen, understand, analyze, and respond to the arguments of others. Identify and explain both correct and flawed logic. Recognize and use counterexamples to refine assumptions or definitions and dispute or disprove an argument.

Standards for Mathematical Practice	Characteristics of Mathematically Proficient Students*
Model with mathematics.	**Mathematically proficient students can:** Use a variety of methods to model, represent, and solve real-world problems. Simplify a complicated problem by making assumptions and approximations. Interpret results in the context of the problem and revise the model if necessary. Choose a model that is both appropriate and efficient to arrive at one or more desired solutions.
Use appropriate tools strategically.	**Mathematically proficient students can:** Identify mathematical tools and recognize their strengths and weaknesses. Select and use appropriate tools to best model/solve problems. Use estimation to predict reasonable solutions and/or detect errors. Identify and successfully use external mathematical resources to pose or solve problems. Use a variety of technologies, including digital content, to explore, confirm, and deepen conceptual understanding.
Attend to precision.	**Mathematically proficient students can:** Understand symbols and use them consistently within the context of a problem. Calculate answers efficiently and accurately and label them appropriately. Formulate precise explanations (orally and in written form) using both mathematical representations and words. Communicate using clear mathematical definitions, vocabulary, and symbols.
Look for and make use of structure.	**Mathematically proficient students can:** Look for, identify, and accept patterns or structure within relationships. Use patterns or structure to make sense of mathematics and connect prior knowledge to similar situations and extend to novel situations. Analyze a complex problem by breaking it down into smaller parts. Reflect on the problem as a whole and shift perspective as needed.
Look for and express regularity in repeated reasoning.	**Mathematically proficient students can** Recognize similarities and patterns in repeated trials with a process. Generalize the process to create a shortcut which may lead to developing rules or creating a formula. Evaluate the reasonableness of results throughout the mathematical process while attending to the details.

* *collaborative project with Cedarburg, Franklin, Fox Point-Bayside, Grafton, Greendale, Kettle Moraine, Menomonee Falls, Oconomowoc, Pewaukee, Waukesha, & Whitefish Bay School Districts and CESA 1.*

Design Features of the Common Core State Standards for Mathematics

The design of the CCSSM has several specific features. Additional resources to support the CCSSM are available online at: http://dpi.wi.gov/standards/stds.html

- The *Standards for Mathematical Practice* must be addressed at all levels and intertwined with the *Standards for Mathematical Content*.
- K-8 grade level content standards illustrate a **coherent and rigorous curriculum** to be completed in each of these grades.
- The high school *Standards for Mathematical Content* are not by grade or course, rather they are grouped in **conceptual categories** and can be clustered in multiple ways to design courses and programs of study.
- The CCSSM are designed to provide **focus**, by identifying two to four critical areas at each K-8 grade level. These are found in the short narrative section of grades K-8, immediately before each grade level's content standards. They present the areas that should be the primary focus for instruction in that grade. Critical areas for each of the high school conceptual categories are described in the narratives.
- The CCSSM were designed to provide **coherence**, through connections and progressions both within and across grade levels. The authors of the CCSSM have developed *Progressions* documents that provide in-depth discussion of the domain progressions across grades, highlight connections across domains, elaborate on the learning expectations for students, and provide instructional suggestions.
- The CCSSM were designed to be **rigorous**, which is provided by a focus on College and Career Readiness and by emphasizing the *Standards for Mathematical Practice* across K-12. The high school CCSSM also specify additional mathematics (+ standards) that students pursuing mathematics-intensive STEM careers should accomplish.

How to use Appendix A of the Common Core State Standards for Mathematics

The *CCSSM Standards for Mathematical Content* are organized by grade level in grades K-8. A similar organization was not possible for the high school content standards, since schools and curricula do not all introduce high school content in the same order. The high school content standards are therefore organized by conceptual categories, leaving open the question of how the required content is to be distributed among high school courses. There are two commonly-used approaches: traditional/non-integrated U.S. curriculum in which content is typically divided into courses named Algebra I, Geometry, and Algebra II; and the integrated approach, more commonly used in other countries, in which the strands of mathematics are interwoven in courses which might simply be named Mathematics I, Mathematics II, and Mathematics III. The CCSSM should be fully acquired through either course sequences.

CCSSM Appendix A, *Designing High School Mathematics Courses Based on the Common Core State Standards*, provides four suggested pathways as to how this distribution might be accomplished (http://corestandards.org/assets/CCSSI_Mathematics_Appendix_A.pdf). In considering this appendix, it is important to keep in mind comments from the CCSSM authors:

> *The **pathways and courses are models, not mandates**. They illustrate possible approaches to organizing the content of the CCSS into coherent and rigorous courses that lead to college and career readiness. States and districts are not expected to adopt these courses as is; rather, they are encouraged to use these pathways and courses as a starting point for developing their own (CCSSM, Appendix A, p.2).*

SECTION 3

Common Core Essential Elements for Mathematics

DYNAMIC LEARNING MAPS ESSENTIAL ELEMENTS

FOR

Mathematics

Common Core Essential Elements for Mathematics

Table of Contents

BACKGROUND ON THE DYNAMIC LEARNING MAPS ESSENTIAL ELEMENTS

The Dynamic Learning Maps Essential Elements are specific statements of knowledge and skills linked to the grade-level expectations identified in the Common Core State Standards. The purpose of the Dynamic Learning Maps Essential Elements is to build a bridge from the content in the Common Core State Standards to academic expectations for students with the most significant cognitive disabilities. The initial draft of the Dynamic Learning Maps Essential Elements (then called the Common Core Essential Elements) was released in the spring of 2012.

The initial version of the Dynamic Learning Maps Essential Elements (DLM EEs) was developed by a group of educators and content specialists from the 12 member states of the Dynamic Learning Maps Alternate Assessment Consortium (DLM) in the spring of 2011. Led by Edvantia, Inc., a sub-contractor of DLM, representatives from each state education agency and the educators and content specialists they selected developed the original draft of DLM EEs. Experts in mathematics and English language arts, as well as individuals with expertise in instruction for students with significant cognitive disabilities reviewed the draft documents. Edvantia then compiled the information into the version released in the spring of 2012.

Concurrent with the development of the DLM EEs, the DLM consortium was actively engaged in building learning maps in mathematics and English language arts. The DLM learning maps are highly connected representations of how academic skills are acquired, as reflected in research literature. In the case of the DLM project, the Common Core State Standards helped to specify academic targets, while the surrounding map content clarified how students could reach the specified standard. Learning maps of this size had not been previously developed, and as a result, alignment between the DLM EEs and the learning maps was not possible until the fall of 2012, when an initial draft of the learning maps was available for review.

ALIGNMENT OF THE DLM EES TO THE DLM LEARNING MAPS

Teams of content experts worked together to revise the initial version of the DLM EEs and the learning maps to ensure appropriate alignment of these two critical elements of the project. Alignment involved horizontal alignment of the DLM EEs with the Common Core State Standards and vertical alignment of the DLM EEs with meaningful progressions in the learning maps. The alignment process began when researchers Caroline Mark and Kelli Thomas compared the learning maps with the initial version of the DLM EEs to determine how the map and the DLM EEs should be adjusted to improve their alignment.

The teams of content experts most closely involved with this alignment work included:

Mathematics	**English Language Arts**
Kelli Thomas, Ph.D. (co-lead)	Caroline Mark, Ph.D. (lead)
Angela Broaddus, Ph.D. (co-lead)	Jonathan Schuster, Ph.D.
Perneet Sood	Russell Swinburne Romine, Ph.D.
Kristin Joannou	Suzanne Peterson
Bryan Candea Kromm	

These teams worked in consultation with Sue Bechard, Ph.D. and Karen Erickson, Ph.D., who offered guidance based on their experience in alternate assessments of students with significant cognitive disabilities.

THE ALIGNMENT PROCESS

The process of aligning the learning map and the DLM EEs began by identifying nodes in the maps that represented the essential elements in mathematics and English language arts. This process revealed areas in the maps where additional nodes were needed to account for incremental growth reflected from an essential element in one grade to the next. Also identified were areas in which an essential element was out of place developmentally, according to research, with other essential elements. For example, adjustments were made when an essential element related to a higher-grade map node appeared earlier on the map than an essential element related to a map node from a lower grade (e.g., a fifth-grade skill preceded a third-grade skill). Finally, the alignment process revealed DLM EEs that were actually written as instructional tasks rather than learning outcomes.

This initial review step provided the roadmap for subsequent revision of both the learning maps and the DLM EEs. The next step in the DLM project was to develop the claims document, which served as the basis for the evidence-centered design of the DLM project and helped to further refine both the modeling of academic learning in the maps and the final revisions to the DLM EEs.

CLAIMS AND CONCEPTUAL AREAS

The DLM system uses a variant of evidence-centered design (ECD) as the framework for developing the DLM Alternate Assessment System. While ECD is multifaceted, it starts with a set of claims regarding important knowledge in the domains of interest (mathematics and English language arts), as well as an understanding of how that knowledge is acquired. Two sets of claims have been developed for DLM that identify the major domains of interest within mathematics and English language arts for students with significant cognitive disabilities. These claims are broad statements about expected student learning that serve to focus the scope of the assessment. Because the learning map identifies particular paths to the acquisition of academic skills, the claims also help to organize the structures in the learning map for this population of students. Specifically, conceptual areas within the map further define the knowledge and skills required to meet the broad claims identified by DLM.

The claims are also significant because they provide another means through which to evaluate alignment between the DLM EEs and the learning map nodes, and serve as the foundation for evaluating the validity of inferences made from test scores. DLM EEs related to a particular claim and conceptual area must clearly link to one another, and the learning map must reflect how that knowledge is acquired. Developing the claims and conceptual areas for DLM provided a critical framework for organizing nodes on the learning maps and, accordingly, the DLM EEs that align with each node.

The table below reveals the relationships among the claims, conceptual areas, and DLM EEs in mathematics. The DLM EEs are represented with codes that reflect the domains in mathematics. For example, the first letter or digit represents the grade of record, the next code reflects the domain, followed by the number that aligns with the Common Core State Standard grade level expectation. As such, K.CC.1 is the code for the DLM EE that aligns with kindergarten (K), counting and cardinality (CC), standard 1. Keys to the codes can be found under the table.

Clearly articulated claims and conceptual areas for DLM served as an important evidence-centered framework within which this version of the DLM EEs was developed. With the claims and conceptual areas in place, the relationship between DLM EEs within a claim and conceptual area or across grade levels is easier to track and strengthen. The learning maps, as well as the claims and conceptual areas, had not yet been developed when the original versions of the DLM EEs were created. As such, the relationship of DLM EEs within and across grade levels was more difficult to evaluate at that time.

Table 1. Dynamic Learning Maps Claims and Conceptual Areas for Students with Significant Cognitive Disabilities in Mathematics

Claim 1	**Number Sense: Students demonstrate increasingly complex understanding of number sense.** Conceptual Areas in the Dynamic Learning Map: **MC 1.1 Understand number structures (counting, place value, fraction)** *<u>Essential Elements Included</u>: K.CC.1, 4 ,5; 1.NBT.1a-b; 2.NBT.2a-b,3; 3.NBT.1,2,3; 3.NF.1-3; 4.NF.1-2,3; 5.NF.1,2; 6.RP.1; 7.RP.1-3; 7.NS.2.c-d; 8.NS.2.a* **MC 1.2 Compare, compose, and decompose numbers and sets** *<u>Essential Elements Included:</u> K.CC.6; 1.NBT.2, 3, 4, 6; 2.NBT.1, 4, 5b; 4.NBT.2, 3; 5.NBT.1, 2, 3, 4; 6.NS.1, 5-8; 7.NS.3; 8.NS.2.b; 8.EE.3-4;* **MC 1.3 Calculate accurately and efficiently using simple arithmetic operations** *<u>Essential Elements Included:</u> 2.NBT.5.a, 6-7; 3.OA.4; 4.NBT.4, 5.NBT.5, 6-7; 6.NS.2, 3; 7.NS.1, 2.a, 2.b; 8.NS.1; 8.EE.1; N-CN.2.a, 2.b, 2.c; N-RN.1; S-CP.1-5; S-IC.1-2*

Claim 2	**Geometry: Students demonstrate increasingly complex spatial reasoning and understanding of geometric principles.** Conceptual Areas in the Dynamic Learning Map: **MC 2.1 Understand and use geometric properties of two- and three-dimensional shapes** *Essential Elements Included: K.MD.1-3; K.G.2-3; 1.G.1, 2; 2.G.1; 3.G.1; 4.G.1, 2; 4.MD.5, 6; 5.G.1-4; 5.MD.3; 7.G.1, 2, 3, 5; 8.G.1, 2, 4, 5; G-CO.1, 4-5, 6-8; G-GMD.4; G-MG.1-3* **MC 2.2 Solve problems involving area, perimeter, and volume** *Essential Elements Included: 1.G.3; 3.G.2; 4.G.3; 4.MD.3; 5.MD.4-5; 6.G.1, 2; 7.G.4, 6; 8.G.9; G-GMD.1-3; G-GPE.7*
Claim 3	**Measurement Data and Analysis: Students demonstrate increasingly complex understanding of measurement, data, and analytic procedures.** Conceptual Areas in the Dynamic Learning Map: **MC 3.1 Understand and use measurement principles and units of measure** *Essential Elements Included: 1.MD.1-2, 3.a, 3.b, 3.c, 3.d; 2.MD.1, 3-4, 5, 6, 7, 8; 3.MD.1, 2, 4; 4.MD.1, 2.a, 2.b, 2.c, 2.d; 5.MD.1.a, 1.b, 1.c; N-Q.1-3* **MC 3.2 Represent and interpret data displays** *Essential Elements Included: 1.MD.4; 2.MD.9-10; 3.MD.3; 4.MD.4.a, 4.b; 5.MD.2; 6.SP.1-2, 5; 7.SP.1-2, 3, 5-7; 8.SP.4; S-ID.1-2, 3, 4*
Claim 4	**Algebraic and functional reasoning: Students solve increasingly complex mathematical problems, making productive use of algebra and functions.** Conceptual Areas in the Dynamic Learning Map: **MC 4.1. Use operations and models to solve problems** *Essential Elements Included: K.OA.1, 1.a, 1.b, 2, 5.a, 5.b; 2.OA.3, 4; 3.OA.1-2, 8; 4.OA.1-2, 3, 4; 6.EE.1-2, 3, 5-7; 7.EE.1, 4; 8.EE.7; A-CED.1, 2-4; A-SSE.1, 3* **MC 4.2 Understand patterns and functional thinking** *Essential Elements Included: 3.OA.9; 4.OA.5; 5.OA.3; 7.EE.2; 8.EE.5-6; 8.F.1-3, 4, 5; A-REI.10-12; A-SSE.4; F-BF.1, 2; F-IF.1-3, 4-6; F-LE.1*

A-CED = creating equations; A-SSE = seeing structure in equations BF = building functions; CC = counting & cardinality; EE = expressions & equations; F-BF = basic fractions; F-IF = interpreting functions; G = geometry; G-GMD = geometric measurement & dimension; G—MG = geometry: modeling with geometry; G-GPE = general properties & equations; MD = measurement & data; NBT = numbers & operations in base ten; N-CN = complex number system; NF = numbers & operations - fractions; N-RN = real number system; NS = number systems; N-Q = number & quantity; OA = operations & algebraic thinking; RP = ratios & proportional relationships; S-IC- statistics & probability - making inferences/justifying conclusions; S-ID = statistics & probability - interpreting categorical & quantitative data; SP = statistics & probability

RESULTING CHANGES TO THE DLM ESSENTIAL ELEMENTS

The development of the entire DLM Alternate Assessment System guided a final round of revisions to the DLM EEs, which can be organized into four broad categories: alignment across grade levels, language specificity, common core alignment, and defining learning expectations (rather than instructional tasks). The first type of revision was required to align the DLM EEs across grade levels, both vertically and horizontally. The maps, and the research supporting them, were critical in determining the appropriate progression of skills and understandings from grade to grade. This alignment across grade levels was important within and across standards, strands, and domains. For example, in determining when it was appropriate to introduce concepts in mathematics regarding the relative position of objects, we had to consider the grade level at which prepositions that describe relative position were introduced in English language arts. Examining the research-based skill development outlined in the learning map aided in these kinds of determinations.

The articulation of the claims and conceptual areas reinforced the need for specific language in the DLM EEs to describe learning within an area. Because teams assigned to grade bands developed the first round of DLM EEs, the language choices from one grade to the next were not consistent. Even when closely related skills, concepts, or understandings were targeted, the same terms were not always selected to describe the intended learning outcome. The teams of content experts who worked on this revised version of the DLM EEs were very intentional in selecting a common set of terms to reflect the claims and conceptual areas and applied them consistently across the entire set of DLM EEs.

Another important change in this version of the DLM EEs involved alignment to the Common Core State Standards (CCSS). Given that the DLM EEs are intended to clarify the bridge to the CCSS expectations for students with the most significant cognitive disabilities, it is critical that alignment be as close as possible without compromising learning and development over time. While there was never a one-to-one correspondence between the CCSS and the DLM EEs, the revisions have made the alignment between the two more precise than it was in the first version.

Finally, revisions to the DLM EEs involved shifting the focus of a small number of DLM EEs that were written in the form of instructional tasks rather than learning expectations, and adding “With guidance and support” to the beginning of a few of the DLM EEs in the primary grades in English language arts to reflect the expectations articulated in the CCSS.

Members of the DLM consortium reviewed each of the changes to the original version of the DLM EEs. Four states provided substantive feedback on the revisions, and this document incorporates the changes those teams suggested.

ACCESS TO INSTRUCTION AND ASSESSMENT

The DLM EEs specify learning targets for students with significant cognitive disabilities; however, they do not describe all of the ways that students can engage in instruction or demonstrate understanding through an assessment. Appropriate modes of communication, both for presentation or response, are not stated in the DLM EEs unless a specific mode is an expectation. Where no limitation has been stated, no limitation should be inferred. Students’ opportunities to learn and to demonstrate learning during assessment should be maximized by providing whatever communication, assistive technologies, augmentative and alternative communication (AAC) devices, or other access tools that are necessary and routinely used by the student during instruction.

Students with significant cognitive disabilities include a broad range of students with diverse disabilities and communication needs. For some students with significant cognitive disabilities, a range of assistive technologies is required to access content and demonstrate achievement. For other students, AAC devices or accommodations for hearing and visual impairments will be needed. During instruction, teams should meet individual student needs using whatever technologies and accommodations are required. Examples of some of the ways that students may use technology while learning and demonstrating learning are topics for professional development, and include:

- communication devices that compensate for a student’s physical inability to produce independent speech.
- alternate access devices that compensate for a student’s physical inability to point to responses, turn pages in a book, or use a pencil or keyboard to answer questions or produce writing.

GUIDANCE AND SUPPORT

The authors of the CCSS use the words “prompting and support” at the earliest grade levels to indicate when students are not expected to achieve standards completely independently. Generally, “prompting” refers to “the action of saying something to persuade, encourage, or remind someone to do or say something” (McKean, 2005). However, in special education, prompting is often used to mean a system of structured cues to elicit desired behaviors that otherwise would not occur. In order to clearly communicate that teacher assistance is permitted during instruction of the DLM EEs and is not limited to structured prompting procedures, the decision was made by the stakeholder group to use the more general term *guidance* throughout the DLM EEs.

Guidance and support during instruction should be interpreted as teacher encouragement, general assistance, and informative feedback to support the student in learning. Some examples of the kinds of teacher behaviors that would be considered guidance and support include verbal supports, such as

- getting the student started (e.g., “Tell me what to do first.”),
- providing a hint in the right direction without revealing the answer (e.g., Student wants to write *dog* but is unsure how, so the teacher might say, “See if you can write the first letter in the word, /d/og [phonetically pronounced].”),
- using structured technologies such as task-specific word banks, or
- providing structured cues such as those found in prompting procedures (e.g., least-to-most prompts, simultaneous prompting, and graduated guidance).

Guidance and support as described above applies to instruction and is also linked to demonstrating learning relative to DLM EEs, where guidance and support is specifically called out within the standards.

CONCLUSION

Developing the research-based model of knowledge and skill development represented in the DLM Learning Maps supported the articulation of assessment claims for mathematics and English language arts. This articulation subsequently allowed for a careful revision of the DLM EEs to reflect both horizontal alignment with the CCSS and vertical alignment across the grades, with the goal of moving students toward more sophisticated understandings in both domains. Though the contributions made by Edvantia and our state partners in developing the initial set of DLM EEs were a critical first step, additional revisions to the DLM EEs were required to ensure consistency across all elements of the Dynamic Learning Maps Alternate Assessment System.

DYNAMIC LEARNING MAPS ESSENTIAL ELEMENTS FOR KINDERGARTEN

Kindergarten Mathematics Domain: Counting and Cardinality

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Know number names and the count sequence.	
K.CC.1. Count to 100 by ones and by tens.	**EE.K.CC.1.** Starting with one, count to 10 by ones.
K.CC.2. Count forward beginning from a given number within the known sequence (instead of having to begin at 1).	Not applicable. See **EE.2.NBT.2.b**.
K.CC.3. Write numbers from 0 to 20. Represent a number of objects with a written numeral 0–20 (with 0 representing a count of no objects).	Not applicable. See **EE.2.NBT.3**.
CLUSTER: Count to tell the number of objects.	
K.CC.4. Understand the relationship between numbers and quantities; connect counting to cardinality.	**EE.K.CC.4.** Demonstrate one-to-one correspondence, pairing each object with one and only one number and each number with one and only one object.
K.CC.4.a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object.	
K.CC.4.b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.	
K.CC.4.c. Understand that each successive number name refers to a quantity that is one larger.	
K.CC.5. Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects.	**EE.K.CC.5.** Count out up to three objects from a larger set, pairing each object with one and only one number name to tell how many.
CLUSTER: Compare numbers.	
K.CC.6. Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies.[1]	**EE.K.CC.6.** Identify whether the number of objects in one group is more or less than (when the quantities are clearly different) or equal to the number of objects in another group.
K.CC.7. Compare two numbers between 1 and 10 presented as written numerals.	Not applicable. See **EE.2.NBT.4**.

[1] Include groups with up to ten objects.

Kindergarten Mathematics Domain: Operations and Algebraic Thinking

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.	
K.OA.1. Represent addition and subtraction with objects, fingers, mental images, drawings[2], sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.	**EE.K.OA.1.** Represent addition as "putting together" or subtraction as "taking from" in everyday activities.
K.OA.2. Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.	Not applicable See **EE.2.NBT.6–7**.
K.OA.3. Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).	Not applicable. See **EE.1.NBT.6**.
K.OA.4. For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.	Not applicable. See **EE.1.NBT.2**.
K.OA.5. Fluently add and subtract within 5.	Not applicable. See **EE.3.OA.4**.

[2] Drawings need not show details but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.)

Kindergarten Mathematics Domain: Number and Operations in Base Ten

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Work with numbers 11–19 to gain foundations for place value.	
K.NBT.1. Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (such as 18 = 10 + 8); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.	Not applicable. See **EE.1.NBT.4** and **EE.1.NBT.6**.

Kindergarten Mathematics Domain: Measurement and Data

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Describe and compare measurable attributes.	
K.MD.1. Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object.	**EE.K.MD.1-3.** Classify objects according to attributes (big/small, heavy/light).
K.MD.2. Directly compare two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute, and describe the difference. *For example, directly compare the heights of two children, and describe one child as taller/shorter.*	
CLUSTER: Classify objects and count the number of objects in each category.	
K.MD.3. Classify objects into given categories; count the numbers of objects in each category and sort the categories by count.[3]	**EE.K.MD.1-3.** Classify objects according to attributes (big/small, heavy/light).

[3] Limit category counts to be less than or equal to 10.

Kindergarten Mathematics Domain: Geometry

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).	
K.G.1. Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as *above*, *below*, *beside*, *in front of*, *behind*, and *next to*.	Not applicable. See **EE.1.G.a**.
K.G.2. Correctly name shapes regardless of their orientations or overall size.	**EE.K.G.2–3.** Match shapes of same size and orientation (circle, square, rectangle, triangle).
K.G.3. Identify shapes as two-dimensional (lying in a plane, “flat”) or three-dimensional (“solid”).	
CLUSTER: Analyze, compare, create, and compose shapes.	
K.G.4. Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/“corners”) and other attributes (e.g., having sides of equal length).	Not applicable. See **EE.7.G.1**.
K.G.5. Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes.	Not applicable.
K.G.6. Compose simple shapes to form larger shapes. *For example, “Can you join these two triangles with full sides touching to make a rectangle?”*	Not applicable. See **EE.1.G.3**.

