WISCONSIN STANDARDS for

Mathematics

Wisconsin Department of Public Instruction

This publication is available from:

Wisconsin Department of Public Instruction
125 South Webster Street
Madison, WI 53703
http://dpi.wi.gov

Printed on recycled paper

Table of Contents

** Separate page numbering is used for this section*

SECTION I

Wisconsin's Approach to Academic Standards

Foreword

On June 2, 2010, I formally adopted the Common Core State Standards for Mathematics and English Language Arts, including the Literacy in History/Social Studies, Science, and the Technical Subjects for Wisconsin.

The adoption of the Common Core State Standards capped a one year effort led by the Council of Chief State School Officers (CCSSO) and the National Governors Association Center for Best Practices (NGA) to define K-12 academic standards that are aligned with college and work expectations, inclusive of rigorous content and application, and are internationally benchmarked. Staff from state departments of education reviewed and provided feedback on early drafts leading to a public comment period for citizens and educators. As of June 2011, 42 states have adopted the Common Core State Standards in this voluntary effort to bring academic consistency across the states.

Adoption of the standards, however, is the easy task. Implementing them through engaging instruction coupled with rigorous learning activities and assessment is the hard work. I applaud the efforts that are underway at the DPI, local school districts, Cooperative Educational Service Agencies (CESAs), professional organizations, and colleges and universities to bring the Common Core State Standards to teachers across Wisconsin.

The first step to implementation requires that teachers know and understand the Common Core State Standards. This document provides guidance on the relationship between the Common Core State Standards and our vision of Every Child a Graduate, supporting all students through Response to Intervention, and the responsibility that all teachers have for developing reading, writing, thinking, speaking, and listening skills.

One of the most distinguishing features of the Common Core State Standards is the emphasis directed to literacy in all of the disciplines. For students to be career and college ready, they must be proficient in reading and writing complex informational and technical text. This means that instruction in every classroom focuses on both the content and the reading and writing skills that students need to demonstrate learning in the discipline.

To support and ensure implementation, we will partner with school districts, universities, professional organizations, CESAs, and CCSSO to develop curriculum resources and highlight effective practices. Wisconsin educators are the best, both in their content knowledge and commitment to high-quality instruction. Combining helpful resources with effective practices used by quality educators leads to success for Wisconsin students.

Tony Evers, PhD
State Superintendent

"The adoption of Common Core State Standards defines K-12 academic standards that are aligned with college and work expectations, inclusive of rigorous content and application."

Acknowledgements

A special thanks to the Council of Chief State School Officers and the National Governors Association for having the vision to undertake the massive state-led project, the Common Core State Standards.

Thanks to Great Lakes West Comprehensive Center and Director Linda Miller for the generous support of Wisconsin's standards projects, and to Rachel Trimble and Beth Ratway for their guidance during the last year.

Thanks also to the CESA Statewide Network and Commissioner Jesse Harness for partnering to keep the CCSS message consistent statewide, and to the CESA School Improvement Specialists Network for their role in producing and providing high quality professional development statewide.

Also thanks to the many staff members across divisions and teams at DPI who have collaboratively contributed their time and talent to this project.

Finally, a special thanks to Wisconsin educators and citizens who provided public comment and feedback to drafts of the Common Core State Standards, served on statewide standards leadership groups, and supported implementation of standards.

Purpose of the Document

To assist Wisconsin education stakeholders in understanding and implementing the **Common Core State Standards (CCSS),** Wisconsin Department of Public Instruction (DPI) has developed guidance to be used along with the CCSS. These materials are intended to provide further direction and should not be viewed as administrative rule. This publication provides a vision for student success, guiding principles for teaching and learning, and locates the standards within a multi-level system of support where high quality instruction, balanced assessment, and collaboration function together for student learning. Information on the design and content of the CCSS is included, as is a guide to assist with facilitating local conversations about these internationally-benchmarked standards and how they impact instruction.

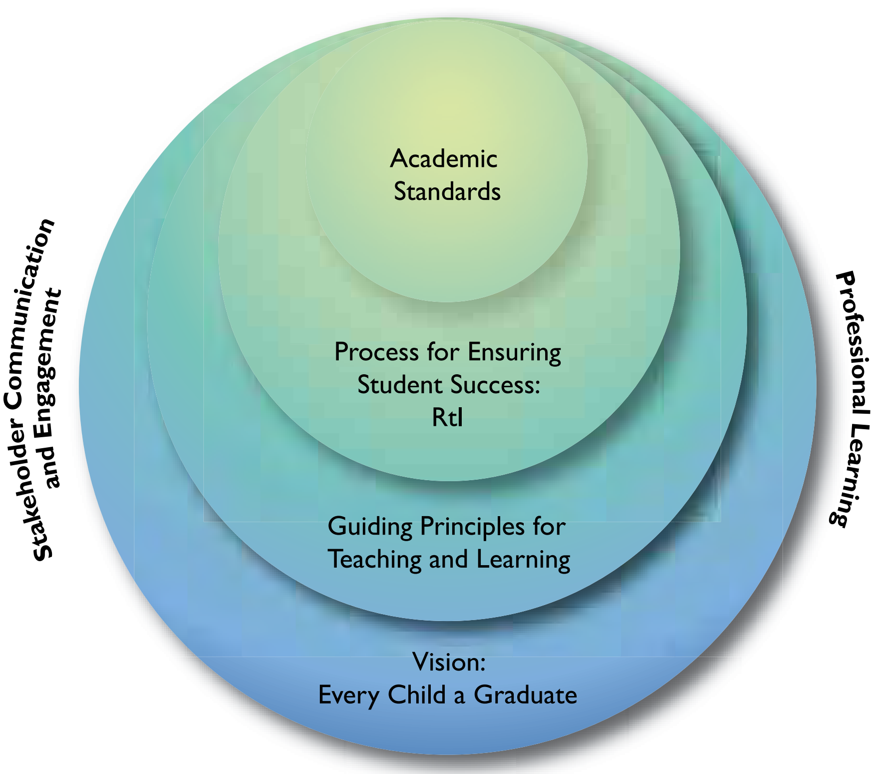
Aligning for Student Success

To build and sustain schools that support every student in achieving success, educators must work together with families, community members, and business partners to connect the most promising practices in the most meaningful contexts. Major statewide initiatives focus on high school graduation, Response to Intervention (RtI), and the *Common Core State Standards for English Language Arts, Disciplinary Literacy, and Mathematics*. While these are often viewed as separate efforts or initiatives, each of them is connected to a larger vision of every child graduating college and career ready. The graphic below illustrates how these initiatives function together for a common purpose. Here, the vision and set of guiding principles form the foundation for building a supportive process for teaching and learning rigorous and relevant content. The following sections articulate this integrated approach to increasing student success in Wisconsin schools and communities.

A Vision: Every Child a Graduate

In Wisconsin, we are committed to ensuring every child is a graduate who has successfully completed a rigorous, meaningful, 21st century education that will prepare him or her for careers, college and citizenship. Though our public education system continues to earn nation-leading graduation rates, a fact we can be proud of, one in ten students drop out of school, achievement gaps are too large, and overall achievement could be even higher. This vision for every child a graduate guides our beliefs and approaches to education in Wisconsin.

Relationship Between Vision, Principles, Process, Content



Guided By Principles

All educational initiatives are guided and impacted by important and often unstated attitudes or principles for teaching and learning. *The Guiding Principles for Teaching and Learning* emerge from research and provide the touchstone for practices that truly affect the vision of every child a graduate prepared for college and career. When made transparent, these principles inform what happens in the classroom, the implementation and evaluation of programs, and most important, remind us of our own beliefs and expectations for students.

"Educators must work together with families, community members, and business partners to connect the most promising practices in the most meaningful contexts."