DYNAMIC LEARNING MAPS ESSENTIAL ELEMENTS FOR FIRST GRADE

First Grade Mathematics Domain: Operations and Algebraic Thinking

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Represent and solve problems involving addition and subtraction.	
1.OA.1. Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.	**EE.1.OA.1.a**. Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), or acting out situations.
	EE.1.OA.1.b. Recognize two groups that have the same or equal quantity.
1.OA.2. Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.	**EE.1.OA.2.** Use "putting together" to solve problems with two sets.
CLUSTER: Understand and apply properties of operations and the relationship between addition and subtraction.	
1.OA.3. Apply properties of operations as strategies to add and subtract.[4] *Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known. (Commutative property of addition.) To add 2 + 6 + 4, the second two numbers can be added to make a 10, so 2 + 6 + 4 = 2 + 10 = 12. (Associative property of addition.)*	Not applicable. See **EE.6.EE.3** and **EE.N-CN.2**.
1.OA.4. Understand subtraction as an unknown-addend problem. *For example, subtract 10 – 8 by finding the number that makes 10 when added to 8.*	Not applicable. See **EE.1.NBT.4** and **EE.1.NBT.6**.
CLUSTER: Add and subtract within 20.	
1.OA.5. Relate counting to addition and subtraction (e.g., by counting on 2 to add 2).	**EE.1.OA.5.a.** Use manipulatives or visual representations to indicate the number that results when adding one more.
	EE.1.OA.5.b. Apply knowledge of "one less" to subtract one from a number.
1.OA.6. Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).	Not applicable. See **EE.3.OA.4**.

[4] Students need not use formal terms for these properties.

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Work with addition and subtraction equations.	
1.OA.7. Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. *For example, which of the following equations are true and which are false? 6 = 6, 7 = 8 – 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2.*	Not applicable. See **EE.1.OA.1.b** and **EE.2.NBT.5.a**.
1.OA.8. Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 + ? = 11, 5 = _ – 3, 6 + 6 = _.*	Not applicable. See **EE.3.OA.4**.

First Grade Mathematics Domain: Number and Operations in Base Ten

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Extend the counting sequence.	
1.NBT.1. Count to 120, starting at any number less than 120. In this range, read and write numerals, and represent a number of objects with a written numeral.	**EE.1.NBT.1.a.** Count by ones to 30.
	EE.1.NBT.1.b. Count as many as 10 objects and represent the quantity with the corresponding numeral.
CLUSTER: Understand place value.	
1.NBT.2. Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases:	**EE.1.NBT.2.** Create sets of 10.
1.NBT.2.a. 10 can be thought of as a bundle of ten ones—called a “ten.”	
1.NBT.2.b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.	
1.NBT.2.c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).	
1.NBT.3. Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <.	**EE.1.NBT.3.** Compare two groups of 10 or fewer items when the number of items in each group is similar.
CLUSTER: Use place value understanding and properties of operations to add and subtract.	
1.NBT.4. Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.	**EE.1.NBT.4.** Compose numbers less than or equal to five in more than one way.
1.NBT.5. Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.	Not applicable. See **EE.1.OA.5.a** and **EE.1.OA.5.b**.
1.NBT.6. Subtract multiples of 10 in the range 10–90 from multiples of 10 in the range 10–90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.	**EE.1.NBT.6.** Decompose numbers less than or equal to five in more than one way.

First Grade Mathematics Domain: Measurement and Data

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Measure lengths indirectly and by iterating length units.	
1.MD.1. Order three objects by length; compare the lengths of two objects indirectly by using a third object.	**EE.1.MD.1–2**. Compare lengths to identify which is longer/shorter, taller/shorter.
1.MD.2. Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. *Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.*	
CLUSTER: Tell and write time.	
1.MD.3. Tell and write time in hours and half-hours using analog and digital clocks.	**EE.1.MD.3.a.** Demonstrate an understanding of the terms *tomorrow*, *yesterday*, and *today*.
	EE.1.MD.3.b. Demonstrate an understanding of the terms *morning*, *afternoon*, *day*, and *night*.
	EE.1.MD.3.c. Identify activities that come before, next, and after.
	EE.1.MD.3.d. Demonstrate an understanding that telling time is the same every day.
CLUSTER: Represent and interpret data.	
1.MD.4. Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.	**EE.1.MD.4.** Organize data into categories by sorting.

First Grade Mathematics Domain: Geometry

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Reason with shapes and their attributes.	
1.G.1. Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes.	**EE.1.G.1.** Identify the relative position of objects that are on, off, in, and out.
1.G.2. Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape.[5]	**EE.1.G.2.** Sort shapes of same size and orientation (circle, square, rectangle, triangle).
1.G.3. Partition circles and rectangles into two and four equal shares, describe the shares using the words *halves*, *fourths*, and *quarters*, and use the phrases *half of*, *fourth of*, and *quarter of*. Describe the whole as *two of* or *four of* the shares. Understand for these examples that decomposing into more equal shares creates smaller shares.	**EE.1.G.3.** Put together two pieces to make a shape that relates to the whole (i.e., two semicircles to make a circle, two squares to make a rectangle).

[5] Students do not need to learn formal names such as "right rectangular prism."

DYNAMIC LEARNING MAPS ESSENTIAL ELEMENTS FOR SECOND GRADE

Second Grade Mathematics Domain: Operations and Algebraic Thinking

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Represent and solve problems involving addition and subtraction.	
2.OA.1. Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.	Not applicable. See **EE.3.OA.4**.
CLUSTER: Add and subtract within 20.	
2.OA.2. Fluently add and subtract within 20 using mental strategies.[6] By end of Grade 2, know from memory all sums of two one-digit numbers.	Not applicable. See **EE.2.NBT.6–7** and **EE.3.OA.4**.
CLUSTER: Word with equal groups of objects to gain foundations for multiplication.	
2.OA.3. Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.	**EE.2.OA.3.** Equally distribute even numbers of objects between two groups.
2.OA.4. Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.	**EE.2.OA.4.** Use addition to find the total number of objects arranged within equal groups up to a total of 10.

[6] See standard 1.OA.C.6 for a list of mental strategies.

Second Grade Mathematics: Number and Operations in Base Ten

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Understand place value.	
2.NBT.1. Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases:	**EE.2.NBT.1.** Represent numbers up to 30 with sets of tens and ones using objects in columns or arrays.
2.NBT.1.a. 100 can be thought of as a bundle of ten tens—called a "hundred."	
2.NBT.1.b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).	
2.NBT.2. Count within 1000; skip-count by 5s, 10s, and 100s.	**EE.2.NBT.2.a.** Count from 1 to 30 (count with meaning; cardinality).
	EE.2.NBT.2.b. Name the next number in a sequence between 1 and 10.
2.NBT.3. Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.	**EE.2.NBT.3.** Identify numerals 1 to 30.
2.NBT.4. Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.	**EE.2.NBT.4.** Compare sets of objects and numbers using appropriate vocabulary (more, less, equal).
CLUSTER: Use place value understanding and properties of operations to add and subtract.	
2.NBT.5. Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.	**EE.2.NBT.5.a.** Identify the meaning of the "+" sign (i.e., combine, plus, add), "–" sign (i.e., separate, subtract, take), and the "=" sign (equal).
	EE.2.NBT.5.b. Using concrete examples, compose and decompose numbers up to 10 in more than one way.
2.NBT.6. Add up to four two-digit numbers using strategies based on place value and properties of operations.	**EE.2.NBT.6-7.** Use objects, representations, and numbers (0–20) to add and subtract.
2.NBT.7. Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.	
2.NBT.8. Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900.	Not applicable.

CCSS Grade-Level Standards	DLM Essential Elements
2.NBT.9. Explain why addition and subtraction strategies work, using place value and the properties of operations.[7]	Not applicable.

[7] Explanations may be supported by drawings or objects.

Second Grade Mathematics Domain: Measurement and Data

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Measure and estimate lengths in standard units.	
2.MD.1. Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.	**EE.2.MD.1.** Measure the length of objects using non-standard units.
2.MD.2. Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.	Not applicable.
2.MD.3. Estimate lengths using units of inches, feet, centimeters, and meters.	**EE.2.MD.3–4.** Order by length using non-standard units.
2.MD.4. Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.	
CLUSTER: Relate addition and subtraction to length.	
2.MD.5. Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem.	**EE.2.MD.5.** Increase or decrease length by adding or subtracting unit(s).
2.MD.6. Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number line diagram.	**EE.2.MD.6.** Use a number line to add one more unit of length.
CLUSTER: Work with time and money.	
2.MD.7. Tell and write time from analog and digital clocks to the nearest five minutes, using *a.m.* and *p.m.*	**EE.2.MD.7**. Identify on a digital clock the hour that matches a routine activity.
2.MD.8. Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately. *Example: If you have 2 dimes and 3 pennies, how many cents do you have?*	**EE.2.MD.8.** Recognize that money has value.
CLUSTER: Represent and interpret data.	
2.MD.9. Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units.	**EE.2.MD.9-10.** Create picture graphs from collected measurement data.
2.MD.10. Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems using information presented in a bar graph.	

Second Grade Mathematics Domain: Geometry

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Reason with shapes and their attributes.	
2.G.1. Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces.[8] Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.	**EE.2.G.1.** Identify common two-dimensional shapes: square, circle, triangle, and rectangle.
2.G.2. Partition a rectangle into rows and columns of same-size squares, and count to find the total number of them.	Not applicable.
2.G.3. Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words *halves*, *thirds*, *half of*, *a third of*, etc., and describe the whole as *two halves*, *three thirds*, *four fourths*. Recognize that equal shares of identical wholes need not have the same shape.	Not applicable. See **EE.4.G.3** and **EE.4.NF.1–2**.

[8] Sizes are compared directly or visually, not compared by measuring.

DYNAMIC LEARNING MAPS ESSENTIAL ELEMENTS FOR THIRD GRADE

Third Grade Mathematics Domain: Operations and Algebraic Thinking

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Represent and solve problems involving multiplication and division.	
3.OA.1. Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5 × 7.*	**EE.3.OA.1-2**. Use repeated addition to find the total number of objects and determine the sum.
3.OA.2. Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.*	
3.OA.3. Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.	Not applicable See **EE.3.OA.1** and **EE.5.NBT.5**.
3.OA.4. Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = _ ÷ 3, 6 × 6 = ?*	**EE.3.OA.4.** Solve addition and subtraction problems when result is unknown, limited to operands and results within 20.
CLUSTER: Understand properties of multiplication and the relationship between multiplication and division.	
3.OA.5. Apply properties of operations as strategies to multiply and divide.[9] *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)*	Not applicable. See **EE.N-CN.2**.
3.OA.6. Understand division as an unknown-factor problem. *For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.*	Not applicable. See **EE.5.NBT.6–7**.
CLUSTER: Multiply and divide within 100.	
3.OA.7. Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.	Not applicable. See **EE.7.NS.2.a** and **EE.7.NS.2.b**.
CLUSTER: Solve problems involving the four operations, and identify and explain patterns in arithmetic.	

[9] Students need not use formal terms for these properties.

CCSS Grade-Level Standards	DLM Essential Elements
3.OA.8. Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.[10]	**EE.3.OA.8.** Solve one-step real-world problems using addition or subtraction within 20.
3.OA.9. Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.*	**EE.3.OA.9.** Identify arithmetic patterns.

[10] This standard is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the convention when there are no parentheses to specify a particular order.

Third Grade Mathematics Domain: Number and Operations in Base Ten

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Use place value understanding and properties of operations to perform multi-digit arithmetic.[11]	
3.NBT.1. Use place value understanding to round whole numbers to the nearest 10 or 100.	**EE.3.NBT.1.** Use decade numbers (10, 20, 30) as benchmarks to demonstrate understanding of place value for numbers 0–30.
3.NBT.2. Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.	**EE.3.NBT.2.** Demonstrate understanding of place value to tens.
3.NBT.3. Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations.	**EE.3.NBT.3**. Count by tens using models such as objects, base ten blocks, or money.

[11] A range of algorithms may be used.

Third Grade Mathematics Domain: Number and Operations—Fractions[12]

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Develop understanding of fractions as numbers.	
3.NF.1. Understand a fraction 1/*b* as the quantity formed by 1 part when a whole is partitioned into *b* equal parts; understand a fraction *a/b* as the quantity formed by *a* parts of size 1/*b.*	**EE.3.NF.1–3.** Differentiate a fractional part from a whole.
3.NF.2. Understand a fraction as a number on the number line; represent fractions on a number line diagram.	
3.NF.2.a. Represent a fraction 1/*b* on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size 1/*b* and that the endpoint of the part based at 0 locates the number 1/*b* on the number line.	
3.NF.2.b. Represent a fraction *a*/*b* on a number line diagram by marking off a lengths 1/*b* from 0. Recognize that the resulting interval has size *a*/*b* and that its endpoint locates the number *a*/*b* on the number line.	
3.NF.3. Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.	
3.NF.3.a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.	
3.NF.3.b. Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3. Explain why the fractions are equivalent, e.g., by using a visual fraction model.	
3.NF.3.c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.*	
3.NF.3.d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.	

[12] Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, 8.

Third Grade Mathematics Domain: Measurement and Data

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.	
3.MD.1. Tell and write time to the nearest minute, and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.	**EE.3.MD.1.** Tell time to the hour on a digital clock.
3.MD.2. Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l).[13] Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.[14]	**EE.3.MD.2.** Identify the appropriate measurement tool to solve one-step word problems involving mass and volume.
CLUSTER: Represent and interpret data.	
3.MD.3. Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. *For example, draw a bar graph in which each square in the bar graph might represent 5 pets.*	**EE.3.MD.3.** Use picture or bar graph data to answer questions about data.
3.MD.4. Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters.	**EE.3.MD.4.** Measure length of objects using standard tools, such as rulers, yardsticks, and meter sticks.
CLUSTER: Geometric measurement: understand concepts of area, and relate area to multiplication and to addition.	
3.MD.5. Recognize area as an attribute of plane figures and understand concepts of area measurement.	Not applicable. See **EE.4.MD.2**.
3.MD.5.a. A square with side length of 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area.	
3.MD.5.b. A plane figure, which can be covered without gaps or overlaps by *n* unit squares, is said to have an area of *n* square units.	
3.MD.6. Measure areas by counting unit squares (square cm, square m, square in., square ft, and improvised units).	
3.MD.7. Relate area to the operations of multiplication and addition.	
3.MD.7.a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.	

[13] Excludes compound units such as cm^3 and finding the geometric volume of a container.
[14] Excludes multiplicative comparison problems (problems involving notions of "times as much"; see Glossary, Table 2).

CCSS Grade-Level Standards	DLM Essential Elements
3.MD.7.b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real-world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.	
3.MD.7.c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning.	
3.MD.7.d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real-world problems.	
CLUSTER: Geometric measurement: recognize perimeter as an attribute of plane figures, and distinguish between linear and area measures.	
3.MD.8. Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.	Not applicable. See **EE.7.G.4** and **EE.8.G.9**.

Third Grade Mathematics Domain: Geometry

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Reason with shapes and their attributes.	
3.G.1. Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.	**EE.3.G.1.** Describe attributes of two-dimensional shapes.
3.G.2. Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. *For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape.*	**EE.3.G.2.** Recognize that shapes can be partitioned into equal areas.

DYNAMIC LEARNING MAPS ESSENTIAL ELEMENTS FOR FOURTH GRADE

Fourth Grade Mathematics Domain: Operations and Algebraic Thinking

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Use the four operations with whole numbers to solve problems.	
4.OA.1. Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.	**EE.4.OA.1-2.** Demonstrate the connection between repeated addition and multiplication.
4.OA.2. Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.	
4.OA.3. Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.	**EE.4.OA.3**. Solve one-step real-world problems using addition or subtraction within 100.
CLUSTER: Gain familiarity with factors and multiples.	
4.OA.4. Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.	**EE.4.OA.4.** Show one way to arrive at a product.
CLUSTER: Generate and analyze patterns.	
4.OA.5. Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.*	**EE.4.OA.5.** Use repeating patterns to make predictions.

Fourth Grade Mathematics Domain: Numbers and Operations in Base Ten[15]

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Generalize place value understanding for multi-digit whole numbers.	
4.NBT.1. Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.*	Not applicable. See **EE.5.NBT.1**.
4.NBT.2. Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.	**EE.4.NBT.2**. Compare whole numbers to 10 using symbols (<, >, =).
4.NBT.3. Use place value understanding to round multi-digit whole numbers to any place.	**EE.4.NBT.3.** Round any whole number 0-30 to the nearest ten.
CLUSTER: Use place value understanding and properties of operations to perform multi-digit arithmetic.	
4.NBT.4. Fluently add and subtract multi-digit whole numbers using the standard algorithm.	**EE.4.NBT.4.** Add and subtract two-digit whole numbers.
4.NBT.5. Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	Not applicable. See **EE.4.OA.1**.
4.NBT.6. Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	Not applicable.

[15] Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.

Fourth Grade Mathematics Domain: Number and Operations—Fractions[16]

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Extend understanding of fraction equivalence and ordering.	
4.NF.1. Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.	**EE.4.NF.1–2.** Identify models of one half (1/2) and one fourth (1/4).
4.NF.2. Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.	
CLUSTER: Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.	
4.NF.3. Understand a fraction a/b with $a > 1$ as a sum of fractions $1/b$.	**EE.4.NF.3.** Differentiate between whole and half.
4.NF.3.a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.	
4.NF.3.b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples: 3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.*	
4.NF.3.c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.	
4.NF.3.d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.	
4.NF.4. Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.	Not applicable. See **EE.4.OA.1–2** and **EE.5.NBT.5**.
4.NF.4.a. Understand a fraction a/b as a multiple of $1/b$. *For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).*	

[16] Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

CCSS Grade-Level Standards	DLM Essential Elements
4.NF.4.b. Understand a multiple of *a/b* as a multiple of 1/*b*, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)*	
4.NF.4.c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*	
CLUSTER: Understand decimal notation for fractions, and compare decimal fractions.	
4.NF.5. Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.[17] *For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.*	
4.NF.6. Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.*	Not applicable. See **EE.7.NS.2.c-d**.
4.NF.7. Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.	

[17] Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade.

Fourth Grade Mathematics Domain: Measurement and Data

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.	
4.MD.1. Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. *For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...*	**EE.4.MD.1.** Identify the smaller measurement unit that comprises a larger unit within a measurement system (inches/foot, centimeter/meter, minutes/hour).
4.MD.2. Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.	**EE.4.MD.2.a.** Tell time using a digital clock. Tell time to the nearest hour using an analog clock.
	EE.4.MD.2.b. Measure mass or volume using standard tools.
	EE.4.MD.2.c. Use standard measurement to compare lengths of objects.
	EE.4.MD.2.d. Identify coins (penny, nickel, dime, quarter) and their values.
4.MD.3. Apply the area and perimeter formulas for rectangles in real-world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length by viewing the area formula as a multiplication equation with an unknown factor.*	**EE.4.MD.3.** Determine the area of a square or rectangle by counting units of measure (unit squares).
CLUSTER: Represent and interpret data.	
4.MD.4. Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. *For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.*	**EE.4.MD.4.a.** Represent data on a picture or bar graph given a model and a graph to complete.
	EE.4.MD.4.b. Interpret data from a picture or bar graph.
CLUSTER: Geometric measurement: understand concepts of angle and measure angles.	
4.MD.5. Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:	**EE.4.MD.5.** Recognize angles in geometric shapes.
4.MD.5.a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles.	
4.MD.5.b. An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees.	
4.MD.6. Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.	**EE.4.MD.6.** Identify angles as larger and smaller.

CCSS Grade-Level Standards	DLM Essential Elements
4.MD.7. Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real-world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.	Not applicable. See **EE.4.G.2.a**.

Fourth Grade Mathematics Domain: Geometry

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Draw and identify lines and angles, and classify shapes by properties of their lines and angles.	
4.G.1. Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.	**EE.4.G.1.** Recognize parallel lines and intersecting lines.
4.G.2. Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.	**EE.4.G.2.** Describe the defining attributes of two-dimensional shapes.
4.G.3. Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures, and draw lines of symmetry.	**EE.4.G.3.** Recognize that lines of symmetry partition shapes into equal areas.

DYNAMIC LEARNING MAPS ESSENTIAL ELEMENTS FOR FIFTH GRADE

Fifth Grade Mathematics Domain: Operations and Algebraic Thinking

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Write and interpret numerical expressions.	
5.OA.1. Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.	Not applicable.
5.OA.2. Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. *For example, express the calculation "add 8 and 7, then multiply by 2" as 2 × (8 + 7). Recognize that 3 × (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product.*	Not applicable.
CLUSTER: Analyze patterns and relationships.	
5.OA.3. Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. *For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.*	**EE.5.OA.3.** Identify and extend numerical patterns.

Fifth Grade Mathematics Domain: Number and Operations in Base Ten

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Understand the place value system.	
5.NBT.1. Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.	**EE.5.NBT.1.** Compare numbers up to 99 using base ten models.
5.NBT.2. Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.	**EE.5.NBT.2.** Use the number of zeros in numbers that are powers of 10 to determine which values are equal, greater than, or less than.
5.NBT.3. Read, write, and compare decimals to thousandths.	**EE.5.NBT.3.** Compare whole numbers up to 100 using symbols (<, >, =).
5.NBT.3.a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., 347.392 = 3 × 100 + 4 × 10 + 7 × 1 + 3 × (1/10) + 9 × (1/100) + 2 × (1/1000).	
5.NBT.3.b. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.	
5.NBT.4. Use place value understanding to round decimals to any place.	**EE.5.NBT.4.** Round two-digit whole numbers to the nearest 10 from 0—90.
CLUSTER: Perform operations with multi-digit whole numbers and with decimals to hundredths.	
5.NBT.5. Fluently multiply multi-digit whole numbers using the standard algorithm.	**EE.5.NBT.5.** Multiply whole numbers up to 5 × 5.
5.NBT.6. Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	**EE.5.NBT.6–7.** Illustrate the concept of division using fair and equal shares.
5.NBT.7. Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.	

Fifth Grade Mathematics Domain: Number and Operations—Fractions

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Use equivalent fractions as a strategy to add and subtract fractions.	
5.NF.1. Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)*	**EE.5.NF.1.** Identify models of halves (1/2, 2/2) and fourths (1/4, 2/4, 3/4, 4/4).
5.NF.2. Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2.*	**EE.5.NF.2.** Identify models of thirds (1/3. 2/3, 3/3) and tenths (1/10, 2/10, 3/10, 4/10, 5/10, 6/10, 7/10, 8/10, 9/10, 10/10).
CLUSTER: Apply and extend previous understandings of multiplication and division to multiply and divide fractions.	
5.NF.3. Interpret a fraction as division of the numerator by the denominator (*a*/*b* = *a* ÷ *b*). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?*	Not applicable. See **EE.6.RP.1**.
5.NF.4. Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.	Not applicable.
5.NF.4.a. Interpret the product (*a*/*b*) × *q* as *a* parts of a partition of *q* into *b* equal parts; equivalently, as the result of a sequence of operations *a* × *q* ÷ *b*. *For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = ac/bd.)*	
5.NF.4.b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.	
5.NF.5. Interpret multiplication as scaling (resizing), by:	Not applicable.
5.NF.5.a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.	

CCSS Grade-Level Standards	DLM Essential Elements
5.NF.5.b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1.	
5.NF.6. Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.	Not applicable. See **EE.10.N-CN.2.b**.
5.NF.7. Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.[18]	Not applicable. See **EE.7.NS.2.b**.
5.NF.7.a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. *For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3.*	
5.NF.7.b. Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4.*	
5.NF.7.c. Solve real-world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?*	

[18] Students able to multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between multiplication and division. But division of a fraction by a fraction is not a requirement at this grade.

Fifth Grade Mathematics Domain: Measurement and Data

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Convert like measurement units within a given measurement system.	
5.MD.1. Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real-world problems.	**EE.5.MD.1.a.** Tell time using an analog or digital clock to the half or quarter hour.
	EE.5.MD.1.b. Use standard units to measure weight and length of objects.
	EE.5.MD.1.c. Indicate relative value of collections of coins.
CLUSTER: Represent and interpret data.	
5.MD.2. Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.*	**EE.5.MD.2.** Represent and interpret data on a picture, line plot, or bar graph.
CLUSTER: Geometric measurement: understand concepts of volume, and relate volume to multiplication and to addition.	
5.MD.3. Recognize volume as an attribute of solid figures and understand concepts of volume measurement.	**EE.5.MD.3.** Identify common three-dimensional shapes.
5.MD.3.a. A cube with side length 1 unit, called a “unit cube,” is said to have “one cubic unit” of volume, and can be used to measure volume.	
5.MD.3.b. A solid figure, which can be packed without gaps or overlaps using *n* unit cubes, is said to have a volume of *n* cubic units.	
5.MD.4. Measure volumes by counting unit cubes, using cubic cm, cubic in., cubic ft, and improvised units.	**EE.5.MD.4–5.** Determine the volume of a rectangular prism by counting units of measure (unit cubes).
5.MD.5. Relate volume to the operations of multiplication and addition, and solve real-world and mathematical problems involving volume.	
5.MD.5.a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.	
5.MD.5.b. Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real-world and mathematical problems.	
5.MD.5.c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real-world problems.	

Fifth Grade Mathematics Domain: Geometry

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Graph points on the coordinate plane to solve real-world and mathematical problems.	
5.G.1. Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., *x*-axis and *x*-coordinate, *y*-axis and *y*-coordinate).	**EE.5.G.1-4.** Sort two-dimensional figures and identify the attributes (angles, number of sides, corners, color) they have in common.
5.G.2. Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.	
CLUSTER: Classify two-dimensional figures into categories based on their properties.	
5.G.3. Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. *For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.*	**EE.5.G.1-4.** Sort two-dimensional figures and identify the attributes (angles, number of sides, corners, color) they have in common.
5.G.4. Classify two-dimensional figures in a hierarchy based on properties.	

DYNAMIC LEARNING MAPS ESSENTIAL ELEMENTS FOR SIXTH GRADE

Sixth Grade Mathematics Domain: Ratios and Proportional Relationships

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Understand ratio concepts, and use ratio reasoning to solve problems.	
6.RP.1. Understand the concept of a ratio, and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."*	**EE.6.RP.1.** Demonstrate a simple ratio relationship.
6.RP.2. Understand the concept of a unit rate *a/b* associated with a ratio *a:b* with $b \neq 0$, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."*[19]	Not applicable. See **EE.7.RP.1–3**.
6.RP.3. Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.	Not applicable. See **EE.8.F.1–3**.
6.RP.3.a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.	
6.RP.3.b. Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?*	
6.RP.3.c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.	
6.RP.3.d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.	

[19] Expectations for unit rates in this grade are limited to non-complex fractions.

Sixth Grade Mathematics Domain: The Number System

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Apply and extend previous understandings of multiplication and division to divide fractions by fractions.	
6.NS.1. Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a story context for (2/3) ÷ (3/4), and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share 1/2 lb. of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?*	**EE.6.NS.1.** Compare the relationships between two unit fractions.
CLUSTER: Compute fluently with multi-digit numbers, and find common factors and multiples.	
6.NS.2. Fluently divide multi-digit numbers using the standard algorithm.	**EE.6.NS.2.** Apply the concept of fair share and equal shares to divide.
6.NS.3. Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.	**EE.6.NS.3.** Solve two-factor multiplication problems with products up to 50 using concrete objects and/or a calculator.
6.NS.4. Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express 36 + 8 as 4(9 + 2).*	Not applicable.
CLUSTER: Apply and extend previous understandings of numbers to the system of rational numbers.	
6.NS.5. Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.	**EE.6.NS.5–8.** Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero).
6.NS.6. Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.	
6.NS.6.a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., –(–3) = 3, and that 0 is its own opposite.	
6.NS.6.b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.	
6.NS.6.c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.	

CCSS Grade-Level Standards	DLM Essential Elements
6.NS.7. Understand ordering and absolute value of rational numbers.	
6.NS.7.a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. *For example, interpret –3 > –7 as a statement that –3 is located to the right of –7 on a number line oriented from left to right.*	
6.NS.7.b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write -3^{o} C > -7^{o} C to express the fact that -3^{o}C is warmer than -7^{o}C.*	
6.NS.7.c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of –30 dollars, write |–30| = 30 to describe the size of the debt in dollars.*	
6.NS.7.d. Distinguish comparisons of absolute value from statements about order. *For example, recognize that an account balance less than –30 dollars represents a debt greater than 30 dollars.*	
6.NS.8. Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.	**EE.6.NS.5–8.** Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero).

Sixth Grade Mathematics Domain: Expressions and Equations

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Apply and extend previous understandings of arithmetic to algebraic expressions.	
6.EE.1. Write and evaluate numerical expressions involving whole-number exponents.	**EE.6.EE.1–2.** Identify equivalent number sentences.
6.EE.2. Write, read, and evaluate expressions in which letters stand for numbers.	
6.EE.2.a. Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract* y *from 5" as 5 –* y.	
6.EE.2.b. Identify parts of an expression using mathematical terms (*sum, term, product, factor, quotient, coefficient*); view one or more parts of an expression as a single entity. *For example, describe the expression 2(8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.*	
6.EE.2.c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas* $V = s^3$ *and* $A = 6s^2$ *to find the volume and surface area of a cube with sides of length* $s = 1/2$.	
6.EE.3. Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3(2 +* x*) to produce the equivalent expression 6 + 3*x*; apply the distributive property to the expression 24*x *+ 18*y *to produce the equivalent expression 6(4*x *+ 3*y*); apply properties of operations to* y + y + y *to produce the equivalent expression 3*y.	**EE.6.EE.3.** Apply the properties of addition to identify equivalent numerical expressions.
6.EE.4. Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions* y + y + y *and 3*y *are equivalent because they name the same number regardless of which number* y *stands for.*	Not applicable.
CLUSTER: Reason about and solve one-variable equations and inequalities.	
6.EE.5. Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.	**EE.6.EE.5–7.** Match an equation to a real-world problem in which variables are used to represent numbers.
6.EE.6. Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.	
6.EE.7. Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.	
6.EE.8. Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.	Not applicable.

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Represent and analyze quantitative relationships between dependent and independent variables.	
6.EE.9. Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.*	Not applicable.

Sixth Grade Mathematics Domain: Geometry

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Solve real-world and mathematical problems involving area, surface area, and volume.	
6.G.1. Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.	**EE.6.G.1.** Solve real-world and mathematical problems about area using unit squares.
6.G.2. Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = lwh$ and $V = bh$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.	**EE.6.G.2.** Solve real-world and mathematical problems about volume using unit cubes.
6.G.3. Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.	Not applicable.
6.G.4. Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.	Not applicable.

Sixth Grade Mathematics Domain: Statistics and Probability

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Develop understanding of statistical variability.	
6.SP.1. Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. *For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.*	**EE.6.SP.1–2.** Display data on a graph or table that shows variability in the data.
6.SP.2. Understand that a set of data collected to answer a statistical question has a distribution, which can be described by its center, spread, and overall shape.	
6.SP.3. Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.	Not applicable. See **EE.S-ID.4**.
CLUSTER: Summarize and describe distributions.	
6.SP.4. Display numerical data in plots on a number line, including dot plots, histograms, and box plots.	Not applicable. See **EE.6.SP.1–2**.
6.SP.5. Summarize numerical data sets in relation to their context, such as by:	**EE.6.SP.5.** Summarize data distributions shown in graphs or tables.
6.SP.5.a. Reporting the number of observations.	
6.SP.5.b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.	
6.SP.5.c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.	
6.SP.5.d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.	

DYNAMIC LEARNING MAPS ESSENTIAL ELEMENTS FOR SEVENTH GRADE

Seventh Grade Mathematics Domain: Ratios and Proportional Relationships

<table>
<tr><th>CCSS Grade-Level Standards</th><th>DLM Essential Elements</th></tr>
<tr><td colspan="2">CLUSTER: Analyze proportional relationships and use them to solve real-world and mathematical problems.</td></tr>
<tr><td>7.RP.1. Compute unit rates associated with ratios of fractions, including ratios of lengths, areas, and other quantities measured in like or different units. For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction $^{1/2}/_{1/4}$ miles per hour, equivalently 2 miles per hour.</td><td rowspan="8">EE.7.RP.1–3. Use a ratio to model or describe a relationship.</td></tr>
<tr><td>7.RP.2. Recognize and represent proportional relationships between quantities.</td></tr>
<tr><td>7.RP.2.a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.</td></tr>
<tr><td>7.RP.2.b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.</td></tr>
<tr><td>7.RP.2.c. Represent proportional relationships by equations. For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn.</td></tr>
<tr><td>7.RP.2.d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate.</td></tr>
<tr><td>7.RP.3. Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.</td></tr>
</table>

Seventh Grade Mathematics Domain: The Number System

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.	
7.NS.1. Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.	**EE.7.NS.1.** Add fractions with like denominators (halves, thirds, fourths, and tenths) with sums less than or equal to one.
7.NS.1.a. Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.*	
7.NS.1.b. Understand $p + q$ as the number located a distance $\lvert q \rvert$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.	
7.NS.1.c. Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.	
7.NS.1.d. Apply properties of operations as strategies to add and subtract rational numbers.	
7.NS.2. Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.	See below.
7.NS.2.a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.	**EE.7.NS.2.a.** Solve multiplication problems with products to 100.
7.NS.2.b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts.	**EE.7.NS.2.b.** Solve division problems with divisors up to five and also with a divisor of 10 without remainders.
7.NS.2.c. Apply properties of operations as strategies to multiply and divide rational numbers	**EE.7.NS.2.c–d**. Express a fraction with a denominator of 10 as a decimal.
7.NS.2.d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.	
7.NS.3. Solve real-world and mathematical problems involving the four operations with rational numbers.[20]	**EE.7.NS.3.** Compare quantities represented as decimals in real-world examples to tenths.

[20] Computations with rational numbers extend the rules for manipulating fractions to complex fractions.

Seventh Grade Mathematics Domain: Expressions and Equations

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Use properties of operations to generate equivalent expressions.	
7.EE.1. Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.	**EE.7.EE.1.** Use the properties of operations as strategies to demonstrate that expressions are equivalent.
7.EE.2. Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example,* a + 0.05a = 1.05a *means that "increase by 5%" is the same as "multiply by 1.05."*	**EE.7.EE.2.** Identify an arithmetic sequence of whole numbers with a whole number common difference.
CLUSTER: Solve real-life and mathematical problems using numerical and algebraic expressions and equations.	
7.EE.3. Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form, convert between forms as appropriate, and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.*	Not applicable.
7.EE.4. Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.	**EE.7.EE.4.** Use the concept of equality with models to solve one-step addition and subtraction equations.
7.EE.4.a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*	
7.EE.4.b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.*	

Seventh Grade Mathematics Domain: Geometry

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Draw, construct, and describe geometrical figures and describe the relationships between them	
7.G.1. Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.	**EE.7.G.1.** Match two similar geometric shapes that are proportional in size and in the same orientation.
7.G.2. Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.	**EE.7.G.2.** Recognize geometric shapes with given conditions.
7.G.3. Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.	**EE.7.G.3.** Match a two-dimensional shape with a three-dimensional shape that shares an attribute.
CLUSTER: Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.	
7.G.4. Know the formulas for the area and circumference of a circle, and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.	**EE.7.G4.** Determine the perimeter of a rectangle by adding the measures of the sides.
7.G.5. Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.	**EE.7.G.5.** Recognize angles that are acute, obtuse, and right.
7.G.6. Solve real-world and mathematical problems involving area, volume, and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.	**EE.7.G.6.** Determine the area of a rectangle using the formula for length × width, and confirm the result using tiling or partitioning into unit squares.

Seventh Grade Mathematics Domain: Statistics and Probability

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Use random sampling to draw inferences about a population.	
7.SP.1. Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.	**EE.7.SP.1–2.** Answer a question related to the collected data from an experiment, given a model of data, or from data collected by the student.
7.SP.2. Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. *For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.*	
CLUSTER: Draw informal comparative inferences about two populations.	
7.SP.3. Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. *For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.*	**EE.7.SP.3.** Compare two sets of data within a single data display such as a picture graph, line plot, or bar graph.
7.SP.4. Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.*	Not applicable. See **EE.S-ID.4**.
CLUSTER: Investigate chance processes, and develop, use, and evaluate probability models.	
7.SP.5. Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.	**EE.7.SP.5–7.** Describe the probability of events occurring as possible or impossible.
7.SP.6. Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.*	
7.SP.7. Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.	

CCSS Grade-Level Standards	DLM Essential Elements
7.SP.7.a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. *For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.*	
7.SP.7.b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. *For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?*	
7.SP.8. Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.	Not applicable.
7.SP.8.a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.	
7.SP.8.b. Represent sample spaces for compound events using methods such as organized lists, tables, and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event.	
7.SP.8.c. Design and use a simulation to generate frequencies for compound events. *For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?*	

DYNAMIC LEARNING MAPS ESSENTIAL ELEMENTS FOR EIGHTH GRADE

Eighth Grade Mathematics Domain: The Number System

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Know that there are numbers that are not rational, and approximate them by rational numbers.	
8.NS.1. Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.	**EE.8.NS.1.** Subtract fractions with like denominators (halves, thirds, fourths, and tenths) with minuends less than or equal to one.
8.NS.2. Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2). *For example, by truncating the decimal expansion of √2, show that √2 is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.*	**EE.8.NS.2.a.** Express a fraction with a denominator of 100 as a decimal.
	EE.8.NS.2.b. Compare quantities represented as decimals in real-world examples to hundredths.

Eighth Grade Mathematics Domain: Expressions and Equations

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Work with radicals and integer exponents.	
8.EE.1. Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example,* $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.	**EE.8.EE.1.** Identify the meaning of an exponent (limited to exponents of 2 and 3).
8.EE.2. Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and x^3 = p, where *p* is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.	**EE.8.EE.2.** Identify a geometric sequence of whole numbers with a whole number common ratio.
8.EE.3. Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as* 3×10^8 *and the population of the world as* 7×10^9*, and determine that the world population is more than 20 times larger.*	**EE.8.EE.3–4.** Compose and decompose whole numbers up to 999.
8.EE.4. Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation, and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.	
CLUSTER: Understand the connections between proportional relationships, lines, and linear equations.	
8.EE.5. Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*	**EE.8.EE.5–6.** Graph a simple ratio by connecting the origin to a point representing the ratio in the form of *y/x*. *For example, when given a ratio in standard form (2:1), convert to 2/1, and plot the point (1,2).*
8.EE.6. Use similar triangles to explain why the slope *m* is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at *b*.	
CLUSTER: Analyze and solve linear equations and pairs of simultaneous linear equations.	
8.EE.7. Solve linear equations in one variable.	**EE.8.EE.7.** Solve simple algebraic equations with one variable using addition and subtraction.
8.EE.7.a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where *a* and *b* are different numbers).	
8.EE.7.b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.	
8.EE.8. Analyze and solve pairs of simultaneous linear equations.	Not applicable. See **EE.8.EE.5–6**.
8.EE.8.a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.	

CCSS Grade-Level Standards	DLM Essential Elements
8.EE.8.b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example, 3*x *+ 2*y *= 5 and 3*x *+ 2*y *= 6 have no solution because 3*x *+ 2*y *cannot simultaneously be 5 and 6.*	
8.EE.8.c. Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*	

Eighth Grade Mathematics Domain: Functions

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Define, evaluate, and compare functions.	
8.F.1. Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.[21]	**EE.8.F.1–3.** Given a function table containing at least 2 complete ordered pairs, identify a missing number that completes another ordered pair (limited to linear functions).
8.F.2. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*	
8.F.3. Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function* $A = s^2$ *giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.*	
CLUSTER: Use functions to model relationships between quantities.	
8.F.4. Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (*x*, *y*) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.	**EE.8.F.4.** Determine the values or rule of a function using a graph or a table.
8.F.5. Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.	**EE.8.F.5.** Describe how a graph represents a relationship between two quantities.

[21] Function notation is not required in Grade 8.

Eighth Grade Mathematics Domain: Geometry

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Understand congruence and similarity using physical models, transparencies, or geometry software.	
8.G.1. Verify experimentally the properties of rotations, reflections, and translations:	**EE.8.G.1.** Recognize translations, rotations, and reflections of shapes.
8.G.1.a. Lines are taken to lines, and line segments to line segments of the same length.	
8.G.1.b. Angles are taken to angles of the same measure.	
8.G.1.c. Parallel lines are taken to parallel lines.	
8.G.2. Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.	**EE.8.G.2.** Identify shapes that are congruent.
8.G.3. Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.	Not applicable.
8.G.4. Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.	**EE.8.G.4.** Identify similar shapes with and without rotation.
8.G.5. Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*	**EE.8.G.5.** Compare any angle to a right angle, and describe the angle as greater than, less than, or congruent to a right angle.
CLUSTER: Understand and apply the Pythagorean Theorem.	
8.G.6. Explain a proof of the Pythagorean Theorem and its converse.	Not applicable.
8.G.7. Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.	Not applicable.
8.G.8. Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.	Not applicable.
CLUSTER: Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.	
8.G.9. Know the formulas for the volumes of cones, cylinders, and spheres, and use them to solve real-world and mathematical problems.	**EE.8.G.9.** Use the formulas for perimeter, area, and volume to solve real-world and mathematical problems (limited to perimeter and area of rectangles and volume of rectangular prisms).

Eighth Grade Mathematics Domain: Statistics and Probability

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Investigate patterns of association in bivariate data.	
8.SP.1. Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.	Not applicable.
8.SP.2. Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.	Not applicable. See **EE.10.S-ID.1–2** and **EE.10.S-ID.3**.
8.SP.3. Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.*	Not applicable.
8.SP.4. Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*	**EE.8.SP.4.** Construct a graph or table from given categorical data, and compare data categorized in the graph or table.

DYNAMIC LEARNING MAPS ESSENTIAL ELEMENTS FOR HIGH SCHOOL

High School Mathematics Domain: Number and Quantity—The Real Number System

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Extend the properties of exponents to rational exponents.	
N-RN.1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.*	**EE.N-RN.1.** Determine the value of a quantity that is squared or cubed.
N-RN.2. Rewrite expressions involving radicals and rational exponents using the properties of exponents.	Not applicable.
CLUSTER: Use properties of rational and irrational numbers.	
N-RN.3. Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.	Not applicable.

High School Mathematics Domain: Number and Quantity—Quantities★

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Reason quantitatively, and use units to solve problems.	
N-Q.1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.	**EE.N-Q.1–3.** Express quantities to the appropriate precision of measurement.
N-Q.2. Define appropriate quantities for the purpose of descriptive modeling.	
N-Q.3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.	

High School Mathematics Domain: Number and Quantity—The Complex Number System

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Perform arithmetic operations with complex numbers.	
N-CN.1. Know there is a complex number *i* such that $i^2 = -1$, and every complex number has the form $a + bi$ with *a* and *b* real.	Not applicable.
N-CN.2. Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.	**EE.N-CN.2.a.** Use the commutative, associative, and distributive properties to add, subtract, and multiply whole numbers.
	EE.N-CN.2.b. Solve real-world problems involving addition and subtraction of decimals, using models when needed.
	EE.N-CN.2.c. Solve real-world problems involving multiplication of decimals and whole numbers, using models when needed.
N-CN.3. (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.	Not applicable.
CLUSTER: Represent complex numbers and their operations on the complex plane.	
N-CN.4. (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.	Not applicable.
N-CN.5. (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example,* $(-1 + \sqrt{3}i)^3 = 8$ *because* $(-1 + \sqrt{3}i)$ *has modulus 2 and argument 120°.*	Not applicable.
N-CN.6. (+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.	Not applicable.
CLUSTER: Use complex numbers in polynomial identities and equations.	
N-CN.7. Solve quadratic equations with real coefficients that have complex solutions.	Not applicable.
N-CN.8. (+) Extend polynomial identities to the complex numbers. *For example, rewrite* $x^2 + 4$ *as* $(x + 2i)(x - 2i)$.	Not applicable.
N-CN.9. (+) Know the Fundamental Theorum of Algebra; show that it is true for quadratic polynomials	Not applicable.

High School Mathematics Domain: Number and Quantity – Vector and Matrix Quantities

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Represent and model with vector quantities.	
N-VM.1. (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., ***v***, |***v***|, ||***v***||, *v*).	Not applicable.
N-VM.2. (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.	Not applicable.
N-VM.3. (+) Solve problems involving velocity and other quantities that can be represented by vectors.	Not applicable.
CLUSTER: Perform operations on vectors.	
N-VM.4. (+) Add and subtract vectors.	Not applicable.
N-VM.4.a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.	
N-VM.4.b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.	
N-VM.4.c. Understand vector subtraction ***v*** – ***w*** as ***v*** + (–***w***), where –***w*** is the additive inverse of ***w***, with the same magnitude as ***w*** and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.	
N-VM.5. (+) Multiply a vector by a scalar.	Not applicable.
N-VM.5.a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$.	
N-VM.5.b. Compute the magnitude of a scalar multiple *c**v*** using ||*c**v***|| = |*c*|*v*. Compute the direction of *c**v*** knowing that when |*c*|*v* ≠ 0, the direction of *c**v*** is either along ***v*** (for *c* > 0) or against ***v*** (for *c* < 0).	
CLUSTER: Perform operations on matrices, and use matrices in applications.	
N-VM.6. (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.	Not applicable.
N-VM.7. (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.	Not applicable.
N-VM.8. (+) Add, subtract, and multiply matrices of appropriate dimensions.	Not applicable.
N-VM.9. (+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.	Not applicable.

CCSS Grade-Level Standards	DLM Essential Elements
N-VM.10. (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.	Not applicable.
N-VM.11. (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.	Not applicable.
N-VM.12. (+) Work with 2 × 2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.	Not applicable.

High School Mathematics Domain: Algebra—Seeing Structure in Expressions

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Interpret the structure of expressions.	
A-SSE.1. Interpret expressions that represent a quantity in terms of its context.★	**EE.A-SSE.1.** Identify an algebraic expression involving one arithmetic operation to represent a real-world problem.
A-SSE.1.a. Interpret parts of an expression, such as terms, factors, and coefficients.	
A-SSE.1.b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret* $P(1+r)^n$ *as the product of* P *and a factor not depending on* P.	
A-SSE.2. Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$*, thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$.	Not applicable.
CLUSTER: Write expressions in equivalent forms to solve problems.	
A-SSE.3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.★	**EE.A-SSE.3.** Solve simple algebraic equations with one variable using multiplication and division.
A-SSE.3.a. Factor a quadratic expression to reveal the zeros of the function it defines.	
A-SSE.3.b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.	
A-SSE.3.c. Use the properties of exponents to transform expressions for exponential functions. *For example the expression* 1.15^t *can be rewritten as* $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ *to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*	
A-SSE.4. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.*★	**EE.A-SSE.4.** Determine the successive term in a geometric sequence given the common ratio.

High School Mathematics Domain: Algebra—Arithmetic with Polynomials and Rational Expressions

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Perform arithmetic operations on polynomials.	
A-APR.1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.	Not applicable.
CLUSTER: Understand the relationship between zeros and factors of polynomials.	
A-APR.2. Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.	Not applicable.
A-APR.3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.	Not applicable.
CLUSTER: Use polynomial identities to solve problems.	
A-APR.4. Prove polynomial identities, and use them to describe numerical relationships. *For example, the polynomial identity* $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ *can be used to generate Pythagorean triples.*	Not applicable.
A-APR.5. (+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.[22]	Not applicable.
CLUSTER: Rewrite rational expressions.	
A-APR.6. Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.	Not applicable.
A.APR.7. (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.	Not applicable.

[22] The Binomial Theorem can be proved by mathematical induction or by a combinatorial argument.