Ensuring a Process for Student Success

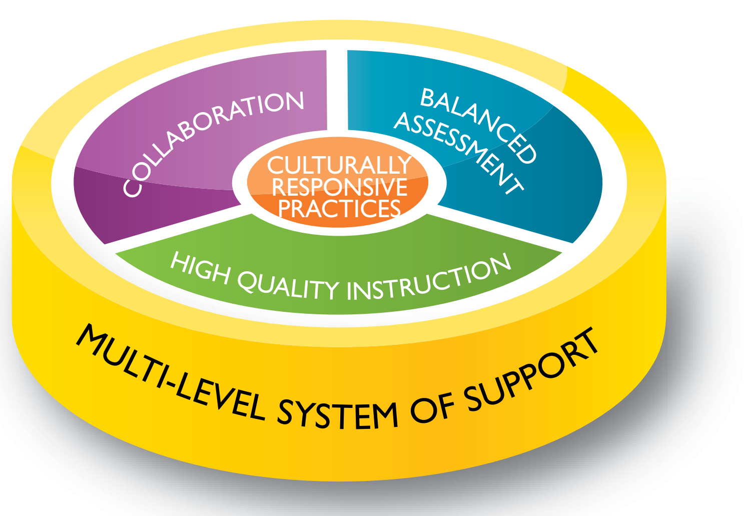
To ensure that every child in Wisconsin graduates prepared for college and career, schools need to provide high quality instruction, balanced assessment, and collaboration reflective of culturally responsive practices. The Wisconsin Response to Intervention (RtI) framework helps to organize the components of a system designed to support student learning. Below, the three essential elements of high quality instruction, balanced assessment and collaboration interact within a multi-level system of support to ensure each student receives what he or she needs to access higher levels of academic and behavioral success.

At the school or district level, programs, initiatives, and practices related to high quality instruction, balanced assessment and collaboration can be more powerful when organized or braided to function systemically to support all students. The focus must be on a comprehensive approach to student learning.

Connecting to Content: The Common Core State Standards

Within this vision for increased student success, rigorous, internationally-benchmarked academic standards provide the content for high quality curriculum and instruction, and for a balanced assessment system aligned to those standards. With the adoption of the CCSS, Wisconsin has the tools to build world-class curriculum, instruction, and assessments for greater student learning. The CCSS articulate what we teach so that educators can focus on how instruction can best meet the needs of each student. When implemented within a multi-level system of support, the CCSS can help to ensure that every child will graduate prepared for college, work, and a meaningful life.

Wisconsin's Vision for RtI



Guiding Principles for Teaching and Learning

These guiding principles are the underpinnings of effective teaching and learning for every Wisconsin teacher and every Wisconsin student. They are larger than any one initiative, process or set of standards. Rather, they are the lens we look through as we identify teaching and learning standards, design assessments and determine what good instruction looks like. These principles recognize that every student has the right to learn and are built upon three essential elements: high quality instruction, balanced assessment, and collaboration. They are meant to align with academic excellence, rigorous instruction, and college and career readiness for every Wisconsin student. For additional research, resources and probing questions to support professional learning on the six principles, please see the Wisconsin Research and Resources section of this document.

Every student has the right to learn.

It is our collective responsibility as an education community to make certain each child receives a high-quality, challenging education designed to maximize potential, an education that reflects and stretches his or her abilities and interests. This belief in the right of every child to learn forms the basis of equitable teaching and learning. The five principles that follow cannot exist without this commitment guiding our work.

Instruction must be rigorous and relevant.

To understand the world in which we live, there are certain things we all must learn. Each school subject is made up of a core of essential knowledge that is deep, rich, and vital. Every student, regardless of age or ability, must be taught this essential knowledge. What students learn is fundamentally connected to how they learn, and successful instruction blends the content of a discipline with processes of an engaging learning environment that changes to meet the dynamic needs of all students.

Purposeful assessment drives instruction and affects learning.

Assessment is an integral part of teaching and learning. Purposeful assessment practices help teachers and students understand where they have been, where they are, and where they might go next. No one assessment can provide sufficient information to plan teaching and learning. Using different types of assessments as part of instruction results in useful information about student understanding and progress. Educators should use this information to guide their own practice and in partnership with students and their families to reflect on learning and set future goals.

Learning is a collaborative responsibility.