High School Mathematics Domain: Algebra—Creating Equations★

<table>
<tr><th>CCSS Grade-Level Standards</th><th>DLM Essential Elements</th></tr>
<tr><td colspan="2">CLUSTER: Create equations that describe numbers or relationships.</td></tr>
<tr><td>A-CED.1. Create equations and inequalities in one variable, and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.</td><td>EE.A-CED.1. Create an equation involving one operation with one variable, and use it to solve a real-world problem.</td></tr>
<tr><td>A-CED.2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.</td><td rowspan="3">EE.A-CED.2–4. Solve one-step inequalities.</td></tr>
<tr><td>A-CED.3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.</td></tr>
<tr><td>A-CED.4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law $V = IR$ to highlight resistance R.</td></tr>
</table>

High School Mathematics Domain: Algebra—Reasoning with Equations and Inequalities

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Understand solving equations as a process of reasoning, and explain the reasoning.	
A-REI.1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	Not applicable.
A-REI.2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.	Not applicable. See **EE.A-CED.1**.
CLUSTER: Solve equations and inequalities in one variable.	
A-REI.3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.	Not applicable. See **EE.A-CED.1**.
A-REI.4. Solve quadratic equations in one variable.	Not applicable.
A-REI.4.a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x – p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.	
A-REI.4.b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions, and write them as $a \pm bi$ for real numbers a and b.	
CLUSTER: Solve systems of equations.	
A-REI.5. Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.	Not applicable.
A-REI.6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.	Not applicable. See **EE.A-REI.10–12**.
A-REI.7. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line* $y = –3x$ *and the circle* $x^2 + y^2 = 3$.	Not applicable. See **EE.A-REI.10–12**.
A-REI.8. (+) Represent a system of linear equations as a single matrix equation in a vector variable.	Not applicable.
A-REI.9. (+) Find the inverse of a matrix if it exists, and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater).	Not applicable.
CLUSTER: Represent and solve equations and inequalities graphically.	
A-REI.10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).	**EE.A-REI.10–12.** Interpret the meaning of a point on the graph of a line. *For example, on a graph of pizza*

CCSS Grade-Level Standards	DLM Essential Elements
A-REI.11. Explain why the *x*-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★	*purchases, trace the graph to a point and tell the number of pizzas purchased and the total cost of the pizzas.*
A-REI.12. Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.	

High School Mathematics Domain: Functions—Interpreting Functions

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Understand the concept of a function, and use function notation.	
F-IF.1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.	**EE.F-IF.1–3.** Use the concept of function to solve problems.
F-IF.2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.	
F-IF.3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by* $f(0) = f(1) = 1$, $f(n + 1) = f(n) + f(n - 1)$ *for* $n \geq 1$.	
CLUSTER: Interpret functions that arise in applications in terms of the context.	
F-IF.4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★	**EE.F-IF.4–6.** Construct graphs that represent linear functions with different rates of change and interpret which is faster/slower, higher/lower, etc.
F-IF.5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function* h(n) *gives the number of person-hours it takes to assemble* n *engines in a factory, then the positive integers would be an appropriate domain for the function.*★	
F-IF.6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★	
CLUSTER: Analyze functions using different representations.	
F-IF.7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★	Not applicable. See **EE.F-IF.1–3**.
F-IF.7.a. Graph linear and quadratic functions, and show intercepts, maxima, and minima.	
F-IF.7.b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.	
F-IF.7.c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.	
F-IF.7.d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.	
F-IF.7.e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.	

CCSS Grade-Level Standards	DLM Essential Elements
F-IF.8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.	Not applicable.
F-IF.8.a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.	
F-IF.8.b. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as* $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, *and classify them as representing exponential growth or decay.*	
F-IF.9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*	Not applicable.

High School Mathematics Domain: Functions—Building Functions

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Build a function that models a relationship between two quantities.	
F-BF.1. Write a function that describes a relationship between two quantities.*	**EE.F-BF.1.** Select the appropriate graphical representation (first quadrant) given a situation involving constant rate of change.
F-BF.1.a. Determine an explicit expression, a recursive process, or steps for calculation from a context.	
F-BF.1.b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*	
F-BF.1.c. (+) Compose functions. *For example, if* T(y) *is the temperature in the atmosphere as a function of height, and* h(t) *is the height of a weather balloon as a function of time, then* T(h(t)) *is the temperature at the location of the weather balloon as a function of time.*	Not applicable.
F-BF.2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.*	**EE.F-BF.2.** Determine an arithmetic sequence with whole numbers when provided a recursive rule.
CLUSTER: Build new functions from existing functions.	
F-BF.3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases, and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*	Not applicable.
F-BF.4. Find inverse functions.	Not applicable.
F-BF.4.a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. *For example,* $f(x) = 2x^3$ *or* $f(x) = (x+1)/(x–1)$ *for* $x \neq 1$.	
F-BF.4.b. (+) Verify by composition that one function is the inverse of another.	
F-BF.4.c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.	
F-BF.4.d. (+) Produce an invertible function from a non-invertible function by restricting the domain.	
F-BF.5. (+) Understand the inverse relationship between exponents and logarithms, and use this relationship to solve problems involving logarithms and exponents.	Not applicable.

High School Mathematics Domain: Functions—Linear, Quadratic, and Exponential Models★

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Construct and compare linear, quadratic, and exponential models, and solve problems.	
F-LE.1. Distinguish between situations that can be modeled with linear functions and with exponential functions.	**EE.F-LE.1–3.** Model a simple linear function such as $y = mx$ to show that these functions increase by equal amounts over equal intervals.
F-LE.1.a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.	
F-LE.1.b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.	
F-LE.1.c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.	
F-LE.2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).	
F-LE.3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.	
F-LE.4. For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.	Not applicable.
CLUSTER: Interpret expressions for functions in terms of the situation they model.	
F-LE.5. Interpret the parameters in a linear or exponential function in terms of a context.	Not applicable. See **EE.F-IF.1–3**.

High School Mathematics Domain: Functions—Trigonometric Functions

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Extend the domain of trigonometric functions using the unit circle.	
F-TF.1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.	Not applicable.
F-TF.2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.	Not applicable.
F-TF.3. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π/4, and π/6, and use the unit circle to express the values of since, cosine, and tangent for π – *x*, π + *x*, and 2π – *x* in terms of their values for *x*, where *x* is any real number.	Not applicable.
F-TF.4. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.	Not applicable.
CLUSTER: Model periodic phenomena with trigonometric functions.	
F-TF.5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.★	Not applicable.
F-TF.6. (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.	Not applicable.
F-TF.7. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology; and interpret them in terms of the context.★	Not applicable.
CLUSTER: Prove and apply trigonometric identities	
F-TF.8. Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$, and use it to find sin(θ), cos(θ), or tan(θ) given sin(θ), cos(θ), or tan(θ) and the quadrant of the angle.	Not applicable.
F-TF.9. (+) Prove the addition and subtraction formulas for sine, cosine, and tangent, and use them to solve problems.	Not applicable.

High School Mathematics Domain: Geometry—Congruence

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Experiment with transformations in the plane.	
G.CO.1. Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.	**EE.G-CO.1.** Know the attributes of perpendicular lines, parallel lines, and line segments; angles; and circles.
G-CO.2. Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).	Not applicable.
G-CO.3. Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.	Not applicable.
G-CO.4. Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.	**EE.G-CO.4–5.** Given a geometric figure and a rotation, reflection, or translation of that figure, identify the components of the two figures that are congruent.
G-CO.5. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.	
CLUSTER: Understand congruence in terms of rigid motions.	
G-CO.6. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.	**EE.G-CO.6–8.** Identify corresponding congruent and similar parts of shapes.
G-CO.7. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.	
G-CO.8. Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.	
CLUSTER: Prove geometric theorems.	
G-CO.9. Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*	Not applicable.
G-CO.10. Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*	Not applicable.

CCSS Grade-Level Standards	DLM Essential Elements
G-CO.11. Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.*	Not applicable.
CLUSTER: Make geometric constructions.	
G-CO.12. Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.*	Not applicable.
G-CO.13. Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.	Not applicable.

High School Mathematics Domain: Geometry—Similarity, Right Triangles, and Trigonometry

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Understand similarity in terms of similarity transformations.	
G-SRT.1. Verify experimentally the properties of dilations given by a center and a scale factor:	Not applicable. See **EE.G-CO.6–8**.
G-SRT.1.a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.	
G-SRT.1.b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.	
G-SRT.2. Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.	Not applicable. See **EE.G-CO.6–8**.
G-SRT.3. Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.	Not applicable. See **EE.G-CO.6–8**.
CLUSTER: Prove theorems involving similarity.	
G-SRT.4. Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.*	Not applicable.
G-SRT.5. Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.	Not applicable. See **EE.G-CO.6–8**.
CLUSTER: Define trigonometric ratios, and solve problems involving right triangles.	
G-SRT.6. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.	Not applicable.
G-SRT.7. Explain and use the relationship between the sine and cosine of complementary angles.	Not applicable.
G-SRT.8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.★	Not applicable.
CLUSTER: Apply trigonometry to general triangles.	
G-SRT.9. (+) Derive the formula $A = ½ ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.	Not applicable.
G-SRT.10. (+) Prove the Laws of Sines and Cosines, and use them to solve problems.	Not applicable.
G-SRT.11. (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).	Not applicable.

High School Mathematics Domain: Geometry—Circles

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Understand and apply theorems about circles.	
G-C.1. Prove that all circles are similar.	Not applicable.
G-C.2. Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*	Not applicable.
G-C.3. Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.	Not applicable.
G-C.4. (+) Construct a tangent line from a point outside a give circle to the circle.	Not applicable.
CLUSTER: Find arc lengths and areas of sectors of circles.	
G-C.5. Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.	Not applicable.

High School Mathematics Domain: Geometry––—Expressing Geometric Properties with Equations

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Translate between the geometric description and the equation for a conic section.	
G-GPE.1. Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.	Not applicable.
G-GPE.2. Derive the equation of a parabola given a focus and directrix.	Not applicable.
G-GPE.3. (+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.	Not applicable.
CLUSTER: Use coordinates to prove simple geometric theorems algebraically.	
G-GPE.4. Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).*	Not applicable.
G-GPE.5. Prove the slope criteria for parallel and perpendicular lines, and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).	Not applicable. See **EE.G.CO.1**.
G-GPE.6. Find the point on a directed line segment between two given points that partitions the segment in a given ratio.	Not applicable. See **EE.G.CO.1**.
G-GPE.7. Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.*	**EE.G-GPE.7.** Find perimeters and areas of squares and rectangles to solve real-world problems.

High School Mathematics Domain: Geometry—Geometric Measurement and Dimension

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Explain volume formulas, and use them to solve problems.	
G-GMD.1. Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.*	**EE.G-GMD.1–3.** Make a prediction about the volume of a container, the area of a figure, and the perimeter of a figure, and then test the prediction using formulas or models.
G-GMD.2. (+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.	Not applicable.
G-GMD.3. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.*	Not applicable. See **EE.8.G.9** and **EE.G-GPE.7**.
CLUSTER: Visualize relationships between two-dimensional and three-dimensional objects.	
G-GMD.4. Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.	**EE.G-GMD.4.** Identify the shapes of two-dimensional cross-sections of three-dimensional objects.

High School Mathematics Domain: Geometry—Modeling with Geometry

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Apply geometric concepts in modeling situations.	
G-MG.1. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).*	**EE.G-MG.1–3.** Use properties of geometric shapes to describe real-life objects.
G-MG.2. Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).*	
G-MG.3. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).*	

High School Mathematics Domain: Statistics and Probability★ —Interpreting Categorical and Quantitative Data

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Summarize, represent, and interpret data on a single count or measurement variable.	
S-ID.1. Represent data with plots on the real number line (dot plots, histograms, and box plots).	**EE.S-ID.1–2.** Given data, construct a simple graph (line, pie, bar, or picture) or table, and interpret the data.
S-ID.2. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.	
S-ID.3. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).	**EE.S-ID.3.** Interpret general trends on a graph or chart.
S-ID.4. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.	**EE.S-ID.4.** Calculate the mean of a given data set (limit the number of data points to fewer than five).
CLUSTER: Summarize, represent, and interpret data on two categorical and quantitative variables.	
S-ID.5. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.	Not applicable. See **EE.F-IF.1** and **EE.A-REI.6–7**.
S-ID.6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.	Not applicable.
S-ID.6.a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions, or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.*	
S-ID.6.b. Informally assess the fit of a function by plotting and analyzing residuals.	
S-ID.6.c. Fit a linear function for a scatter plot that suggests a linear association.	
CLUSTER: Interpret linear models.	
S-ID.7. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.	Not applicable. See **EE.F-IF.4–6**.
S-ID.8. Compute (using technology), and interpret the correlation coefficient of a linear fit.	Not applicable.
S-ID.9. Distinguish between correlation and causation.	Not applicable.

High School Mathematics Domain: Statistics and Probability—Making Inferences and Justifying Conclusions

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Understand and evaluate random processes underlying statistical experiments.	
S-IC.1. Understand statistics as a process for making inferences about population parameters based on a random sample from that population.	**EE.S-IC.1–2.** Determine the likelihood of an event occurring when the outcomes are equally likely to occur.
S-IC.2. Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?*	
CLUSTER: Make inferences and justify conclusions from sample surveys, experiments, and observational studies.	
S-IC.3. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.	Not applicable. See **EE.S-ID.1–2**.
S-IC.4. Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.	Not applicable. See **EE.S-ID.1–2**.
S-IC.5. Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.	Not applicable. See **EE.S-ID.1–2**.
S-IC.6. Evaluate reports based on data.	Not applicable. See **EE.S-ID.1–2**.

High School Mathematics Domain: Statistics and Probability—Conditional Probability and the Rules of Probability

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Understand independence and conditional probability, and use them to interpret data.	
S-CP.1. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").	**EE.S-CP.1–5.** Identify when events are independent or dependent.
S-CP.2. Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.	
S-CP.3. Understand the conditional probability of *A* given *B* as $P(A \text{ and } B)/P(B)$, and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*.	
S-CP.4. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.*	
S-CP.5. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.*	
CLUSTER: Use the rules of probability to compute probabilities of compound events in a uniform probability model.	
S-CP.6. Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A*, and interpret the answer in terms of the model.	Not applicable. See **EE.S-IC.1–2**.
S-CP.7. Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms of the model.	Not applicable. See **EE.S-IC.1–2**.
S-CP.8. (+) Apply the general Multiplication Rule in a uniform probability model, $P(A \text{ and } B) = P(A)P(B\|A) = P(B)P(A\|B)$, and interpret the answer in terms of the model.	Not applicable.
S-CP.9. (+) Use permutations and combinations to compute probabilities of compound events and solve problems.	Not applicable.

High School Mathematics Domain: Statistics and Probability—Using Probability to Make Decisions

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Calculate expected values, and use them to solve problems.	
S-MD.1. (+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.	Not applicable.
S-MD.2. (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.	Not applicable.
S-MD.3. (+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. *For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.*	Not applicable.
S-MD.4. (+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. *For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?*	Not applicable.
CLUSTER: Use probability to evaluate outcomes of decisions.	
S-MD.5. (+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.	Not applicable.
S-MD.5.a. Find the expected payoff for a game of chance. *For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant.*	
S-MD.5.b. Evaluate and compare strategies on the basis of expected values. *For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.*	
S-MD.6. (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).	Not applicable.
S-MD.7. (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).	Not applicable.

GLOSSARY AND EXAMPLES OF MATHEMATICS TERMS

Acute triangle. A triangle with all acute angles (acute means measuring less than 90°). See Hhttp://www.mathsisfun.com/definitions/acute-triangle.html

Angles. A shape formed by two lines or rays that diverge from a common point or vertex.

Area. The size of a region enclosed by the figure. Area is measured in square units (e.g., the area of this rectangle is six square units).

Associative property for addition. The sum of three or more numbers which are always the same when added together, no matter what order they are in. This is illustrated by $a + (b + c) = (a + b) + c$; $2 + (3 + 4) = (2 + 3) + 4$.

Associative property for multiplication. The product of three or more numbers which are always the same when multiplied together, regardless of their grouping. This is illustrated by $a(bc) = (ab)c$; $2(3×4) = (2×3)4$.

Attributes. For math purposes, "attributes" refer to characteristics of an object or geometric shape. These include qualities of shape, color, size, side, length, etc.

Base ten blocks. Blocks used to learn place value, addition, subtraction, multiplication, and division. Base ten blocks consist of cubes (ones place), rods (tens place), flats (hundreds place), and blocks (thousands place).

Categorical data. Types of data, which may be divided into groups such as race, sex, age group, and educational level when categorized into a small number of groups.

Commutative property of addition. The sum of numbers are always the same when added together, no matter if the order of the addends are changed. This is illustrated by $a + b = b + a$ ($2 + 1 = 1 + 2$).

Commutative property of multiplication. The product of numbers are always the same when multiplied together, even if the order of factors are changed (i.e., if a and b are two real numbers, then $a × b = b × a$.)

Compose numbers. To combine parts/components to form a number (adding parts to obtain a number).

Congruent figures. Figures that have the same size and shape.

Congruent/congruence. The same.

Decompose numbers. The process of separating numbers into their components (to divide a number into smaller parts). *Example:* 456 can be decomposed as 456 = 400 + 50 + 6.

Denominator. The "bottom" number of a fraction; the number that represents the total number of parts into which one whole is divided (e.g., in 3/4, the 4 is the denominator and indicates that one whole is divided into 4 parts).

Dividend. The number that is being divided (e.g., In the problem, there are 550 pencils; each pack has 10 pencils; how many packs are there? 550 ÷ 10 = 55, *550* is the dividend because it tells how many pencils there are in all to be divided.).

Divisor. A number by which another number is divided (e.g., In the problem, there are 550 pencils; each pack has 10 pencils; how many packs are there? 550 ÷ 10 = 55, *10* is the divisor because it tells how many times 550 is to be divided.

Edge. The line segment where two faces of a solid figure meet (i.e., a cube has 12 edges).

ELA. English Language Arts

Equation. A mathematical sentence of equality between two expressions; equations have an equal sign (e.g., n + 50 = 75 or 75 = n + 50 means that n + 50 must have the same value as 75).

Equilateral triangle. A triangle with all three sides of equal length, corresponding to what could also be known as a "regular" triangle – an equilateral triangle is therefore a special case of an isosceles triangle having not just two but all three sides equal. An equilateral triangle also has three equal angles. See Hhttp://www.mathsisfun.com/definitions/equilateral-triangle.html

Expression. An operation between numbers that represents a single numeric quantity; expressions do not have an equal sign (e.g., *4r, x+2, y-1*).

Face. A plane surface of a three-dimensional figure.

Fact families. Sets of related math facts. For example:

Addition fact family: 3 + 5 = 8; 8 - 3 = 5; 5 + 3 = 8; and 8 - 5 = 3
Multiplication fact family: 5 x 4 = 20; 20 ÷ 5 = 4; 4 x 5=20; and 20 ÷ 4 = 5

Fair share. In division meaning splitting into equal parts or groups with nothing left over.

Frequency table. A table that lists items and uses tally marks to record and show the number of times they occur.

Functions. A special kind of relation where each x-value has one and only one y-value.

Function table. A table that lists pairs of numbers that show a function.

Inequality. A mathematical sentence in which the value of the expressions on either side of the relationship symbol are unequal; relation symbols used in inequalities include > (greater than) and < (less than) symbols (e.g., $7 > 3$, $x < y$).

Input/output table. A table that lists pairs of numbers that show a function.

Integers. Positive and negative whole numbers.

Interlocking cubes. Manipulatives that help students learn number and math concepts - cubes represent "units" and link in one direction. Interlocking cubes are used for patterning, grouping, sorting, counting, numbers, addition, subtraction, multiplication, division, and measurement.

Intersecting lines. Lines that cross.

Inverse operations. Opposite/reverse operations (e.g., subtraction is the inverse operation of addition, which is why 4 + 5 = 9 and 9 – 5 = 4; division is the inverse operation of multiplication, which is why 4 x 5 = 20 and 20 ÷ 5 = 4).

Linear equation. An equation that is made up of two expressions set equal to each other (e.g., $y = 2x + 5$) - A linear equation has only one or two variables and graph as a straight line. See Hhttp://www.eduplace.com/math/mathsteps/7/d/index.html

Line graph. A graphical representation using points connected by line segments to show how something changes over time.

Lines of symmetry. Any imaginary line along which a figure could be folded so that both halves match exactly.

Manipulatives. Objects that are used to explore mathematical ideas and solve mathematical problems (e.g., tools, models, blocks, tiles cubes, geoboards, colored rods, M&M's).

Mathematical structures.

Addition – compare-total unknown
Ex. If Anita has 10 sheets of paper and you have 10 more sheets than Anita. How many sheets do you have?

Addition – start unknown
Ex. Sam gave away 10 apples and has five apples left. How many apples did he start have before he gave 10 apples?

Addition join-part/part – whole
Ex. Jessie had 20 cakes and bought five more. How many does he have now?

Subtraction – classic take away
Ex. If Judy had $50 and spent $10, how much does she have left?

Subtraction – difference unknown
Ex. Sandi has 10 cats and 20 dogs. Which does she have more of, cats or dogs? How many more?

Subtraction – deficit missing amount
Ex. Sandy wants to collect 35 cards and she already has 15. How many more cards does she need?

Multiplication – repeated addition
Ex. James got paid $5 each day for five days. How much money did he have at the end of the five days?

Multiplication – array
Ex. Carlos wanted to cover his rectangular paper with one-inch tiles. If his paper is five inches long and four inches wide, how many tiles will it take to cover the paper?

Multiplication – fundamental counting principle
Ex. Julie packed four shirts and four jeans for her trip. How many outfits can she make?

Division – repeated subtraction
Ex. James pays $5 each day to ride the bus. How many days can he ride for $20?

Division – factor/area – side length
Ex. Tim wants to know the width of a rectangular surface covered in 20 one-inch tiles. He knows the length is five inches, but what is the width?

Division – partitive/fair share
Ex. Julie has 20 different outfits. She has five shirts – how many pair of jeans does she have to make 20 different outfits?

Mean. The "average" – To find the mean, add up all the numbers and then divide by the number of numbers.

Median. The "middle" value in the list of numbers - To find the median, your numbers have to be listed in numerical order, so you may have to rewrite your list.

Minuend. The number one is subtracting from (e.g., 9 in 9 – 2 = __).

Mode. The value that occurs most often - If no number is repeated, then there is no mode for the list. See Hhttp://www.purplemath.com/modules/meanmode.htm

Models. Pictorial or tactile aids used explore mathematical ideas and solve mathematical problems – Manipulatives can be used to model situations.

Non-numeric patterns. Using symbols, shapes, designs, and pictures to make patterns (e.g., □□ΔΔ◊◊□□ΔΔ◊◊).

Non-standard units of measure. Measurements that are neither metric nor English (e.g., number of footsteps used to measure distance or using a piece of yarn used to measure length).

Number line. A diagram that represents numbers as points on a line; a number line must have the arrows at the end.

Number sentence. An equation or inequality using numbers and symbols that is written horizontally (e.g., $5 < 7$ or 5 +7+12).

Numerals. 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9.

Numeric patterns. A pattern that uses skip counting, often starting with the number 1 or 2 – Counting by tens and twos may also be presented to students beginning with different numbers such as 7 or 23; this is more difficult for students but indicates a deeper understanding of skip counting (e.g., 7, 17, 27, 37, 47, . . . or 7, 9, 11, 13, 15, 17).

Numerical expression. A mathematical phrase that involves only numbers and one or more operational symbols.

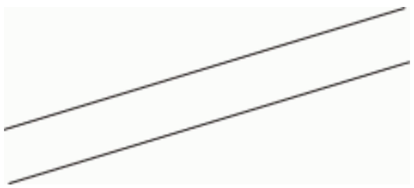
Obtuse triangle. A triangle that has one obtuse angle (obtuse means measuring more than 90°). See Hhttp://www.mathsisfun.com/definitions/obtuse-triangle.html

Operations. Addition, subtraction, multiplication, and division.

Ordered pair. In the ordered pair (1, 3), the first number is called the x-coordinate; the second number is called the y-coordinate; this ordered pair represents the coordinates of point A.

- The x-coordinate tells the distance right (positive) or left (negative).
- The y-coordinate tells the distance up (positive) or down (negative).

Parallel Lines. Lines that are the same distance apart and that never intersect – Lines that have the same slope are parallel.



Pattern. Patterns with a minimum of three terms

- using numbers by repeatedly adding or subtracting (i.e., 2, 4, 6, 8, 10, 12; 0, 3, 6, 9, 12, 15; or 50, 45, 40, 35, 30, 25).
- using objects, figures, colors, sound, etc. - a repeated pattern needs to be at least six terms.