Teaching and learning are both collaborative processes. Collaboration benefits teaching and learning when it occurs on several levels: when students, teachers, family members, and the community collectively prioritize education and engage in activities that support local schools, educators, and students; when educators collaborate with their colleagues to support innovative classroom practices and set high expectations for themselves and their students; and when students are given opportunities to work together toward academic goals in ways that enhance learning.

Students bring strengths and experiences to learning.

Every student learns. Although no two students come to school with the same culture, learning strengths, background knowledge, or experiences, and no two students learn in exactly the same way, every student's unique personal history enriches classrooms, schools, and the community. This diversity is our greatest education asset.

Responsive environments engage learners.

Meaningful learning happens in environments where creativity, awareness, inquiry, and critical thinking are part of instruction. Responsive learning environments adapt to the individual needs of each student and encourage learning by promoting collaboration rather than isolation of learners. Learning environments, whether classrooms, schools, or other systems, should be structured to promote engaged teaching and learning.

Reaching Every Student

The CCSS set high, clear and consistent expectations for all students. In order to ensure that all students can meet and exceed those expectations, Wisconsin educators provide flexible and fluid support based on student need. Each student brings a complex system of strengths and experiences to learning. One student may have gifts and talents in mathematics and need additional support to reach grade-level standards in reading. A student may be learning English as a second language while remaining identified for gifted services in science. The following statements provide guidance for how to ensure that the CCSS provide the foundation for learning for every student in Wisconsin, regardless of their unique learning needs.

Application of Common Core State Standards for English Language Learners

The National Governors Association Center for Best Practices and the Council of Chief State School Officers strongly believe that all students should be held to the same high expectations outlined in the Common Core State Standards. This includes students who are English language learners (ELLs). However, these students may require additional time, appropriate instructional support, and aligned assessments as they acquire both English language proficiency and content area knowledge.

ELLs are a heterogeneous group with differences in ethnic background, first language, socioeconomic status, quality of prior schooling, and levels of English language proficiency. Effectively educating these students requires pre-assessing each student instructionally, adjusting instruction accordingly, and closely monitoring student progress. For example, ELLs who are literate in a first language that shares cognates with English can apply first-language vocabulary knowledge when reading in English; likewise ELLs with high levels of schooling can often bring to bear conceptual knowledge developed in their first language when reading in English. However, ELLs with limited or interrupted schooling will need to acquire background knowledge prerequisite to educational tasks at hand. Additionally, the development of native-like proficiency in English takes many years and may not be achieved by all ELLs especially if they start schooling in the US in the later grades. Teachers should recognize that it is possible to achieve the standards for reading and literature, writing and research, language development and speaking and listening without manifesting native-like control of conventions and vocabulary.

English Language Arts

The Common Core State Standards for English Language Arts (ELA) articulate rigorous grade-level expectations in the areas of reading, writing, speaking, listening to prepare all students to be college and career ready, including English language learners. Second-language learners also will benefit from instruction about how to negotiate situations outside of those settings so they are able to participate on equal footing with native speakers in all aspects of social, economic, and civic endeavors.

ELLs bring with them many resources that enhance their education and can serve as resources for schools and society. Many ELLs have first language and literacy knowledge and skills that boost their acquisition of language and literacy in a second language; additionally, they bring an array of talents and cultural practices and perspectives that enrich our schools and society. Teachers must build on this enormous reservoir of talent and provide those students who need it with additional time and appropriate instructional support. This includes language proficiency standards that teachers can use in conjunction with the ELA standards to assist ELLs in becoming proficient and literate in English. To help ELLs meet high academic standards in language arts it is essential that they have access to:

- Teachers and personnel at the school and district levels who are well prepared and qualified to support ELLs while taking advantage of the many strengths and skills they bring to the classroom;
- Literacy-rich school environments where students are immersed in a variety of language experiences;
- Instruction that develops foundational skills in English and enables ELLs to participate fully in grade-level coursework;
- Coursework that prepares ELLs for postsecondary education or the workplace, yet is made comprehensible for students learning content in a second language (through specific pedagogical techniques and additional resources);
- Opportunities for classroom discourse and interaction that are well-designed to enable ELLs to develop communicative strengths in language arts;
- Ongoing assessment and feedback to guide learning; and
- Speakers of English who know the language well enough to provide ELLs with models and support.

Application to Students with Disabilities

The Common Core State Standards articulate rigorous grade-level expectations in the areas of mathematics and English language arts. These standards identify the knowledge and skills students need in order to be successful in college and careers.

Students with disabilities, students eligible under the Individuals with Disabilities Education Act (IDEA), must be challenged to excel within the general curriculum and be prepared for success in their post-school lives, including college and/or careers. These common standards provide an historic opportunity to improve access to rigorous academic content standards for students with disabilities. The continued development of understanding about research-based instructional practices and a focus on their effective implementation will help improve access to mathematics and English language arts (ELA) standards for all students, including those with disabilities. Students with disabilities are a heterogeneous group with one common characteristic: the presence of disabling conditions that significantly hinder their abilities to benefit from general education (IDEA 34 CFR §300.39, 2004). Therefore, how these high standards are taught and assessed is of the utmost importance in reaching this diverse group of students.

In order for students with disabilities to meet high academic standards and to fully demonstrate their conceptual and procedural knowledge and skills in mathematics, reading, writing, speaking and listening (English language arts), their instruction must incorporate supports and accommodations, including:

- Supports and related services designed to meet the unique needs of these students and to enable their access to the general education curriculum (IDEA 34 CFR §300.34, 2004).
- An Individualized Education Program (IEP) which includes annual goals aligned with and chosen to facilitate their attainment of grade-level academic standards.
- Teachers and specialized instructional support personnel who are prepared and qualified to deliver high-quality, evidence-based, individualized instruction and support services.

Promoting a culture of high expectations for all students is a fundamental goal of the Common Core State Standards. In order to participate with success in the general curriculum, students with disabilities, as appropriate, may be provided additional supports and services, such as:

- Instructional supports for learning, based on the principles of Universal Design for Learning (UDL), which foster student engagement by presenting information in multiple ways and allowing for diverse avenues of action and expression.

- Instructional accommodations (Thompson, Morse, Sharpe & Hall, 2005), changes in materials or procedures, which do not change the standards but allow students to learn within the framework of the Common Core.

- Assistive technology devices and services to ensure access to the general education curriculum and the Common Core State Standards.

Some students with the most significant cognitive disabilities will require substantial supports and accommodations to have meaningful access to certain standards in both instruction and assessment, based on their communication and academic needs. These supports and accommodations should ensure that students receive access to multiple means of learning and opportunities to demonstrate knowledge, but retain the rigor and high expectations of the Common Core State Standards.

Implications for the Common Core State Standards for Students with Gifts and Talents

The CCSS provide a roadmap for what students need to learn by benchmarking expectations across grade levels. They include rigorous content and application of knowledge through higher-order skills. As such, they can serve as a foundation for a robust core curriculum, however, students with gifts and talents may need additional challenges or curricular options. In order to recognize what adaptations need to be made or what interventions need to be employed, we must understand who these students are.

According to the National Association for Gifted Children (2011), "Giftedness, intelligence, and talent are fluid concepts and may look different in different contexts and cultures" (para. 1). This means that there are students that demonstrate high performance or have the potential to do so in academics, creativity, leadership, and/or the visual and performing arts. Despite this diversity there are common characteristics that are important to note.

Students with gifts and talents:

- Learn at a fast pace.

- Are stimulated by depth and complexity of content.

- Make connections.

These traits have implications for how the Common Core State Standards are used. They reveal that as curriculum is designed and instruction, is planned there must be:

- Differentiation based on student readiness, interest, and learning style:

  – Pre-assessing in order to know where a student stands in relation to the content that will be taught (readiness), then teach those standards that the student has not mastered and enrich, compact, and/or accelerate when standards have been mastered. This might mean using standards that are beyond the grade level of the student.

– Knowledge of our students so we are familiar with their strengths, background knowledge, experiences, interests, and learning styles.

– Flexible grouping to provide opportunities for students to interact with peers that have similar abilities, similar interests, and similar learning styles (homogenous grouping), as well as different abilities, different interests, and different learning styles (heterogeneous grouping).