  Extend a pattern - When a student is asked to continue a pattern, the pattern is presented, and the student is asked, “What comes next?” before a student can extend or describe a pattern, the given pattern must be comprised of a minimum of three terms so that the student can see the regularities of the situation and extend or describe the pattern based on those regularities.

Percent. A way of expressing a fraction as “out of 100” (e.g., 50% means 50 out of 100 or 50/100).

Perpendicular lines. Lines that intersect, forming right angles.

Polygon. A closed plane figure made by line segments.

Prediction. A guess based on available information.

Quadrilateral. A four-sided polygon.

Rational numbers. Any number that can be expressed as a/b (b≠0) where a and b are integers; also, in decimal form, any terminating or ultimately repeating decimal.

Ratios. A comparison between two things. For instance, someone can look at a group of people and refer to the "ratio of boys to girls" in the class. Suppose there are 35 students, 15 of whom are boys; the ratio of boys to girls is 15 to 20. See Hhttp://www.purplemath.com/modules/ratio.htm

Real-life situations. Ways in which mathematical concepts are used in real life.

Real numbers. All numbers on a number line, including negative and positive integers, fractions, and irrational numbers.

Real-world applications. Ways in which mathematical concepts are used in real-life situations.

Rectangle. A four-sided polygon (a flat shape with straight sides) where every angle is a right angle (90°); opposite sides are parallel and of equal length.

Right triangle. A triangle that has one right angle (a right angle measures exactly 90°) – Only a single angle in a triangle can be a right angle or it would not be a triangle. A small square is used to mark which angle in the figure is the right angle.

Sets. A group or collection of things that go together (e.g., a group of four stars).

Side. In most general terms, a line segment that is part of the figure - it is connected at either end to another line segment, which, in turn, may or may not be connected to still other line segments.

Similar figures. Figures that have the same shape but different sizes.

Similar shapes. Objects of the same shape but different sizes in which the corresponding angles are the same.

Slope. The steepness/incline/grade of a line.

> ***Positive slope*** – the condition in which a line inclines from left to right.
> ***Negative slope*** – the condition in which a line declines from left to right.

Square. A four-sided polygon (a flat shape with straight sides) where all sides have equal length and every angle is a right angle (90°).

Square root. A value that can be multiplied by itself to give the original number (e.g., the square root of 25 is 5 because 5 x 5 = 25).

Square root notation. Numbers written using a radical √.

Subitize. To judge the number of objects in a group accurately without counting.

Three-dimensional geometric figures. The study of solid figures in three-dimensional space: cube, rectangular prism, sphere, cone, cylinder, and pyramid.

Two-dimensional figures. The study of two-dimensional figures in a plane; drawings of square, rectangle, circle, triangle, pentagon, hexagon, and octagon.

Unknown fixed quantities. A constant that is a quantity; a value that does not change.

Variable. A symbol for an unknown number to be solved; it is usually a letter like x or y (e.g., in x + 3 = 7, x is the variable).

Venn diagram. Made up of two or more overlapping circles. It is often used in mathematics to show relationships between sets. A Venn diagram enables students to organize similarities and differences visually.

Vertex (vertices, pl.). The point(s) where two or more edges meet (corners).

Volume. The amount of three-dimensional space an object occupies; capacity.

GLOSSARY OF SPECIAL EDUCATION TERMS

Accommodations. Changes in the administration of an assessment, such as setting, scheduling, timing, presentation format, response mode, or others, including any combination of these that does not change the construct intended to be measured by the assessment or the meaning of the resulting scores. Accommodations are used for equity, not advantage, and serve to level the playing field. To be appropriate, assessment accommodations must be identified in the student's Individualized Education Plan (IEP) or Section 504 plan and used regularly during instruction and classroom assessment.

Achievement descriptors. Narrative descriptions of performance levels that convey student performance at each achievement level and further defines content standards by connecting them to information that describes how well students are doing in learning the knowledge and skills contained in the content standards. (See also "performance descriptors.")

Achievement levels. A measurement that distinguishes an adequate performance from a Level I or expert performance. Achievement levels provide a determination of the extent to which a student has met the content standards. (See also Performance levels.)

Achievement standard .A system that includes performance levels (e.g., unsatisfactory, Level III, advanced), descriptions of student performance for each level, examples of student work representing the entire range of performance for each level, and cut scores. A system of performance standards operationalizes and further defines content standards by connecting them to information that describes how well students are doing in learning the knowledge and skills contained in the content standards. (See also "performance standards.")

Achievement test. An instrument designed to efficiently measure the amount of academic knowledge and/or skill a student has acquired from instruction. Such tests provide information that can be compared to either a norm group or a measure of performance, such as a standard.

Age appropriate. The characteristics of the skills taught, the activities and materials selected, and the language level employed that reflect the chronological age of the student.

Alignment. The similarity or match between or among content standards, achievement (performance) standards, curriculum, instruction, and assessments in terms of equal breadth, depth, and complexity of knowledge and skill expectations.

Alternate assessment. An instrument used in gathering information on the standards-based performance and progress of students whose disabilities preclude their valid and reliable participation in general assessments. Alternate assessments measure the performance of a relatively small population of students who are unable to participate in the general assessment system, even with accommodations, as determined by the IEP team.

Assessment. The process of collecting information about individuals, groups, or systems that relies upon a number of instruments, one of which may be a test. Therefore, assessment is a more comprehensive term than *test*.

Assessment literacy. The knowledge of the basic principles of sound assessment practice including terminology, development, administration, analysis, and standards of quality.

Assistance (vs. support). The degree to which the teacher provides aid to the student's performance that provides direct assistance in the content or skill being demonstrated by the student. That is, the assistance involves the teacher performing the cognitive work required. Assistance results in an invalidation of the item or score. (See also "support.")

Assistive technology. A device, piece of equipment, product system, or service that is used to increase, maintain, or improve the functional capabilities of a student with a disability. (See 34 CFR §300.5 and 300.6.)

Cues. Assistance, words, or actions provided to a student to increase the likelihood that the student will give the desired response.

Curriculum. A document that describes what teachers do in order to convey grade-level knowledge and skills to a student.

Depth. The level of cognitive processing (e.g., recognition, recall, problem solving, analysis, synthesis, and evaluation) required for success relative to the performance standards.

Disaggregation. The collection and reporting of student achievement results by particular subgroups (e.g., students with disabilities, limited English Level III students) to ascertain the subgroup's academic progress. Disaggregation makes it possible to compare subgroups or cohorts.

Essence of the standard. That which conveys the same ideas, skills, and content of the standard, expressed in simpler terms.

Essential Elements (EEs or CCEEs). The Common Core Essential Elements are specific statements of the content and skills that are linked to the Common Core State Standards (CCSS) grade level specific expectations for students with significant cognitive disabilities.

Grade Band Essential Element. A statement of essential precursor content and skills linked to the Common Core State Standards (CCSS) grade level clusters and indicators that maintain the essence of that standard, thereby identifying the grade-level expectations for students with significant cognitive disabilities to access and make progress in the general curriculum.

Grade level. The grade in which a student is enrolled.

Instructional Achievement Level Descriptors (IALDs). Describes student achievement and illustrates student performance. IALDs operationalize and further define Essential Elements by connecting them to information that describes how well students are doing in learning the knowledge and skills contained in the Essential Elements.

Individualized Education Program (IEP). An IEP is a written plan, developed by a team of regular and special educators, parents, related service personnel, and the student, as appropriate, describing the specially designed instruction needed for an eligible exceptional student to progress in the content standards and objectives and to meet other educational needs.

Linked. A relationship between a grade level indicator for Common Core State Standards (CCSS) and Common Core Essential Elements (EEs or CCEEs) that reflects similar content and skills but does not match the breadth, depth, and complexity of the standards.

Multiple measures. Measurement of student or school performance through more than one form or test.

- For students, these might include teacher observations, performance assessments or portfolios.
- For schools, these might include dropout rates, absenteeism, college attendance or documented behavior problems

Natural cue. Assistance given to a student that provides a flow among the expectations presented by the educator, opportunities to learn, and the desired outcome exhibited by the student.

Opportunity to learn. The provision of learning conditions, including suitable adjustments, to maximize a student's chances of attaining the desired learning outcomes, such as the mastery of content standards.

Readability. The formatting of presented material that considers the organization of text; syntactic complexity of sentences; use of abstractions; density of concepts; sequence and organization of ideas; page format; sentence length; paragraph length; variety of punctuation; student background knowledge or interest; and use of illustrations or graphics in determining the appropriate level of difficulty of instructional or assessment materials.

Real-world application. The opportunity for a student to exhibit a behavior or complete a task that he or she would normally be expected to perform outside of the school environment.

Response requirements. The type, kind, or method of action required of a student to answer a question or testing item. The response may include, but is not limited to, reading, writing, speaking, creating, and drawing.

Stakeholders. A group of individuals perceived to be vested in a particular decision (e.g., a policy decision).

Standardized. An established procedure that assures that a test is administered with the same directions, and under the same conditions and is scored in the same manner for all students to ensure the comparability of scores. Standardization allows reliable and valid comparison to be made among students taking the test. The two major types of standardized tests are norm-referenced and criterion-referenced.

Standards. There are two types of standards, content and achievement (performance).

- **Content standards.** Statements of the subject-specific knowledge and skills that schools are expected to teach students, indicating what students should know and be able to do.

- **Achievement (Performance) standards.** Indices of qualities that specify how adept or competent a student demonstration must be and consist of the following four components:
  - levels that provide descriptive labels or narratives for student performance (i.e., advanced, Level III, etc.);
  - descriptions of what students at each particular level must demonstrate relative to the task;
  - examples of student work at each level illustrating the range of performance within each level; and
  - cut scores clearly separating each performance level.

Standards-based assessments. Assessments constructed to measure how well students have mastered specific content standards or skills.

Test. A measuring device or procedure. Educational tests are typically composed of questions or tasks designed to elicit predetermined behavioral responses or to measure specific academic content standards.

Test presentation. The method, manner, or structure in which test items or assessments are administered to the student.

Universal design of assessment. A method for developing an assessment to ensure accessibility by all students regardless of ability or disability. Universal design of assessment is based on principles used in the field of architecture in which user diversity is considered during the conceptual stage of development.

*Adapted from the *Glossary of Assessment Terms and Acronyms Used in Assessing Special Education Students: A Report from the Assessing Special Education Students (ASES) State Collaborative on Assessment and Student Standards (SCASS)*

APPENDIX

Development of the Dynamic Learning Maps Essential Elements has been a collaborative effort among practitioners, researchers, and our state representatives. Listed below are the reviews and the individuals involved with each round of improvements to the Dynamic Learning Maps Essential Elements. Thank you to all of our contributors.

REVIEW OF DRAFT TWO OF DYNAMIC LEARNING MAPS ESSENTIAL ELEMENTS

A special thanks to all of the experts nominated by their state to review draft two of the Dynamic Learning Maps Essential Elements. We are grateful for your time and efforts to improve these standards for students with significant cognitive disabilities. Your comments have been incorporated into this draft. The states with teams who reviewed draft two include:

Illinois	Oklahoma
Iowa	Utah
Kansas	Virginia
Michigan	West Virginia
Missouri	Wisconsin

DEVELOPMENT OF THE ORIGINAL DYNAMIC LEARNING MAPS COMMON CORE ESSENTIAL ELEMENTS

A special thanks to Edvantia and the team of representatives from Dynamic Learning Maps consortium states who developed the original Common Core Essential Elements upon which the revised Dynamic Learning Maps Essential Elements are based. The team from Edvantia who led the original effort included:

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SECTION 4

Wisconsin's Approach to Literacy in All Subjects

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What is Disciplinary Literacy?

Literacy, the ability to read, write, listen, speak, think critically and perform in different ways and for different purposes, begins to develop early and becomes increasingly important as students pursue specialized fields of study in high school and beyond. The Common Core State Standards (CCSS) for Literacy in Science, Social Studies, History, and the Technical Subjects are connected to College and Career Readiness Standards that guide educators as they strive to help students meet the literacy challenges within each particular field of study. This national effort is referred to as disciplinary literacy.

In Wisconsin, disciplinary literacy is defined as the confluence of content knowledge, experiences, and skills merged with the ability to read, write, listen, speak, think critically and perform in a way that is meaningful within the context of a given field.

These abilities are important in ALL courses and subjects. While the Common Core State Standards (CCSS) for Literacy in Science, Social Studies, History, and the Technical Subjects provide standards for cross-discipline reading and writing in grades 6-12, Wisconsin recognizes the need to broaden this effort and include **all disciplines and every educator in every grade level K-12.** This literacy focus must begin as soon as children have access to formal education and continue intentionally as college and career readiness goals advance for all children in Wisconsin.

To address this expanded definition and approach to disciplinary literacy, excerpts from the K-5 Common Core State Standards for English Language Arts are included in this document. Elementary classroom teachers build the foundational literacy skills necessary for students to access all learning. Additionally, they develop content specific to deep literary study, oratory tradition and linguistic analysis; skills specific to English language arts. Literacy reaches beyond this knowledge in one content area to include reading, writing, listening, speaking and thinking critically in each discipline beginning at an early age. The applicable K-5 standards help educators in Wisconsin build a ladder of skills and dispositions that lead to accelerated achievement across disciplines and will be included in every content-specific standards document into the future.

Why is disciplinary literacy important?

The modern global society, of which our students are a part, requires postsecondary learning. An analysis of workforce trends by Georgetown University economist Anthony Carnevale and his colleagues found that nearly 60 percent of all job openings in 2007 required some postsecondary education; postsecondary success depends on students' ability to comprehend and produce the kinds of complex texts found in all disciplines. Therefore, the economic future of our state, as well as our students and their success as productive citizens and critical thinkers link to disciplinary literacy.

Textbooks, articles, manuals and historical primary source documents create specialized challenges for learners. These texts often include abstracts, figures, tables, diagrams and specialized vocabulary. The ideas are complex and build across a number of paragraphs requiring focus and strategic processing. To comprehend and produce this type of text, students must be immersed in the language and thinking processes of that discipline and they must be supported by an expert guide, their teacher (Carnegie Report, 2010).

A focus at the elementary level on foundational reading, when expanded to include engaging experiences connected to informational texts, vocabulary, and writing for content-specific purposes builds background knowledge and skills in each discipline. This increases opportunities for success as students approach more rigorous content in those disciplines (Alliance for Excellent Education, 2011).

Reading, writing, speaking, listening and critical thinking must be integrated into each discipline across all grades so that all students gradually build knowledge and skills toward college and career readiness. Collaboration among institutes of higher education, CESA Statewide Network, districts, schools, teachers and family and community will guide the implementation of the Common Core State Standards in Wisconsin.

The message is that literacy is integral to attainment of content knowledge and content is essential background knowledge for literacy development.

This interdependent relationship exists in all disciplines.

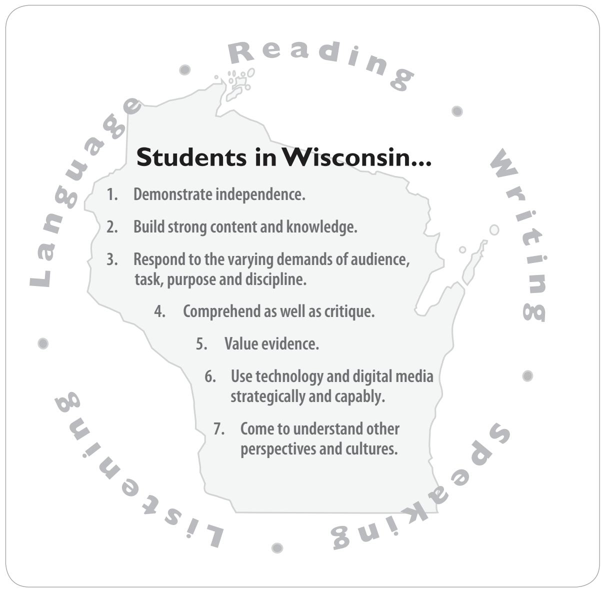
The Common Core State Standards require educators to support literacy in each classroom across the state. Since the impact of this effort is significant, it is essential that resources and supports be accessible to all educators. To build consistent understanding, DPI convened a statewide Disciplinary Literacy Leadership Team in 2011 comprised of educators from many content areas and educational backgrounds. This team was charged with examining the CCSS for Disciplinary Literacy, identifying the needs in the field for support, and gathering materials and resources to address those needs. Resources are available at:
www.dpi.wi.gov/standards

Wisconsin Foundations for Disciplinary Literacy

To guide understanding and professional learning, a set of foundations, developed in concert with Wisconsin's *Guiding Principles for Teaching and Learning,* directs Wisconsin's approach to disciplinary literacy.

Academic learning begins in early childhood and develops across all disciplines.

Each discipline has its own specific vocabulary, text types, and ways of communicating. Children begin learning these context- and content-specific differences early in life and continue through high school and beyond. While gardening, small children observe and learn the form and function of a root, stem, leaf and soil; or measure, mix and blend while baking a cake. School offers all students opportunities to develop the ability to, for example, think like a scientist, write like a historian, critique like an artist, problem-solve like an auto mechanic, or analyze technological advances like a health care technician. As literacy skills develop, educators gradually shift the responsibility for reading, writing, listening, speaking and critical thinking to students through guided supports in both individual and collaborative learning experiences.



Content knowledge is strengthened when educators integrate discipline-specific literacy into teaching and learning.

Educators help students recognize and understand the nuances of a discipline by using strategies that "make their thinking visible." They promote classroom reading, writing, listening, speaking and critical thinking using authentic materials that support the development of content-specific knowledge. They guide students through these complex texts by using strategies that develop conceptual understanding of language and set expectations for relevant application of skills. These literacy practices deepen students' content knowledge, strategies and skills so that their learning transfers to real world situations.

The literacy skills of reading, writing, listening, speaking and critical thinking improve when content-rich learning experiences motivate and engage students.

Educators who foster disciplinary literacy develop experiences that integrate rigorous content with relevant collaborative and creative literacy processes to motivate and engage students. Setting high expectations, they structure routines and supports that empower students to take charge of their own learning. When students work in teams to research science and mathematics concepts in the development of an invention or a graphic arts design; when they collaboratively build a blog that explains their recent marketing venture, they use specific literacy skills and strategies to solidify learning. Students need these opportunities over time to develop the precise and complex reading, writing, listening, speaking and critical thinking skills demanded in today's careers.

Students demonstrate their content knowledge through reading, writing, listening, and speaking as part of a content-literate community.

Students who are literate in a particular discipline are able to successfully read, write, and speak about that discipline and can listen to and think critically as others communicate in that community. Performance tasks that allow students to present the complexity of a content area in a way that is meaningful to the field become authentic approaches to assessing mastery within a discipline. Such tasks empower students to discover the real world connections across disciplines and to actively participate in communities of discipline-literate peers. As Wisconsin moves to the SMARTER Balanced Assessment System these performance tasks will be integral to assessment of student learning.

What research and resources are available to support educators' use of the Common Core State Standards for Literacy in All Subjects?

The Common Core State Standards for Literacy in All Subjects reflect the importance of literacy in both the oral and written language and in both productive (speaking and writing) and receptive (listening and reading) discourse. Clearly, critical and precise thinking are required to develop all of these specific strategies and skills. The standards also address the learning and functioning of language in a technological, media-driven world because the language that we use is selective depending upon the context of the conversation.

The following section will offer relevant research and resources to support professional learning in reading, writing, speaking, listening and language across disciplines. Collegial conversation and learning, both cross-discipline and within-discipline will help make the Common Core State Standards more applicable to schools and districts, and will address the needs of unique programs within those contexts. A collection of online resources will continue to develop as support materials emerge.

Reading Connections

While early reading focuses on learning that letters make sounds, and that words carry meaning, reading quickly develops to a point where the message taken from text depends on what the reader brings to it. The Carnegie Report, *Reading in the Disciplines* (2010) describes this phenomenon:

"The ability to comprehend written texts is not a static or fixed ability, but rather one that involves a dynamic relationship between the demands of texts and prior knowledge and goals of the reader."

Therefore, a musician reading a journal article that describes concepts in music theory will take more information away from the text than a music novice because of their knowledge and experience in music. As well, an individual who spends a significant amount of time reading automotive manuals will more easily navigate a cell phone manual because of familiarity with that type of text.

A chart excerpted from the Carnegie Report (2010) details a few of the generic and more discipline-specific strategies that support students as they attempt to comprehend complex text. While the generic strategies pertain across content areas, discipline-specific ones must be tailored to match the demands of the content area.

Both generic and discipline focused strategies and knowledge must be applied to the comprehension and evaluation of:

- Textbooks
- Journal and magazine articles
- Historically situated primary documents
- Full Length Books
- Newspaper Articles
- Book Chapters
- Multimedia and Digital Texts

Generic Reading Strategies	Discipline-Specific Reading Strategies
Monitor comprehension	Build prior knowledge
Pre-read	Build specialized vocabulary
Set goals	Learn to deconstruct complex sentences
Think about what one already knows	Use knowledge of text structures and genres to predict main and subordinate ideas
Ask questions	Map graphic (and mathematical) representations against explanations in the text
Make predictions	Pose discipline relevant questions
Test predictions against the text	Compare claims and propositions across texts
Re-read	Use norms for reasoning within the discipline (i.e. what counts as evidence) to evaluate claims
Summarize	

Source: Carnegie Report, (2010)

Additional resources that support reading in specific subjects include *Content Counts! Developing Disciplinary Literacy Skills, K–6* by Jennifer L. Altieri (2011). This guide for discipline-specific literacy at the elementary level offers strategies to balance the demands of literacy while continuing to make content count and help students meet the reading, writing, speaking and listening demands of the content areas as they advance in school.

A resource by Doug Buehl (2011) entitled *Developing Readers in the Academic Disciplines* describes what it means to read, write, and think through a disciplinary lens in the adolescent years. This teacher-friendly guide helps connect literacy with disciplinary understandings to bridge academic knowledge gaps, frontload instruction, and build critical thinking through questioning.

Note on range and content of student reading

To become college and career ready, students must grapple with works of exceptional craft and thought whose range extends across genres, cultures, and centuries. Such works offer profound insights into the human condition and serve as models for students' own thinking and writing. Along with high-quality contemporary works, these texts should be chosen from seminal U.S. documents, the classics of American literature, and the timeless dramas of Shakespeare. Through wide and deep reading of literature and literary nonfiction of steadily increasing sophistication, students gain a reservoir of literary and cultural knowledge, references, and images; the ability to evaluate intricate arguments; and the capacity to surmount the challenges posed by complex texts. (*CCSS p. 35 http://www.corestandards.org/assets/CCSSI_ELA%20Standards.pdf*)

The Common Core State Standards require that all students "be able to comprehend texts of steadily increasing complexity as they progress through school" (Appendix A: Research Supporting Key Elements of the Standards, p. 2). More detailed definitions of complex text and examples of complex texts across disciplines are available in Appendix B of the English Language Arts CCSS at: www.dpi.wi.gov/standards.

Writing Connections

The Common Core State Standards call for emphasis on three types of writing: narrative, informational and logical argument. Writing that presents a logical argument is especially appropriate to discipline-specific work since credible evidence differs across content areas. The ability to consider multiple perspectives, assess the validity of claims and present a point of view is required in argumentative writing. These thinking and communication skills are "critical to college and career readiness" (Appendix A: p. 24).

A 2007 report entitled *Writing Next: Effective Strategies to Improve Writing of Adolescents in Middle and High Schools* detailed research on writing to learn, rather than only for assessment, as having a significant impact on content learning.

The study found writing to learn was equally effective for all content areas in the study (social studies, math and science) and at every grade (4-12).

Note on range and content of student writing

For students, writing is a key means of asserting and defending claims, showing what they know about a subject, and conveying what they have experienced, imagined, thought, and felt. To be college- and career-ready writers, students must take task, purpose, and audience into careful consideration, choosing words, information, structures, and formats deliberately. They need to know how to combine elements of different kinds of writing—for example, to use narrative strategies within an argument and explanation within narrative—to produce complex and nuanced writing. They need to be able to use technology strategically when creating, refining, and collaborating on writing. They have to become adept at gathering information, evaluating sources, and citing material accurately, reporting findings from their research and analysis of sources in a clear and cogent manner. They must have flexibility, concentration, and fluency to produce high quality first draft text under a tight deadline as well as the capacity to revisit and make improvements to a piece of writing over multiple drafts when circumstances encourage or require it. *(CCSS p.41 http://www.corestandards.org/assets/CCSSI_ELA%20Standards.pdf*)

When a social studies teacher guides students in taking on the perspective of a person from a specific historical era, she might ask students to write a first person narrative from that perspective. Research into that era leads students to discover personal beliefs of that historical person. They may dig into the personal experiences, ideas, and events involved in the era to visualize life in that period. They then develop a rich understanding of the era and embed language from that era into the texts that they create. (Samples of discipline-specific writing across grades and content areas are available in Appendix C of the English Language Arts CCSS at: www.dpi.wi.gov/standards.

Speaking, Listening and Language Connections

The ability to share ideas and orally communicate with credibility in a specific academic discourse empowers students and allows access to specialized groups. In *Situated Language and Learning: A Critique of Traditional Schooling*, James Paul Gee (2004) describes the need to prioritize these skills so that students are at ease as they enter situations connected to a specific content area and are more likely to continue their learning in that discipline.

As expertise develops, students feel more and more comfortable applying knowledge and skills while speaking and listening in a specific discipline.

- A media course may teach students appropriate expression, tone and rate of speech when addressing a large audience.
- Listening carefully to questions posed is a specialized skill that debate facilitators develop.
- Scientists learn to listen for bias in the perspectives presented by peers to determine the reliability of scientific outcomes.
- Artists have very specialized and specific ways of speaking about the many aspects of a piece.

A policy brief from the Alliance for Excellent Education called, *Engineering Solutions to the National Crisis in Literacy: How to Make Good on the Promise of the Common Core State Standards* describes "a staircase of literacy demands" and emphasizes the importance of a progressive development of language and literacy over time.

The conceptual understanding of "functions" in mathematics may begin to develop in elementary school in its simplest form. As the concept develops over the years, students will use the word "function" in a meaningful way when speaking and writing to describe the mathematical concept they apply. When educators explicitly connect a mathematical term to its application and repeatedly expose students to the concept connected to the term, a specialized language becomes second nature to the mathematics classroom.

Students must have extensive vocabularies, built through reading and explicit instruction embedded in the context of content learning. This enables them to comprehend complex texts, engage in purposeful writing and communicate effectively within a discipline.

Skills in determining or clarifying the meaning of words and phrases encountered, choosing flexibly from an array of strategies, and seeing an individual word as part of a network of other words that, for example, have similar denotations but different connotations allow students to access information and support their own learning.

Literacy in Multiple Languages

Increasing economic, security, cross-cultural and global demands underscore the value of literacy in more than one language. Students who think, read, write, and communicate in multiple languages are an asset to our own country and can more easily interact and compete in the world at large.

English language learners (ELL) in our classrooms face significant challenges as they add a new language and work to grasp content at the same rate as their English-speaking peers. In a report to the Carnegie Corporation entitled *Double the Work: Challenges and Solutions to Acquiring Academic Literacy for Adolescent English Language Learners (2007)* researchers found that a focus on academic literacy is crucial for ELL's success in school. In their description of academic literacy they include reading, writing and oral discourse that:

- Varies from subject to subject.
- Requires knowledge of multiple genres of text, purposes for text use and text media.
- Is influenced by students' literacies in context outside of school.
- Is influenced by students' personal, social, and cultural experiences.

The needs of our English language learners are addressed when we embed disciplinary literacy strategies into our subject area teaching. These high impact strategies and skills allow English language learners and all students to more readily access content knowledge and connect it to the prior knowledge they bring to the classroom. When educators take the initiative to understand and embed these strategies and skills, they offer additional opportunities for success to all of our students.

Who Should Use the Common Core State Standards for Literacy in All Subjects?

The term "disciplinary literacy" may be new to many Wisconsin teachers. The Common Core State Standards for Literacy in All Subjects as excerpted from the Common Core Standards for English Language Arts, are intended for all K-12 educators. Each standard is written broadly in content-neutral language, breaking down the complex skills that comprise reading, writing, speaking, listening, and language. These standards serve as a complement to the specific content-related standards of each individual discipline. Administrators and communities may also find the disciplinary literacy standards helpful in charting a clear and consistent school or district-wide approach to literacy that moves Wisconsin forward toward the goal of every student career and college ready.

References:

Altieri, Jennifer (2011). *Content Counts! Developing Disciplinary Literacy Skills, K–6*. International Reading Association. ISBN 13: 978-0-87207-838-3

Buehl, Doug. (2011). *Developing Readers in the Academic Disciplines*. International Reading Association. ISBN 13: 978-0-87207-845-1

Carnevale, A. (2010) *Center on Education and the Workforce Forecasts of Education Demand to 2018*

College and Career Readiness Standards; http://www.nc4ea.org/files/appropriate_college-readiness_standards_for_all_students-05-03-06.pdf

Common Core Standards for English Language Arts; www.corestandards.org

Washington, DC: Georgetown Center on Education and the Workforce, 2010, available at: http://www9.georgetown.edu/grad/gppi/hpi/cew/pdfs/CEW_press_conference_ppt.pdf (accessed June 7, 2011)

Double the work: Challenges and Solutions to Acquiring Academic Literacy for Adolescent English Language Learners. Carnegie Corporation. New York: 2007.

Engineering Solutions to the National Crisis in Literacy: How to Make Good on the Promise of the Common Core State Standards. Alliance for Excellent Education. Washington D.C. 2011

Gee, James Paul (2004) *Situated Language and Learning: A Critique of Traditional Schooling*

Reading in the Disciplines: The Challenges of Adolescent Literacy. Carnegie Corporation. New York: 2010

State Superintendent's Adolescent Literacy Plan (2008) Wisconsin Department of Public Instruction, Madison, WI

Vygotsky, (1978) *Mind in Society: The Development of Higher Psychological Processes* Harvard University Press; 14th edition

Writing Next: Effective Strategies to Improve Writing of Adolescents in Middle and High Schools (2007)

SECTION 5

Wisconsin Research and Resources

Guiding Principles for Teaching and Learning:

Research, Probing Questions, Resources, and References

1. Every student has the right to learn.

It is our collective responsibility as an education community to make certain each child receives a high-quality, challenging education designed to maximize potential; an education that reflects and stretches his or her abilities and interests. This belief in the right of every child to learn forms the basis of equitable teaching and learning. The five principles that follow cannot exist without this commitment guiding our work.

2. Instruction must be rigorous and relevant.

To understand the world in which we live, there are certain things we all must learn. Each school subject is made up of a core of essential knowledge that is deep, rich, and vital. Every student, regardless of age or ability, must be taught this essential knowledge. What students learn is fundamentally connected to how they learn, and successful instruction blends the content of a discipline with processes of an engaging learning environment that changes to meet the dynamic needs of all students.

3. Purposeful assessment drives instruction and affects learning.

Assessment is an integral part of teaching and learning. Purposeful assessment practices help teachers and students understand where they have been, where they are, and where they might go next. No one assessment can provide sufficient information to plan teaching and learning. Using different types of assessments as part of instruction results in useful information about student understanding and progress. Educators should use this information to guide their own practice and in partnership with students and their families to reflect on learning and set future goals.

4. Learning is a collaborative responsibility.

Teaching and learning are both collaborative processes. Collaboration benefits teaching and learning when it occurs on several levels: when students, teachers, family members, and the community collectively prioritize education and engage in activities that support local schools, educators, and students; when educators collaborate with their colleagues to support innovative classroom practices and set high expectations for themselves and their students; and when students are given opportunities to work together toward academic goals in ways that enhance learning.

5. Students bring strengths and experiences to learning.

Every student learns. Although no two students come to school with the same culture, learning strengths, background knowledge, or experiences, and no two students learn in exactly the same way, every student's unique personal history enriches classrooms, schools, and the community. This diversity is our greatest education asset.

6. Responsive environments engage learners.

Meaningful learning happens in environments where creativity, awareness, inquiry, and critical thinking are part of instruction. Responsive learning environments adapt to the individual needs of each student and encourage learning by promoting collaboration rather than isolation of learners. Learning environments, whether classrooms, schools, or other systems, should be structured to promote engaged teaching and learning.

Guiding Principle 1:
Every student has the right to learn.

It is our collective responsibility as an education community to make certain each child receives a high-quality, challenging education designed to maximize potential, an education that reflects and stretches his or her abilities and interests. This belief in the right of every child to learn forms the basis of equitable teaching and learning. The five principles that follow cannot exist without this commitment guiding our work.

Every student's right to learn provides the overarching vision for Wisconsin's Guiding Principles for education. To be successful, education must be committed to serving the learning needs of students from various social, economic, cultural, linguistic, and developmental backgrounds. For all students to have a guaranteed right to learn, schooling must be equitable.

Research Summary

Focusing on Equity

The belief that each student has the right to learn despite differences in educational needs and backgrounds has important implications for ensuring an equitable education for all students. In the education research literature, the term *educational equality* refers to the notion that all students should have access to an education of similar quality—the proxy for which is frequently educational *inputs* such as funding, facilities, resources, and quality teaching and learning. In contrast, the term *educational equity* connotes the requirement that all students receive an education that allows them to achieve at a standard level or attain standard educational *outcomes* (Brighouse & Swift, 2008). Importantly, equality in terms of educational resources or inputs may not guarantee equity in educational outcomes because not all students reach the same level of achievement with the same access to resources (Brighouse & Swift, 2008). To serve students of varying economic, social, developmental, or linguistic backgrounds, achieving equity in education may require more resources to meet the greater educational needs of certain students (Berne & Stiefel, 1994).

The research literature offers several components that provide a framework for understanding what an equitable education for all students looks like at the classroom level. These components include a call for all students to be provided with the following:

- Access to resources and facilities
- Instruction in all areas tailored to their needs
- Curriculum that is rigorous and relevant
- Educators who are culturally sensitive and respectful
- Interactions with staff and other students that are positive and encouraging in an atmosphere of learning
- Assessment that is varied to give each student the opportunity to demonstrate learning (Education Northwest, 2011)

Access

Access to resources and facilities largely refers to various legal mandates that all children have the right to attend school and participate in all school activities. Since the landmark ruling *Brown v. Board of Education of Topeka* (1954), court decisions and federal regulations have mandated equality of access to all educational opportunities for students regardless of race, ethnicity, or gender
(Civil Rights Act, 1964), disability (Education for All Handicapped Children Act, 1975), or language (*Lau v. Nichols, 1974*). Equity in the provision of educational resources and funding was improved with the passage of Title I of the Elementary and Secondary Education Act (ESEA; 1965), which provided additional resources for economically disadvantaged students to meet their learning needs. Since Title I, research on equity in education has grown, and with the reauthorization of ESEA in the No Child Left Behind Act in 2001, equity in educational outcomes for all students was emphasized in the law. Access to an equitable education is a legal right for all children, and the quality of that access in classroom instruction is a moral and ethical right.

Instruction

Instruction that is tailored to meet all students' needs goes beyond simply providing equal access to education. High-quality instruction has increasingly been defined in the literature as a key factor in student achievement. High-quality instruction includes differentiated instructional strategies, teaching to students' learning styles, and provision of instructional support for students who are educationally, socially, or linguistically challenged. Differentiated instruction involves utilizing unique instructional strategies for meeting individual student needs as well as modifying curriculum for both high- and low-performing students. Assessing and teaching to student learning styles is one form of differentiation. Research has shown the value of adapting instructional strategies to different student learning styles (Gardner, 1999) and supports the practice of classroom differentiation (Mulroy & Eddinger, 2003; Tomlinson, 2005).

Curriculum

Designing curriculum that is rigorous and relevant provides an important foundation for a high-quality learning environment by helping make standards-based content accessible to all students. A relevant, rigorous curriculum has been found to be important for all students. Although advanced and rigorous curriculum is generally viewed to be an important factor of academic success for high-achieving students, research also indicates that using challenging, interesting, and varied curriculum for students of all achievement levels improves student achievement (Daggett, 2005). Rigorous curriculum can be adapted for low-performing students in a way that challenges them and helps them meet learning standards. For example, the universal design for learning (UDL) offers strategies for making the general curriculum accessible to special education students (Rose, Hasselbring, Stahl, & Zabala, 2009). Similarly, research on lesson scaffolding emphasizes strategies for providing a rigorous content curriculum to student who are culturally or linguistically diverse or who need additional context to understand certain concepts (Gibbons, 2002).

Climate

Interactions with staff and students that are positive and focused on learning are part of an emotionally safe school climate, but the literature also supports the need for a climate of high academic expectations (Haycock, 2001). Schools with large numbers of high-poverty and racially diverse students have shown significant academic growth when teachers and staff members create an environment of high expectations for achievement (Reeves, 2010). In addition, research on school climate has asserted the need for students to feel emotionally safe and respected as well as physically safe in school (Gronna & Chin-Chance, 1999).

A positive, respectful learning environment with high expectations and curricular and instructional supports for all students offers an avenue to genuine educational equity.

Probing Questions

- What are some of the needs and challenges your school faces in moving toward a fully equitable education for all students?
- How could you provide leadership in your school to work to ensure an equitable education for all students?

Resources

A variety of resources are available for teachers and leaders on educational equity for all students. A few websites and links are highlighted below:

The School Improvement Center developed activities to help districts develop an equity framework. These resources can be found at Actualizing Equity: The Equity Framework: http://www.gapsc.com/EducatorPreparation/NoChildLeftBehind/Admin/Files/conference_032010/Actualizing_Equity.pdf.

The Education Equality Project developed a website with useful resources for educators. It can be found at http://www.edequality.org.

The Equity Center has a website with a variety of resources. The resources can be found at http://educationnorthwest.org/project/Equity%20Program/resource/.

The Midwest Equity Assistance Center has a website with many resources. It can be found at http://www.meac.org/Publications.html.

The Office for Civil Rights has a useful website for educators. It can be found at http://www2.ed.gov/about/offices/list/ocr/index.html.

Southern Poverty Law Center, Teaching Tolerance Program. Resources can be found at http://www.splcenter.org/what-we-do/teaching-tolerance.

References

Berne, R., & Stiefel, L. (1994). Measuring educational equity at the school level: The finance perspective. *Educational Evaluation and Policy Analysis*, 16(4), 405–421.

Brighouse, H., & Swift, A. (2008). Putting educational equality in its place. *Education, Finance and Policy*, 3(4), 444–446.

Brown v. Board of Education of Topeka, 347 U.S. 483 (1954).

Civil Rights Act, Title IX, Pub. L. No. 88-352, 78 Stat. 241 (1964).

Daggett, W. R. (2005). *Achieving academic excellence through rigor and relevance* [White paper]. Rexford, NY: International Center for Leadership in Education. Retrieved June 3, 2011, from http://www.leadered.com/pdf/Academic_Excellence.pdf

Education Northwest. (2011). *Key components of educational equity* [Website]. Retrieved June 3, 2011, from http://educationnorthwest.org/equity-program/educational

Education for All Handicapped Children Act, Pub. L. No. 94-142 (1975).

Elementary and Secondary Education Act of 1965, Pub. L. No. 89-10) (1965).

Gardner, H. (1999). *Intelligence reframed: Multiple intelligences for the 21st century*. New York: Basic Books.

Gibbons, P. (2002). *Scaffolding language, scaffolding learning: Teaching second language learners in the mainstream classroom*. Westport, CT: Heinemann.

Gronna, S. S., & Chin-Chance, S. A. (1999, April). *Effects of school safety and school characteristics on grade 8 achievement*. Paper presented at the American Educational Research Association, Montreal, Quebec, Canada. (ERIC Document Reproduction Service No. 430292). Retrieved June 3, 2011, from http://www.eric.ed.gov/PDFS/ED430292.pdf

Haycock, K. (2001). Closing the achievement gap. *Educational Leadership*, 58(6), 6–11.

Lau v. Nichols, 414 U.S. 565 (1974).

Mulroy, H., & Eddinger, K. (2003, March). *Differentiation and literacy.* Paper presented at the Institute on Inclusive Education, Rochester, NY.

No Child Left Behind Act of 2001, Pub. L. No. 107-110, 115 Stat. 1425 (2002). Retrieved June 3, 2011, from http://www.ed.gov/policy/elsec/leg/esea02/107-110.pdf

Reeves, D. B. (2010). The 90/90/90 schools: A case study. In D. B. Reeves, *Accountability in action* (2nd ed., 185–196). Denver, CO: Advanced Learning Press.

Rose, D., Hasselbring, T., Stahl, S., & Zabala, J. (2009). Assistive technology, NIMAS, and UDL: From some students to all students. In D. Gordon, J. Gravel, & L. Schifter (Eds.), *A policy reader in universal design for learning* (pp. 133–154). Cambridge, MA: Harvard Education Press.

Tomlinson, C. A. (2005). Grading and differentiation: Paradox or good practice? *Theory Into Practice,* 44(3) 262–269.

Guiding Principle 2:
Instruction must be rigorous and relevant.

To understand the world in which we live, there are certain things we all must learn. Each school subject is made up of a core of essential knowledge that is deep, rich, and vital. Every student, regardless of age or ability, must be taught this essential knowledge. What students learn is fundamentally connected to how they learn, and successful instruction blends the content of a discipline with processes of an engaging learning environment that changes to meet the dynamic needs of all students.

Research Summary

Instruction should connect directly to students' lives and must deeply engage them with the content in order for students to be better prepared for college and careers. To succeed in postsecondary education and in a 21st century economy, students must be afforded opportunities to practice higher-order thinking skills, such as how to analyze an argument, weigh evidence, recognize bias (their own and others' bias), distinguish fact from opinion, balance competing principles, work collaboratively with others, and be able to communicate clearly what they understand (Wagner, 2006). In order to accomplish these goals, instruction must be rigorous and meaningful.

The definition of *rigor* varies greatly in both research and practice. Bower and Powers (2009) conducted a study to determine the essential components of rigor. They defined *rigor* through their research as "how the standard curriculum is delivered within the classroom to ensure students are not only successful on standardized assessments but also able to apply this knowledge to new situations both within the classroom and in the real world." They also identified higher-order thinking and real-world application as two critical aspects of rigor, suggesting that it is not enough for students to know how to memorize information and perform on multiple-choice and short-answer tests. Students must have deep and rich content knowledge, but rigor also includes the ability to apply that knowledge in authentic ways.

Teaching and learning approaches that involve students collaborating on projects that culminate with a product or presentation are a way to bring rigor into the classroom. Students can take on real problems, use what they know and research to come up with real solutions to real problems. They must engage with their subject and with their peers.

In August 2010, the Institutes of Education Sciences reported the results of a randomized control trial showing that a problem-based curriculum boosted high school students' knowledge of economics. This research suggests that students using this learning system and its variants score similarly on standardized tests as students who follow more traditional classroom practices. The research also suggests that students learning through problem-solving and projects are more adept at applying what they know and are more deeply engaged.

The notion of a meaningful curriculum is not a new one. John Dewey (1990), writing in 1902, called for a curriculum that involves a critical but balanced understanding of the culture and the prior knowledge of each child in order to extend learning. According to Spillane (2000), presenting content in more authentic ways—disciplinary and other real-world contexts—has become a central theme of current reform movements. Schools should be places where "the work students are asked to do [is] work worth doing" (Darling-Hammond, 2006, p. 21). Research collected by the International Center for Leadership in Education shows that "students understand and retain knowledge best when they have applied it in a practical, relevant setting" (Daggett, 2005, p. 2). A skilled 21st century educator helps students master learning targets and standards using purposefully crafted lessons and teaches with appropriate instructional strategies incorporated. The students understand why they are learning particular skills and content and are engaged in learning opportunities that allow them to use their inquiry skills, creativity, and critical thinking to solve problems.

According to Brown, Collins, and Duguid (1989), instruction connected to individual contexts has been found to have a significant impact on learning. Research conducted by Sanbonmatsu, Shavitt, and Sherman (1991) and Petty and Cacioppo (1984) also contends that student learning is directly influenced by how well it is connected to a context. Much of this research began with the analysis of how people learn when they find the ideas significant to their own world. It begins to show the importance of connecting content and instruction to the world of the students. Weaver and Cottrell (1988) point out that how content is presented can affect how students retain it. They state instruction that connects the content to the students' lives and experiences helps students to internalize meaning. Sass (1989) and Keller (1987) suggest that if teachers can make the content familiar to the students and link it to what they are familiar with, students' learning will increase. Shulman and Luechauer (1993) contend that these connections must be done by engaging students with rigorous content in interactive learning environments.

Higher-Order Thinking

Higher-order thinking, according to Newmann (1990), "challenges the student to interpret, analyze, or manipulate information" (p. 45). This definition suggests that instruction must be designed to engage students through multiple levels in order for them to gain a better understanding of the content. An analysis of the research by Lewis and Smith (1993) led to their definition of *higher-order thinking*: "when a person takes new information and information stored in memory and interrelates and/or rearranges and extends this information to achieve a purpose or find possible answers in perplexing situations" (p. 44). This definition emphasizes the level of complexity necessary to help students reach a deeper and higher level of understanding of the content. Shulman (1987) points out teachers will need an in-depth knowledge of their content to be able to fit these types of strategies to their instruction.

Real-World Application

VanOers and Wardekker (1999) indicate that connecting instruction to real-world applications gives meaning to learning, makes it practical, and can help to develop connections with the greater community. Incorporating real-world examples becomes more authentic to students because they will be able to connect the learning to the bigger picture rather than just the classroom. Newmann and Wehlage (1993) describe the three criteria developed by Archbald and Newmann (1988) for this type of authentic learning: "Students construct meaning and produce knowledge, students use disciplined inquiry to construct meaning, and students aim their work toward production of discourse, products, and performances that have value or meaning beyond success in school" (p. 8) These criteria, when reflected upon by teachers, can be a useful tool to ensure that instruction is authentic and engaging for all students.

Authentic Learning

Authentic learning builds on the concept of "learning by doing" to increase a student's engagement. To succeed, this method needs to have meaning or value to the student, embody in-depth learning in the subject and allow the student to use what he or she learned to produce something new and innovative (Lemke & Coughlin, 2009). For example, in project-based learning, students collaborate to create their own projects that demonstrate their knowledge (Bell, 2010). Students start by developing a question that will guide their work. The teacher acts as the supervisor. The goal is greater understanding of the topic, deeper learning, higher-level reading, and increased motivation (Bell, 2010). Research has shown that students who engage in project-based learning outscore their traditionally educated peers in standardized testing (Bell, 2010).

Constructivist learning is also a way to bring authenticity to the classroom. Richard Mayer (2004) defines constructivist learning as an "active process in which learners are active sense makers who seek to build coherent and organized knowledge." Students co-construct their learning, with the teacher serving as a guide or facilitator (oftentimes using technology as a facilitating tool). The teacher doesn't function in a purely didactic manner. Neo and Neo (2009) state that constructivism helps students develop problem-solving skills, critical thinking and creative skills and apply them in meaningful ways. Inquiry-based instruction, a type of constructivist learning, has students identify real world problems and then pose and find answers to their own questions. A study by Minner, Levy and Century (2010) has shown this method can improve student performance. They found inquiry-based instruction has a larger impact (approximately 25-30% higher) on a student's initial understanding and retention of content than any other variable.

Another form of authentic learning involves video simulated learning or gaming. Research has shown that video games can provide a rich learning context by fostering creative thinking. The games can show players how to manage complex problems and how their decisions can affect the outcome (Sharritt, 2008). This form of learning also can engage students in collaboration and interaction with peers.

Multimodal Instruction

Multimodal teaching leverages various presentation formats—such as printed material, videos, PowerPoints, and computers—to appeal to different learning styles (Birch, 2009; Moreno & Mayer, 2007). It accommodates a more diverse curriculum and can provide a more engaging and interactive learning environment (Birch, 2009). According to research, an effective way of learning is by utilizing different modalities within the classroom, which can help students understand difficult concepts—therefore improving how they learn (Moreno & Mayer, 2007).

An example of multimodal learning that incorporates technology is digital storytelling. Digital storytelling is the practice of telling stories by using technology tools (e.g., digital cameras, authoring tools, computers) to create multimedia stories (Sadik, 2008). Researchers have found that using this form of learning facilitates student engagement, deep learning, project-based learning, and effective integration of technology into instruction (Sadik, 2008).