- Differentiation of content, process, and product.
  - Use of a variety of materials (differentiating content) to provide challenge. Students may be studying the same concept using different text and resources.
  - Variety of tasks (differentiating process). For example in a science lesson about the relationship between temperature and rate of melting, some students may use computer-enhanced thermometers to record and graph temperature so they can concentrate on detecting patterns while other students may graph temperature at one-minute intervals, then examine the graph for patterns.
  - Variety of ways to demonstrate their learning (differentiating product). These choices can provide opportunities for students with varying abilities, interests, and learning styles to show what they have discovered.
- Adjustment to the level, depth, and pace of curriculum.
  - Compact the curriculum to intensify the pace.

- Vary questioning and use creative and critical thinking strategies to provide depth.
- Use standards beyond the grade level of the students. Since the CCSS provide a K-12 learning progression, this is easily done.
- Accelerate subject areas or whole grades when appropriate.

• Match the intensity of the intervention with the student's needs. This means that we must be prepared to adapt the core curriculum and plan for a continuum of services to meet the needs of all students, including those with gifts and talents.

References

Individuals with Disabilities Education Act (IDEA), 34 CFR §300.34 (a). (2004).

Individuals with Disabilities Education Act (IDEA), 34 CFR §300.39 (b)(3). (2004).

National Association for Gifted Children (2010). Redefining Giftedness for a New Century Shifting the Paradigm. Retrieved from http://www.nagc.org/index.aspx?id=6404.

National Association for Gifted Children (2011). What is giftedness?

Retrieved from http://nagc.org/index.aspx?id=574.

Sousa, D.A. (200). How the gifted brain learns. Thousand Oaks, CA: Corwin Press.

Thompson, Sandra J., Amanda B. Morse, Michael Sharpe, and Sharon Hall. "Accommodations Manual: How to Select, Administer and Evaluate Use of Accommodations and Assessment for Students with Disabilities," 2nd Edition. Council for Chief State School Officers, 2005 http://www.ccsso.org/content/pdfs/AccommodationsManual.pdf . (Accessed January, 29, 2010).

Reaching Every Discipline: Wisconsin's Approach to Disciplinary Literacy

In Wisconsin, disciplinary literacy is defined as the confluence of content knowledge, experiences, and skills merged with the ability to read, write, listen, speak, think critically and perform in a way that is meaningful within the context of a given field.

Background

In Wisconsin, we hold the vision that every child must graduate ready for post-secondary education and the workforce. To achieve this vision, students must develop the skills to think, read, communicate, and perform in many academic contexts. If students must develop these specific skills, every educator must then consider how students learn to read, write, think, speak and listen in their discipline.

The kinds of reading, writing, thinking, speaking and listening required in a marketing course are quite different when compared with the same processes applied in an agriculture, art or history course. For example, a student may have successfully learned the vocabulary and content needed to score an A on a freshman biology test, but finds he still struggles to understand relevant articles from *Popular Science Magazine*, or use his science vocabulary to post respected responses on an environmental blog he reads at home. This student knows biology content, but lacks the disciplinary literacy to think, read, write, and speak with others in this field. Without this ability, his content knowledge is limited only to the classroom, and cannot extend to the real world around him.

Teaching for disciplinary literacy ensures that students develop the skills to use the deep content knowledge they learn in school in ways that are relevant to each of them, and to the world around them.

In 2009, *The State Superintendent's Adolescent Literacy Plan* offered recommendations for how to begin professional conversations about disciplinary literacy in Wisconsin. The plan recommended Wisconsin write standards for literacy that were specific to each discipline, and emphasized the need to accompany these literacy standards with discipline-specific professional learning.

Wisconsin's Approach to Disciplinary Literacy

In 2010, the Council of Chief State School Officers (CCSSO) responded to this need for standards by publishing Common Core State Standards for Literacy in History/Social Studies, Science and Technical Subjects in grades 6-12. These standards were adopted by State Superintendent Tony Evers in June 2010. Wisconsin applauds this bold move to begin a national conversation on disciplinary literacy, and recognizes the need to broaden this effort to include all disciplines, and every educator in every grade level.