Probing Questions

- Research emphasizes the need for higher-order thinking embedded in instructional practice. How might you learn to incorporate higher-order thinking strategies into your practice?
- The research also suggests the need to connect learning experiences to the real world of the students. How can you use real-world examples in your practice to better engage students in their learning?

Resources

The Rigor/Relevance Framework created by Daggett (2005) is a useful tool to create units, lessons, and assessments that ask students to engage with content at a higher, deeper level. The model and examples are available on the following website: http://www.leadered.com/rrr.html.

Newmann's Authentic Intellectual Work Framework (Newmann, Secada & Wehlage, 1995) gives teachers the tools to analyze instructional practices and student work in regard to indicators of rigor. The research and tools are available at the Center for Authentic Intellectual Work website: http://centerforaiw.com/.

References

Archbald, D., & Newmann, F. M. (1988). *Beyond standardized testing: Assessing authentic academic achievement in the secondary school*. Reston, VA: National Association of Secondary School Principals.

Bell, S. (2010). Project-based learning for the 21st century: Skills for the future. *The Clearing House*, 83, 39–43.

Birch, D. (2009). PowerPoint with audio: A breeze to enhance the student learning experience. *E-Journal of Business Education & Scholarship of Teaching*, 3(1), 36–42.

Bower, H. A., & Powers, J. D. (2009, Fall). What is rigor? A qualitative analysis of one school's definition. *Academic Leadership Live: The Online Journal*, 7(4). Retrieved June 3, 2011, from http://www.academicleadership.org/article/What_is_Rigor_A_Qualitative_Analysis_of_One_School_s_Definition

Brown, J. S., Collins, A., & Duguid, P. (1989). Situated cognition and the culture of learning. *Educational Researcher,* 18(1), 32–42.

Daggett, W. R. (2005). *Achieving academic excellence through rigor and relevance*. Rexford, NY: International Center for Leadership in Education.

Darling-Hammond, L. (2006). Securing the right to learn: Policy and practice for powerful teaching and learning. *Educational Researcher*, 35(7), 13–24.

Dewey, J. (1990). *School and society [and] The child and the curriculum*. Chicago: University of Chicago Press.

Finkelstein, Neal, Thomas Hanson, Chun-Wei Huang, Becca Hirschman, and Min Huang. (2010). Effects of problem based economics on high school economics instruction." *Institute For Education Sciences*. West Ed.

Keller, J. M. (1987). Strategies for stimulating the motivation to learn. *Performance & Instruction*, 26(8), 1–7.

Lemke, C., & Coughlin, E. (2009, September). The change agents: Technology is empowering 21st century students in four key ways. *Educational Leadership*, 67(1), 54–59.

Lewis, A., & Smith, D. (1993). Defining higher order thinking. *Theory Into Practice*, 32(3), 131–137.

Mayer, R.E. (2004). *Should There Be a Three-Strikes Rule Against Pure Discovery Learning? The Case for Guided Methods of Instruction*. American Psychologist, 59(1), 14-19.

Minner, Daphne D., Abigail Jurist Levy, and Jeanne Century. "Inquiry-Based Science Instruction—What Is It and Does It Matter? Results from a Research Synthesis Years 1984 to 2002." *JOURNAL OF RESEARCH IN SCIENCE TEACHING* 47.4 (April 2010): 474-96.

Moreno, R., & Mayer, R. (2007). *Interactive multimodal learning environments* [Special issue on interactive learning environment-contemporary issues and trends]. Educational Psychology Review, 19, 309–326.

Neo, M. & Neo, T.K. (2009). *Engaging students in multimedia-mediated Constructivist learning-Students' perceptions*. Educational Technology & Society, 12(2), 254-266.

Newmann, F. M. (1990). Higher order thinking in teaching social studies: A rationale for the assessment of classroom thoughtfulness. *Journal of Curriculum Studies*, 22(2), 41–56.

Newmann, F. M., Secada, W. G., & Wehlage, G. G. (1995). *A guide to authentic instruction and assessment: Vision, standards, and scoring*. Madison, WI: Wisconsin Center for Education Research.

Newmann, F. M., & Wehlage, G. G. (1993, April). Five standards of authentic instruction. *Educational Leadership*, 50(7), 8–12.

Petty, R. E., & Cacioppo, J. T. (1984). The effects of involvement on responses to argument quality: Central and peripheral routes to persuasion. *Journal of Personality and Social Psychology*, 46(1), 69–81.

Sadik, A. (2008). Digital storytelling: A meaningful technology-integrated approach for engaged student learning. *Educational Technology Research & Development*, 56, 487–506.

Sanbonmatsu, D. M., Shavitt, S., & Sherman, S. J. (1991). The role of personal relevance in the formation of distinctiveness-based illusory correlations. *Personality and Social Psychology Bulletin*, 17(2), 124–132.

Sass, E. J. (1989). Motivation in the college classroom: What students tell us. *Teaching of Psychology*, 16(2), 86–88.

Sharritt, M. J. (2008). Forms of learning in collaborative video game play. *Research and Practice in Technology Enhanced Learning*, 3(2), 97–138.

Shulman, L. S. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review*, 57(1), 1–22.

Shulman, G., & Luechauer, D. (1993). The empowering educator: A CQI approach to classroom leadership. In D. L. Hubbard (Ed.), *Continuous quality improvement: Making the transition to education* (pp. 424–453). Maryville, MO: Prescott.

Spillane, J. P. (2000). A fifth-grade teacher's reconstruction of mathematics and literacy teaching: Exploring interactions among identity, learning, and subject matter. *Elementary School Journal*, 100(4), 307–330.

VanOers, B., & Wardekker, K. (1999). On becoming an authentic learner: Semiotic activity in the early grades. *Journal of Curriculum Studies*, 31(2), 229–249.

Wagner, T. (2006, January 11). Rigor on trial [Commentary]. *Education Week*, 25(18), 28–29. Retrieved June 3, 2011, from http://www.edweek.org/ew/articles/2006/01/11/18wagner.h25.html?tkn=NXVFlUJgch3u9KNoYbF2gM%2BinCPa3hvbbWkj&print=1

Weaver, R. L., & Cottrell, H. W. (1988). Motivating students: Stimulating and sustaining student effort. *College Student Journal*, 22, 22–32.

Wentling, R. M., & Waight, C. L. (2001). Initiative that assist and barriers that hinder the successful transition of minority youth into the workplace in the USA. *Journal of Education and Work*, 14(1), 71–89.

Guiding Principle 3:
Purposeful assessment drives instruction and affects learning.

Assessment is an integral part of teaching and learning. Purposeful assessment practices help teachers and students understand where they have been, where they are, and where they might go next. No one assessment can provide sufficient information to plan teaching and learning. Using different types of assessments as part of instruction results in useful information about student understanding and progress. Educators should use this information to guide their own practice and in partnership with students and their families to reflect on learning and set future goals.

Research Summary

Assessment informs teachers, administrators, parents, and other stakeholders about student achievement. It provides valuable information for designing instruction; acts as an evaluation for students, classrooms, and schools; and informs policy decisions. Instruments of assessment can provide formative or summative data, and they can use traditional or authentic designs. Research on assessment emphasizes that the difference between formative and summative assessment has to do with how the data from the assessment is used.

Dunn and Mulvenon (2009) define summative assessment as assessment "data for the purposes of assessing academic progress at the end of a specified time period (i.e., a unit of material or an entire school year) and for the purposes of establishing a student's academic standing relative to some established criterion" (p. 3).

The Council of Chief State School Officers (CCSSO) (2008) define formative assessment as a process "used by teachers and students during instruction that provides feedback to adjust ongoing teaching and learning to improve students' achievement of intended instructional outcomes" (p. 3).

Wisconsin's approach to balanced assessment www.dpi.wi.gov/oea/balanced emphasizes the importance of identifying the purposes for administering an assessment. Identifying the purpose or data needed establishes whether a particular assessment is being used formatively or summatively. There can be multiple purposes for giving a particular assessment, but identifying how the data will be used helps to ensure that the assessment is collecting the data that is needed for educators, students and their families.

Assessments, whether formative or summative, can be designed as traditional or authentic tools. Traditional assessment uses tools such as paper and pencil tests, while authentic assessment focuses on evaluating student learning in a more "real life" situation. The bulk of the research on assessment design focuses on authentic assessment.

Formative Assessment

Using formative assessment as a regular part of instruction has been shown to improve student learning from early childhood to university education. It has been shown to increase learning for both low-performing and high-performing students. Black and Wiliam's (1998) seminal study found that the use of formative assessment produces significant learning gains for low-achieving students. Other researchers have shown similar results for students with special learning needs (McCurdy & Shapiro, 1992; Fuchs & Fuchs, 1986). Research also supports the use of formative assessment in kindergarten classes (Bergan, Sladeczek, Schwarz, & Smith, 1991), and university students (Martinez & Martinez, 1992).

Formative assessment provides students with information on the gaps that exist between their current knowledge and the stated learning goals (Ramaprasad, 1983). By providing feedback on specific errors it helps students understand that their low performance can be improved and is not a result of lack of ability (Vispoel & Austin, 1995). Studies emphasize that formative assessment is most effective when teachers use it to provide specific and timely feedback on errors and suggestions for improvement (Wininger, 2005), when students understand the learning objectives and assessment criteria, and when students have the opportunity to reflect on their work (Ross, 2006; Ruiz-Primo & Furtak, 2006). Recent research supports the use of web-based formative assessment for improving student achievement (Wang, 2007).

A number of studies emphasize the importance of teacher professional development on formative assessment in order to gain maximum student achievement benefits (Atkins, Black & Coffey, 2001; Black & Wiliam, 1998). A 2009 article in *Educational Measurement* asserts that teachers are better at analyzing formative assessment data than at using it to design instruction. Research calls for more professional development on assessment for teachers (Heritage, Kim, Vendlinski, & Herman, 2009).

Authentic Assessment

Generating rich assessment data can be accomplished through the use of an authentic assessment design as well as through traditional tests. Authentic assessments require students to "use prior knowledge, recent learning, and relevant skills to solve realistic, complex problems" (DiMartino & Castaneda, 2007, p. 39). Research on authentic assessment often explores one particular form, such as portfolios (Berryman & Russell, 2001; Tierney et al., 1998); however, several studies examined more than one form of authentic assessment: portfolios, project-based assessment, use of rubrics, teacher observation, and student demonstration (Darling-Hammond, Rustique-Forrester, & Pecheone, 2005; Herman, 1997; Wiggins, 1990). Authentic assessment tools can be used to collect both formative and summative data. These data can provide a more complete picture of student learning.

Balanced Assessment

Wisconsin's Next Generation Assessment Task Force (2009) defines the purpose and characteristics of a balanced assessment system:

Purpose: to provide students, educators, parents, and the public with a range of information about academic achievement and to determine the best practices and policies that will result in improvements to student learning.

Characteristics: includes a continuum of strategies and tools that are designed specifically to meet discrete needs–daily classroom instruction, periodic checkpoints during the year, and annual snapshots of achievement. (p. 6)

A balanced assessment system is an important component of quality teaching and learning. Stiggins (2007) points out that a variety of quality assessments must be available to teachers in order to form a clearer picture of student achievement of the standards. Popham (2008) believes that when an assessment is of high quality, it can accurately detect changes in student achievement and can contribute to continuous improvement of the educational system.

Probing Questions

- How might you use questioning and discussion in your classroom in a way that gives you formative assessment information on all students?
- How can you use assignments and tests as effective formative assessment?
- How could you design and implement a balanced assessment system that includes pre- and post assessments for learning?

Resources

Rick Stiggins, founder and director of the Assessment Training Institute, provides resources on the practice of assessment at http://www.assessmentinst.com/author/rick-stiggins/.

Margaret Heritage's books Formative Assessment for Literacy and Academic Language (2008, coauthored with Alison Bailey) and Formative Assessment: Making It Happen in the Classroom (2010) provide resources and practices. These books are available through bookstores.

ASCD has publications on assessment at http://www.ascd.org/SearchResults.aspx?s=assessment&c=1&n=10&p=0.

The National Middle Schools Association provides assessment information through a search for "assessment" at http://www.nmsa.org/.

Boston (2002) recommends the following resources for assessment:

- A Practical Guide to Alternative Assessment, by J. R. Herman, P. L. Aschbacher, and L. Winters. Available at a variety of booksellers.
- Improving Classroom Assessment: A Toolkit for Professional Developers
  http://educationnorthwest.org/resource/700
- Classroom Assessment and the National Science Education Standards
  http:www.nap.edu/catalog/9847.html

References

Atkins, J. M., Black, P., & Coffey, J. (2001). *Classroom assessment and the National Science Education Standards*. Washington, DC: National Academy Press.

Bergen, J. R., Sladeczek, I. E., Schwarz, R. D., & Smith, A. N. (1991). Effects of a measurement and planning system on kindergartners' cognitive development and educational programming. *American Educational Research Journal*, 28(3), 683–714.

Berryman, L., & Russell, D. R. (2001). Portfolios across the curriculum: Whole school assessment in Kentucky. *English Journal*, 90(6), 76–83.

Black, P., & Wiliam, D. (1998). Assessment and classroom learning. *Assessment in Education*, 5(1), 7–74.

Boston, C. (2002). The concept of formative assessment. *Practical Assessment, Research, and Evaluation*, 8(9). Retrieved June 3, 2011, from http://pareonline.net/getvn.asp?v=8&n=9

Council of Chief State School Officers. (2008). *Attributes of effective formative assessment*. Washington, DC: Author. Retrieved June 3, 2011, from http://www.ccsso.org/Documents/2008/Attributes_of_Effective_2008.pdf

Darling-Hammond, L., Rustique-Forrester, E., & Pecheone, R. (2005). *Multiple measure approaches to high school graduation*. Stanford, CA: School Redesign Network at Stanford University.

DiMartino, J., & Castaneda, A. (2007). Assessing applied skills. *Educational Leadership*, 64(7), 38–42.

Dunn, K. E., & Mulvenon, S. W. (2009). A critical review of research on formative assessment: The limited scientific evidence of the impact of formative assessment in education. *Practical Assessment, Research, and Evaluation*, 14(7). Retrieved June 3, 2011, from http://pareonline.net/pdf/v14n7.pdf

Fuchs, L. S., & Fuchs, D. (1986). Effects of systematic formative evaluation: A meta-analysis. *Exceptional Children*, 52(2), 199–208.

Heritage, M., Kim, J., Vendlinski, T., & Herman, J. (2009). From evidence to action: A seamless process in formative assessment? *Educational Measurement: Issues and Practice*, 28(3), 24–31.

Herman, J. (1997). Assessing new assessments: Do they measure up? *Theory Into Practice*, 36(4), 196–204.

Martinez, J. G. R., & Martinez, N. C. (1992). Re-examining repeated testing and teacher effects in a remedial mathematics course. *British Journal of Educational Psychology*, 62(3), 356–363.

McCurdy, B. L., & Shapiro, E. S. (1992). A comparison of teacher monitoring, peer monitoring, and self-monitoring with curriculum-based measurement in reading among student with learning disabilities. *Journal of Special Education*, 26(2), 162–180.

Next Generation Assessment Task Force. (2009). *Crafting a balanced system of assessment in Wisconsin*. Madison: Wisconsin Department of Public Instruction. Retrieved June 3, 2011, from http://www.dpi.state.wi.us/oea/pdf/NGTFbr.pdf

Popham, W. J. (2008). *Transformative assessment*. Alexandria, VA: Association for Supervision and Curriculum Development.

Ramaprasad, A. (1983). On the definition of feedback. *Behavioral Science*, 28(1), 4–13.

Ross, J. A. (2006). The reliability, validity, and utility of self-assessment. *Practical Assessment, Research and Evaluation*, 11(10). Retrieved June 3, 2011, from http://pareonline.net/pdf/v11n10.pdf

Ruiz-Primo, M. A., & Furtak, E. M. (2006). Informal formative assessment and scientific inquiry: Exploring teachers' practices and student learning. *Educational Assessment*, 11(2), 205–235.

Stiggins, R. J. (2007, November–December). Assessment for learning: A key to student motivation and learning. EDge, 2(2), 1–20.

Tierney, R., Clark, C., Fenner, L., Herter, R. J., Simpson, C. S., & Wiser, B. (1998). Portfolios: Assumptions, tensions, and possibilities. *Reading Research Quarterly*, 33(4), 474–486.

Vispoel, W. P., & Austin, J. R. (1995). Success and failure in junior high school: A critical incident approach to understanding students' attributional beliefs. *American Educational Research Journal*, 32(2), 377–412.

Wang, T. H. (2007). What strategies are effective for formative assessment in a e-learning environment? *Journal of Computer Assisted Learning*, 23(1), 171–186.

Wiggins, G. (1990). The case for authentic assessment. *Practical Assessment, Research, and Evaluation*, 2(2). Retrieved June 3, 2011, from http://pareonline.net/getvn.asp?v=2&n=2

Guiding Principle 4:
Learning is a collaborative responsibility.

Teaching and learning are both collaborative processes. Collaboration benefits teaching and learning when it occurs on several levels: when students, teachers, family members, and the community collectively prioritize education and engage in activities that support local schools, educators, and students; when educators collaborate with their colleagues to support innovative classroom practices and set high expectations for themselves and their students; and when students are given opportunities to work together toward academic goals in ways that enhance learning.

Research Summary

Collaborative learning is an approach to teaching and learning that requires learners to work together to deliberate, discuss, and create meaning. Smith and MacGregor (1992) define the term as follows:

"Collaborative learning" is an umbrella term for a variety of educational approaches involving joint intellectual effort by students, or students and teachers together. Usually, students are working in groups of two or more, mutually searching for understanding, solutions, or meanings, or creating a product. Collaborative learning activities vary widely, but most center on students' exploration or application of the course material, not simply the teacher's presentation or explication of it. (p. 1)

Collaborative learning has been practiced and studied since the early 1900s. The principles are based on the theories of John Dewey (2009), Lev Vygotsky (1980), and Benjamin Bloom (1956). Their collective work focusing on how students learn has led educators to develop more student-focused learning environments that put students at the center of instruction. Vygotsky specifically stated that learning is a social act and must not be done in isolation. This principle is the foundation of collaborative learning.

The research of Vygotsky (1980) and Jerome Bruner (1985) indicates that collaborative learning environments are one of the necessities for learning. Slavin's (1989) research also suggests that students and teachers learn more, are more engaged, and feel like they get more out of their classes when working in a collaborative environment. Totten, Sills, Digby, and Russ (1991) found that those involved in collaborative learning understand content at deeper levels and have higher rates of achievement and retention than learners who work alone. They suggest that collaborative learning gives students opportunities to internalize their learning.

A meta-analysis from the Cooperative Learning Center at the University of Minnesota concluded that having students work collaboratively has significantly more impact on learning than having students work alone (Johnson, Maruyama, Johnson, Nelson, & Skon, 1981). An analysis of 122 studies on cooperative learning revealed:

- More students learn more material when they work together—talking through the material with each other and making sure that all group members understand—than when students compete with one another or work alone individualistically.
- More students are motivated to learn the material when they work together than when students compete or work alone individualistically (and the motivation tends to be more intrinsic).
- Students have more positive attitudes when they work together than when they compete or work alone individualistically.
- Students are more positive about the subject being studied, the teacher, and themselves as learners in that class and are more accepting of each other (male or female, handicapped or not, bright or struggling, or from different ethnic backgrounds) when they work together.

Collaboration can be between teachers, between students, and between teacher and student.

Teacher-Teacher Collaboration

It is critical for teachers to have the time to collaborate. Professional learning communities, which provide teachers with established time to collaborate with other teachers, have become a more common practice in recent years. Louis and Kruse (1995) conducted a case study analysis that highlighted some of the positive outcomes associated with professional learning communities, including a reduction in teacher isolation, increases in teacher commitment and sense of shared responsibility, and a better understanding of effective instructional practices. Professional learning communities encourage collaborative problem solving and allow teachers to gain new strategies and skills to improve and energize their teaching and classrooms.

Another example of teacher-to-teacher collaboration is lesson study. This professional development process began in Japan. Lesson study is a collaborative approach to designing and studying classroom lessons and practice. The most critical components of lesson study are observation of the lesson, collection of data about teaching and learning, and a collaborative analysis of the data to further impact instruction (Lewis, 2002; Lewis & Tsuchida, 1998; Wang-Iverson & Yoshida, 2005). Some of these characteristics are similar to other forms of professional development—analyzing student work, cognitive coaching, and action research, to name a few—but the fact that it focuses on teachers observing a live lesson that was collaboratively developed is different than any other form of professional development. Lesson study is a way for teachers to work together, collect data, and analyze data to reflect on teaching and learning (Lewis, 2002).

Student-Student Collaboration

Collaborative learning not only allows students to engage deeply with content but also helps students build the interpersonal skills needed to be successful in college and careers. Johnson, Johnson, and Holubec (1993) state that collaborative learning provides students with the opportunity to develop social skills. They found that many of the outcomes expected as part of a collaborative learning activity corresponded with goals for student content understanding and skill attainment. The strategies associated with collaborative learning—such as role assignments, collaborative problem solving, and task and group processing—all build the social skills that students need to be successful when working with others. Additionally, these skills are important in preparing students for the world of work, where collaborative writing and problem-solving are key elements of many careers.

There is a plethora of instructional and learning strategies that encourage student collaboration, including peer teaching, peer learning, reciprocal learning, team learning, study circles, study groups, and work groups, to name just a few (Johnson & Johnson, 1986). Collaborative inquiry, which combines many of the elements of student collaboration just mentioned, is a research-based strategy in which learners work together through various phases "of planning, reflection, and action as they explore an issue or question of importance to the group" (Goodnough, 2005 88). Collaborative inquiry brings together many perspectives to solve a problem, engaging students in relevant learning around an authentic question. It allows students to work together toward a common purpose to explore, make meaning, and understand the world around them (Lee & Smagorinsky, 2000).

Teacher-Student Collaboration

The purpose for collaboration in an educational setting is to learn and unpack content together to develop a shared understanding. Harding-Smith (1993) points out that collaborative learning approaches are based on the idea that learning must be a social act. It is through interaction that learning occurs. Johnson and Johnson (1986) similarly emphasize that when students and teachers talk and listen to each other, they gain a deeper understanding of the content and can develop the skills necessary to negotiate meaning throughout their lives.

Collaboration requires a shift from teacher-led instruction to instruction and learning that is designed by both teachers and students. Collaboration between student and teacher plays a critical role in helping students reflect and engage in their own learning experiences. The constructivist learning movement is one current example of efforts to increase the amount of collaboration between student and teacher occurring in the classroom. Mayer (2004) defines constructivist learning as an "active process in which learners are active sense makers who seek to build coherent and organized knowledge" (p. 14). Students co-construct their learning, with the teacher serving as a guide or facilitator. The teacher does not function in a purely didactic (i.e., lecturing) role. Neo and Neo (2009) found that constructivism helps students develop problem-solving skills, critical thinking, and creative skills and apply them in meaningful ways.

Probing Questions

- How can you use collaborative learning processes to engage students in their learning?
- How might you create space for teacher-teacher collaboration within your context?

Resources

All Things PLC website provides a number of resources on professional learning communities. Links to these resources can be found at http://www.allthingsplc.info/.

The Wisconsin Center for Education Research hosts a website with many resources for collaborative and small group learning. It can be found at http://www.wcer.wisc.edu/archive/cl1/cl/..

The Texas Collaborative for Teaching Excellence has created a professional development module about collaborative learning, which provides readings, research, and resources. It can be found at http://www.texascollaborative.org/Collaborative_Learning_Module.htm.

A review of research on professional learning communities, presented at the National School Reform Faculty research forum in 2006, contains findings that outline what is known about professional learning communities and how they should be structured. This paper is available at http://www.nsrfharmony.org/research.vescio_ross_adams.pdf.

References

Bloom, B. S. (Ed.) (1956). *Taxonomy of educational objectives. Handbook 1: Cognitive domain*. White Plains, NY: Longman.

Bruner, J. (1985). Vygotsky: An historical and conceptual perspective. In J. V. Wetsch (Ed.), *Culture, communication, and cognition: Vygotskian perspectives* (pp. 21–34). London: Cambridge University Press.

Dewey, J. (2009). *Democracy and education: An introduction to the philosophy of education*. New York: Cosimo Classics.

Goodnough, Karen. (2005). Fostering teacher learning through collaborative inquiry. *The Clearing House* 79(2), 88-92.

Harding-Smith, T. (1993). *Learning together: An introduction to collaborative learning*. New York: HarperCollins.

Johnson, R. T., & Johnson, D. W. (1986). Action research: Cooperative learning in the science classroom. *Science and Children*, 24(2), 31–32.

Johnson, D. W., Johnson, R. T., & Holubec, E. J. (1993). *Circles of learning: Cooperation in the classroom*. Edina, MN: Interaction.

Johnson, D. W., Maruyama, G., Johnson, R. T., Nelson, D., & Skon, L. (1981). Effects of cooperative, competitive, and individualistic goal structures on achievement: A meta-analysis. *Psychological Bulletin*, 89(1), 47–62.

Lee, C. D., & Smagorinsky, P. (Eds.). (2000). *Vygotskian perspectives on literacy research: Constructing meaning through collaborative inquiry*. Cambridge, England: Cambridge University Press.

Lewis, C. (2002). *Lesson study: A handbook of teacher-led instructional change*. Philadelphia: Research for Better Schools.

Lewis, C., & Tsuchida, I. (1998, Winter). A lesson is like a swiftly flowing river: Research lessons and the improvement of Japanese education. *American Educator*, 14–17, 50–52.

Wang-Iverson, P., & Yoshida, M. (2005). Building our understanding of lesson study. Philadelphia: Research for Better Schools.

Louis, K. S., & Kruse, S. D. (1995). *Professionalism and community: Perspectives on reforming urban schools*. Thousand Oaks, CA: Corwin Press.

Mayer, R. E. (2004). Should there be a three strikes rule against pure discovery? The case for guided methods of instruction. *American Psychologist*, 59(1), 14–19.

Neo, M., & Neo, T.-K. (2009). Engaging students in multimedia-mediated constructivist learning: Students' perceptions. *Educational Technology and Society*, 12(2), 254–266.

Slavin, R. E. (1989). Research on cooperative learning: An international perspective. *Scandinavian Journal of Educational Research*, 33(4), 231–243.

Smith, B. L., & MacGregor, J. T. (1992). *What is collaborative learning?* Olympia, WA: Washington Center for Improving the Quality of Undergraduate Education. Retrieved June 3, 2011, from http://learningcommons.evergreen.edu/pdf/collab.pdf

Totten, S., Sills, T., Digby, A., & Russ, P. (1991). *Cooperative learning: A guide to research*. New York: Garland.

Vygotsky, L. (1980). *Mind in society: The development of higher psychological processes*. Cambridge, MA: Harvard University Press.

Guiding Principle 5:
Students bring strengths and experiences to learning.

Every student learns. Although no two students come to school with the same culture, learning strengths, background knowledge, or experiences, and no two students learn in exactly the same way, every student's unique personal history enriches classrooms, schools, and the community. This diversity is our greatest education asset.

Research Summary

The authors of the groundbreaking work *How People Learn: Brain, Mind, Experience, and School* (Bransford, Brown, & Cocking, 2000) found that students' preconceptions may clash with new concepts and information they learn in school. If those preconceptions are not addressed, students may fail to grasp what is being taught or may learn only to pass a test. In other words, a student might enter kindergarten believing the world is flat because he or she has seen a flat map. Despite the presentation of geographic names and principles, the student still maintains the fundamental preconception about the shape of the world. Developing competence—or in this case, a knowledge of the shape of the world—requires that students have a deep foundation of factual knowledge, a context or conceptual framework to place it in, and the opportunity to explore how it connects to the real world. Ultimately, a metacognitive approach—one that pushes students to think about their own thought processes—can help them take control of their own learning.

As educational research on how people learn advances, so does our approach to teaching and learning. Strategies to advance teaching and learning are constantly evolving into new and innovative ways to reach learners. When a teacher uses students' interests, curiosity, and areas of confidence as starting points in planning instruction, learning is more productive. Teachers who are cognizant of these issues—and reflect on how to use them as strengths upon which they can build—ensure that all students have access to the content. Areas to consider are student strengths, gender, background knowledge, and connections to the home environment.

Building on Student Strengths

Teaching to students' strengths can improve student engagement (Sternberg, 2000, Sternberg & Grigorenko, 2000). Many students have strengths that are unrecognized and neglected in traditional schooling. Students in underrepresented minority groups have culturally relevant knowledge that teachers can use to promote learning. Sternberg et al. (2000) found that conventional instruction in school systematically discriminates against students with creative and practical strengths and tends to favor students with strong memory and analytical abilities. This research, combined with Sternberg's earlier (1988) research showing that teaching for diverse styles of learning produces superior results, suggests that capitalizing on the various strengths that all students bring to the classroom can positively affect students' learning. When students are taught in a way that fits *how* they think, they do better in school (Sternberg, 2000; Sternberg & Grigorenko, 2000). Sternberg and O'Hara (2000) found that when students were taught in a way that incorporated analytical thinking, creative thinking (creating, imagining, and inventing) and practical thinking (applying, implementing, and putting into practice)—students achieved at higher levels than when taught using conventional instructional methods.

Gender Considerations

Changing instruction might help alleviate the gender gap in literacy achievement. Research conducted by Sax (2005) reveals that boys fall behind girls in reading and writing early on and never catch up. Sax (2007) found that this dynamic plays a role in higher high school dropout rates for males, particularly black males. The college graduation rate for females approaches twice that of males in Hispanic and black populations. Many classrooms are a better fit for the verbal-emotive, sit-still, take-notes, listen-carefully, multitasking girl (Sax, 2005). The characteristics that boys bring to learning—impulsivity, single-task focus, spatial-kinesthetic learning, and physical aggression—often are viewed as problems.

Researchers such as Blum (1997) have identified more than 100 structural differences between the male and female brains. Altering strategies to accommodate more typically male assets—for example, the use of multimodal teaching (discussed on pages 10-11 of this report); the use of various display formats, such as printed material, videos, presentations, and computers; and an interactive learning environment to appeal to different learning styles—can help bridge the gap between what students are thinking and what they are able to put down on paper. Sadik's (2008) research suggests that using multimodal instructional strategies like digital storytelling—allowing students to incorporate digital cameras, creative and editing tools, computers, and other technology to design multimedia presentations—deepens students' learning.

Background Knowledge

Bransford et al. (2000) note in *How People Learn*, learning depends on how prior knowledge is incorporated into building new knowledge, and thus teachers must take into account students' prior knowledge. Jensen's (2008) research on the brain and learning demonstrates that expertise cannot be developed merely through exposure to information. Students must connect the information to their prior knowledge to internalize and deepen their understanding. Teachers can connect academic learning with real-life experiences. Service learning, project-based learning, school-based enterprises, and student leadership courses are some examples of how schools are trying to make the curriculum relevant. The key to making the curriculum relevant is asking the students to help connect the academics to their lives; this approach gets students actively engaged in their learning, which builds a stronger connection and commitment to school. Bell (2010) suggests that strategies such as project-based approaches to learning can help ensure that content and skills are taught together and connected to prior knowledge, which helps students understand how to develop and apply new skills in various contexts.

Connections to the Home Environment

Cochran-Smith (2004) emphasizes family histories, traditions, and stories as an important part of education. Often, children enter school and find themselves in a place that does not recognize or value the knowledge or experience they bring from their homes or communities. This situation can create a feeling of disconnect for students—a dissonance obliging them to live in and navigate between two different worlds, each preventing them from full participation or success in the other. Districts and schools can alleviate this dissonance by valuing and taking advantage of the unique experiences that each student brings to the classroom. Emphasizing connections to parents and community, recognizing and utilizing student strengths and experiences, and incorporating varied opportunities within the curriculum can help alleviate this dissonance.

Ferguson (2001) points out that it is particularly important to establish connections that not only bring the parents into the school environment but also encourage school understanding and participation within the community. Social distinctions often grow out of differences in attitudes, values, behaviors, and family and community practices (Ferguson, 2001). Students need to feel their unique knowledge and experience is valued by the school, and parents and community members need to feel they are respected and welcome within the school.

Although much attention has been paid to No Child Left Behind (NCLB) requirements for annual achievement tests and high-quality teachers, the law also includes important requirements for schools, districts, and states to organize programs of parental involvement and to communicate with parents and the public about student achievement and the quality of schools. Epstein (2005) offers perspectives on the NCLB requirements for family involvement; provides a few examples from the field; suggests modifications that are needed in the law; and encourages sociologists of education to take new directions in research on school, family, and community partnerships.

Probing Questions

- What are some ways that you currently use students' background knowledge to inform instruction?
- Does your experience teaching boys to read and write concur with the research? What ideas do you have to address the achievement gaps related to gender?
- What are ways you can uncover, acknowledge, and use students' backgrounds and strengths to enhance learning?
- What are some strategies for valuing and taking advantage of the unique experiences that each student brings to the classroom?

Resources

A good resource still valid today is *Making Assessment Work for Everyone: How to Build on Student Strengths*. See the SEDL website to download this resource: http://www.sedl.org/pubs/tl05/.

A short, easy-to-digest article from Carnegie Mellon University is titled *Theory and Research-Based Principles of Learning*. The article and full bibliography are at http://www.cmu.edu/teaching/principles/learning.html.

References

Bell, S. (2010). Project-based learning for the 21st century: Skills for the future. *The Clearing House*, 83(2), 39–43. Retrieved June 3, 2011, from http://teacherscollegesj.org/resources/publications/PBL%20for%20the%2021%20Century.pdf

Blum, D. (1997). *Sex on the brain: The biological differences between men and women*. New York: Viking.

Bransford, J. D., Brown, A. L. & Cocking, R. R. (Eds.). (2000). How people learn: *Brain, mind, experience, and school* (Expanded ed.). Washington, DC: National Academy Press.

Cochran-Smith, M. (2004). Walking the road: *Race, diversity, and social justice in teacher education*. New York: Teachers College Press.

Epstein, J. (2005). Attainable goals? The spirit and letter of the No Child Left Behind Act on parental involvement. *Sociology of Education*, 78(2), 179–182.

Ferguson, A. A. (2001). *Bad boys: Public schools in the making of black masculinity*. Ann Arbor: University of Michigan Press.

Jensen, E. P. (2008). A fresh look at brain-based education. *Phi Delta Kappan*, 89(6), 408–417.

Sadik, A. (2008). Digital storytelling: A meaningful technology-integrated approach for engaged student learning. *Educational Technology Research and Development*, 56(4), 487–506.

Sax, L. (2005). *Why gender matters: What parents and teachers need to know about the emerging science of sex differences*. New York: Doubleday.

Sax, L. (2007). *Boys adrift: The five factors driving the growing epidemic of unmotivated boys and underachieving young men*. New York: Basic Books.

Sternberg, R. J. (1988). *The triarchic mind: A new theory of human intelligence*. New York: Viking.

Sternberg, R. J. (2000). Wisdom as a form of giftedness. *Gifted Child Quarterly*, 44(4), 252–259.

Sternberg, R. J., & Grigorenko, E. L. (2000). *Teaching for successful intelligence*. Arlington Heights, IL: Skylight Training.

Sternberg, R. J., Grigorenko, E. L., Jarvin, L., Clinkenbeard, P., Ferrari, M., & Torff, B. (2000, Spring). The effectiveness of triarchic teaching and assessment. *NRC/GT Newsletter,* 3–8. Retrieved June, 3, 2011, from http://www.gifted.uconn.edu/nrcgt/newsletter/spring00/spring00.pdf

Sternberg, R. J., & O'Hara, L. A. (2000). Intelligence and creativity. In R. J. Sternberg (Ed.), *Handbook of intelligence* (pp. 611–628). New York: Cambridge University Press.

Vygotsky, L. S. (1980). *Mind in society: The development of higher psychological processes*. Cambridge, MA: Harvard University Press.

Guiding Principle 6:
Responsive environments engage learners.

Meaningful learning happens in environments where creativity, awareness, inquiry, and critical thinking are part of instruction. Responsive learning environments adapt to the individual needs of each student and encourage learning by promoting collaboration rather than isolation of learners. Learning environments, whether classrooms, schools, or other systems, should be structured to promote engaged teaching and learning.

Research Summary

To be effective for all students, classroom learning environments must be responsive to a broad range of needs among a diverse student population. These diverse needs include cultural and linguistic differences as well as developmental levels, academic readiness, and learning styles. A responsive learning environment engages all students by providing a respectful climate where instruction and curriculum are designed to respond to the backgrounds and needs of every student.

Culturally Responsive Teaching

Research on culturally responsive teaching emphasizes the importance of teachers' understanding the cultural characteristics and contributions of various ethnic groups (Smith, 1998) and showing respect toward these students and their culture (Ladson-Billings, 1995; Pewewardy & Cahape, 2003). Culturally responsive teaching is defined by Gay (2002) as "using the cultural characteristics, experiences, and perspectives of ethnically diverse students as conduits for teaching them more effectively" (p. 106).

Research on culturally responsive teaching has found that students both are more engaged in learning and learn more effectively when the knowledge and skills taught are presented within a context of their experience and cultural frames of references (Au & Kawakami, 1994; Gay, 2000; Ladson-Billings, 1995). Areas considered part of creating a culturally responsive learning environments are (1) understanding the cultural lifestyles of their students, such as which ethnic groups give priority to communal living and problem solving; (2) knowing differences in the modes of interaction between children and adults in different ethnic groups; and (3) becoming aware of cultural implications of gender role socialization among different groups (Banks & Banks, 2001). To provide a culturally responsive learning environment teachers need to:

- Communicate high expectations for all students (Gay, 2000; Hollins & Oliver, 1999; Ladson-Billings, 1994, Nieto, 1999).
- Use active teaching methods and act as learning facilitators (Banks & Banks, 2001; Gay, 2000).
- Maintain positive perspectives on families of diverse students (Delgado-Gaitin & Trueba, 1991).
- Gain knowledge of cultures of the students in their classrooms (Banks & Banks, 2001; Nieto, 1999).
- Reshape the curriculum to include culturally diverse topics (Banks & Banks, 2001; Gay, 2000; Hilliard, 1991).
- Use culturally sensitive instruction that includes student-controlled discussion and small-group work (Banks & Banks, 2001; Nieto, 1999).

Further research asserts that culturally responsive teachers help students understand that knowledge is not absolute and neutral but has moral and political elements. This knowledge can help students from diverse groups view learning as empowering (Ladson-Billings, 1995; Tharp & Gallimore, 1988).

Strategies for designing curriculum and instruction for culturally diverse students are similar to the strategies for differentiating curriculum and instruction. In fact, Mulroy and Eddinger (2003) point out that the research on differentiation emerged, in part, because of the demand on schools to serve an increasingly diverse student population. Heacox (2002) asserts that classrooms are diverse in cognitive abilities, learning styles, socioeconomic factors, readiness, learning pace, and gender and cultural influences.

Differentiation

Research on differentiation includes meeting the learning needs of all students through modifying instruction and curriculum to consider developmental level, academic readiness, and socioeconomic backgrounds, as well as cultural and linguistic differences. Tomlinson (2005) defines differentiated instruction as a philosophy of teaching based on the premise that students learn best when their teachers accommodate the difference in their readiness levels, interests, and learning profiles. In a differentiated learning environment, each student is valued for his or her unique strengths while being offered opportunities to learn and demonstrate learning through a variety of strategies (Mulroy & Eddinger, 2003). Hall (2002) states, "To differentiate instruction is to recognize students' varying backgrounds, readiness, language, learning preferences, and interests and to react responsively" (p. 1).

According to Tomlinson (2005), who has written extensively on differentiation, three elements guide differentiated instruction: content, process, and product. *Content* means that all students are given access to the same content but are allowed to master it in different ways. Process refers to the ways in which the content is taught. *Product* refers to how students demonstrate understanding. Corley (2005) provides three questions that drive differentiation: (1) What do you want the student to know? (2) How can each student best learn this? and (3) How can each student most effectively demonstrate learning? Maker (1986) offers a framework through which differentiation can occur in the classroom:

- Create an encouraging and engaging learning environment through student-centered activities, encouraging independent learning, accepting student contributions, using a rich variety of resources, and providing mobility and flexibility in grouping.

- Modify the content according to abstractness and complexity. Provide a variety of content and particularly content focused on people.

- Modify the learning process through use of inquiry, higher-order thinking activities, group interactions, variable pacing, creativity and student risk-taking, and freedom of choice in learning activities.

- Modify the product through facilitating different ways for students to demonstrate learning, such as the use of authentic assessments.

In addition, researchers have found that the use of flexible grouping and tiered instruction for differentiation increases student achievement (Corley, 2005; Tomlinson & Eidson, 2003). Heacox (2002) describes differentiation as follows:

The focus is not on the adjustment of the students, but rather the adjustment of teaching and instructional strategies making it about learning, not teaching. The teacher is the facilitator who…puts students at the center of teaching and learning and lets his or her students' learning needs direct instructional planning (p. 1).

Several studies conducted in elementary and middle school classroom have found that student achievement is increased in differentiated classrooms (Connor, Morrison, & Katch 2004; McAdamis, 2001). Tomlinson and Eidson (2003) emphasize the need to include the components of student readiness, student interest, and student learning profile in differentiating instruction. Students' interests and learning profiles are often tied to their learning styles.

Learning Styles

The body of research on learning styles has coalesced around the work of Howard Gardner, who introduced the theory of multiple intelligences in 1983. Gardner's work suggests that the concept of a pure intelligence that can be measured by a single I.Q. score is flawed, and he has identified nine intelligences that people possess to various degrees. His theory asserts that a person's type of intelligence determines how he or she learns best (Gardner, 1999).

Learning style refers to how a student learns, and the concept takes into account cultural background and social and economic factors as well as multiple intelligences. Beishuizen and Stoutjesdjik (1999) define *learning style* as a consistent mode of acquiring knowledge through study, or experience. Research has shown that the quality of learning at all levels of education (primary, secondary, and higher education) is enhanced when instruction and curriculum take into account individual learning styles (Dunn, Griggs, Olsen, Beasley & Gorman, 1995). Another study found that student learning improved when the learning environment was modified to allow students to construct personally relevant knowledge and to engage in the materials at different levels and from different points of view (Dearing, 1997).

A responsive classroom environment considers the individual learning needs of all students. These learning needs include a variety of factors that influence how students learn: culture, language, developmental level, readiness, social and economic background, and learning style.

Creativity

Creativity is an essential component for creating an engaging and accessible classroom environment. The Wisconsin Task Force on Arts and Creativity in Education (2009) defines *creativity* as a process that combines "imagination, creativity, and innovation to produce something novel that has value" (p. 14). Sir Ken Robinson (2011) and Daniel Pink (2006) both support the need for schools to focus on creating classroom that foster this type of creativity in students. According to Robinson (2011), classrooms that foster creativity and allow students to question assumptions, look at content through various lenses, and create new understandings can help students be more successful in postsecondary education and the workplace.

Probing Questions

- Describe two or three ways you might differentiate the instruction in your classroom. How might you share this with a new teacher?
- How might you implement a simple strategy for assessing your students' learning styles?

Resources

ASCD offers a number of resources on differentiated instruction, including work by Carol Ann Tomlinson, at http://www.ascd.org.

For resources on culturally responsive teaching, the Center for Culturally Responsive Teaching and Learning can be accessed at http://www.culturallyresponsive.org/.

The website of the National Center for Culturally Responsive Education Systems (NCCREST) can be accessed at http://www.nccrest.org.

For learning styles and resources on multiple intelligences, Thomas Armstrong hosts a website with information on Gardner's Theory of Multiple Intelligences and related teaching resources at http://www.thomasarmstrong.com/multiple_intelligences.php.

Creativity: Its Place in Education is a report that offers suggestions for creative classrooms and teaching. This report can be found at http://www.jpb.com/creative/Creativity_in_Education.pdf.

The report of the Wisconsin Task Force on Arts and Creativity in Education offers recommendations for policy and practice. This report can be found at ftp://doaftp04.doa.state.wi.us/doadocs/taskforce_report_final2009pdf.

References

Au, K. H., & Kawakami, A. J. (1994). Cultural congruence in instruction. In E. R. Hollins, J. E. King, & W. C. Hayman (Eds.), *Teaching diverse populations: Formulating a knowledge base* (p. 5–23). Albany: State University of New York Press.

Banks, J. A., & Banks, C. A. (2001). *Multicultural education: Issues and perspectives* (4th ed.). New York: Wiley.

Beishuizen, J. J., & Stoutjesdjik, E. T. (1999). Study strategies in a computer assisted study environment. *Learning and Instruction*, 9(3), 281–301.

Connor, C. M., Morrison, F. J., & Katch, L. E. (2004). Beyond the reading wars: Exploring the effect of child-instruction interactions on growth in early reading. *Scientific Studies of Reading*, 8(2), 305–336.

Corley, M. (2005). Differentiated instruction: Adjusting to the needs of all learners. *Focus on Basics: Connecting Research and Practice*, 7(C), 13–16.

Dearing, R. (1997). *Higher education in the learning society: Report of the National Committee*. London: HMSO.

Delgado-Gaitan, C., & Trueba, H. (1991). *Crossing cultural borders: Education for immigrant families in America*. London: Falmer.

Dunn, R., Griggs, S., Olsen, J., Beasley, M., & Gorman, B. (1995). A meta-analytic validation of the Dunn and Dunn model of learning-style preferences. *Journal of Educational Research*, 88(6), 353–362.

Gardner, H. (1999). *Intelligence reframed: Multiple intelligences for the 21st century*. New York: Basic Books.

Gay, G. (2000). *Culturally responsive teaching: Theory, research, and practice*. New York: Teachers College Press.

Gay, G. (2002). Preparing for culturally responsive teaching. *Journal of Teacher Education*, 53(2) 106–116.

Heacox, D. (2002). *Differentiating instruction in the regular classroom: How to reach and teach all learners, Grades 3–12*. Minneapolis, MN: Free Spirit.

Hilliard, A. G., III. (1991). Why we must pluralize the curriculum. *Educational Leadership*, 49(4), 12–16.

Hollins, E. R., & Oliver, E. I. (1999). *Pathways to success in school: Culturally responsive teaching*. Mahwah, NJ: Erlbaum.

Ladson-Billings, G. (1994). *The dreamkeepers: Successful teachers of African American children*. San Francisco: Jossey-Bass.

Ladson-Billings, G. (1995). Toward a theory of culturally relevant pedagogy. American *Educational Research Journal*, 32(3), 465–491.

Maker, C. J. (1986). *Critical issues in gifted education: Defensible programs for the gifted.* Rockville, MD: Aspen.

McAdamis, S. (2001). Teachers tailor their instruction to meet a variety of student needs. *Journal of Staff Development*, 22(2), 1–5.

Mulroy, H., & Eddinger, K. (2003, June). *Differentiation and literacy*. Paper presented at the Institute on Inclusive Education, Nazareth College of Rochester, Rochester, NY.

Nieto, S. (1999). *The light in their eyes: Creating multicultural learning opportunities*. New York: Teachers College Press.

Pewewardy, C. H., & Cahape, P. (2003). Culturally responsive teaching for American Indian students. *ERIC Digest*. Retrieved June 3, 2011, from http://www.ericdigests.org/2005-1/teaching.htm

Pink, D. H. (2006). *A whole new mind: Why right-brainers will rule the future*. New York: Riverhead.

Robinson, Ken. (2011). *Out of our minds: Learning to be creative*. West Sussex, United Kingdom: Capstone.

Smith, G. P. (1998). *Common sense about common knowledge*: The knowledge bases for diversity. Washington, DC: American Association of Colleges for Teacher Education.

Tharp, R. G., & Gallimore, R. (1988). *Rousing minds to life: Teaching, learning, and schooling in social context*. Cambridge: England: Cambridge University Press.

Tomlinson, C. A. (2005). Grading and differentiation: Paradox or good practice? *Theory Into Practice*, 44(3) 262–269.

Tomlinson, C. A., & Eidson, C. C. (2003). *Differentiation in practice: A resource guide for differentiating curriculum. Grades 5–9*. Alexandria, VA: Association for Supervision and Curriculum Development.

Wisconsin Task Force on Arts and Creativity in Education. (2009). *A plan for action*. Madison: Wisconsin Department of Public Instruction. Retrieved June 3, 2011, from ftp://doaftp04.doa.state.wi.us/doadocs/taskforce_report_final2009.pdf