The ability to read, write, think, speak, and listen, in different ways and for different purposes begins early and becomes increasingly important as students pursue specialized fields of study in high school and beyond. These abilities are as important in mathematics, engineering and art courses as they are in science, social studies and English.

To further solidify Wisconsin's expanded approach to disciplinary literacy, a statewide leadership team comprised of K-16 educators from diverse subject areas was convened. A set of foundations, was established and directs Wisconsin's approach to disciplinary literacy.

This document begins the conversation about literacy in all subjects. It will come to life when presented to teachers and they are able to showcase their subjects' connection to literacy in all subjects which will bring the literacy standards to life for their community of learners.

Wisconsin Foundations for Disciplinary Literacy

To guide understanding and professional learning, a set of foundational statements, developed in concert with *Wisconsin's Guiding Principles for Teaching and Learning*, directs Wisconsin's approach to disciplinary literacy.

- Academic learning begins in early childhood and develops across all disciplines.
- Content knowledge is strengthened when educators integrate discipline-specific literacy into teaching and learning.

- The literacy skills of reading, writing, listening, speaking and critical thinking improve when content-rich learning experiences motivate and engage students.
- Students demonstrate their content knowledge through reading, writing, listening, and speaking as part of a content literate community.

Wisconsin's Common Core Standards for Literacy in All Subjects

With the Wisconsin Foundations for Disciplinary Literacy, Wisconsin expands the Common Core State Standards for Literacy in History/Social Studies, Science and Technical Subjects, to include every educator in every discipline and at every level. The Common Core Standards for English Language Arts include the Literacy Standards in History/Social Studies, Science and Technical Subjects as well as other relevant standards materials, resources, and research that support discipline-specific conversations across all content areas and grade levels.

The Common Core State Standards for Literacy in all Subjects is included as part of every set of Wisconsin standards as each discipline is reviewed in accordance with the process for Wisconsin standards revision http://www.dpi.wi.gov/standards. This document includes relevant resources and research that may be helpful in advancing school and district conversations, and can also be downloaded at www.dpi.wi.gov/standards or purchased as a stand-alone document through www.dpi.wi.gov/publications.

SECTION 2

Wisconsin's Approach to Mathematics

Acknowledgements

Mathematics Standards Leadership Team

Cathy Burge
Grade 2-3 Looping Teacher
Viking Elementary School
School District of Holmen

Doug Burge
K-12 Mathematics Coordinator
School District of Holmen

Sue Chmielinski
PK-12 Mathematics Coordinator
Wauwatosa School District

Ken Davis
District Mathematics Coach
School District of Beloit

Dave Ebert
Mathematics Teacher
Oregon High School

Astrid Fossum
Mathematics Teaching Specialist
Milwaukee Public Schools

Laura Godfrey
Mathematics Resource Teacher
Madison Metropolitan School District

Jodean Grunow
Senior Lecturer
Department of Mathematics
University of Wisconsin-Platteville

Sue Hanson-Otis
K-12 Mathematics Coordinator
Franklin Public Schools

DeAnn Huinker
Director
Center for Mathematics and Science Education Research
University of Wisconsin-Milwaukee

Henry Kepner
Professor
Department of Curriculum and Instruction
University of Wisconsin-Milwaukee
Past-President, National Council of Teachers of Mathematics

John Korth
Mathematics Instructor
Mid-State Technical College

Jennifer Kosiak
Associate Professor of Mathematics
University of Wisconsin-La Crosse

Henry Kranendonk
Mathematics Curriculum Specialist, Retired
Milwaukee Public Schools

Sarah Lord
Instructional Resource Teacher
Madison Metropolitan School District

Gloria Mari-Beffa
Mathematics Professor
University of Wisconsin-Madison

Kevin McLeod
Associate Professor
Department of Mathematical Sciences
University of Wisconsin-Milwaukee

Michelle Parks
Educational Consultant
Mathematics/Science
CESA 10
Chippewa Falls, Wisconsin

Mary Richards
Mathematics Coach
School District of New London

Beth Schefelker
Mathematics Teaching Specialist
Milwaukee Public Schools

Billie Earl Sparks
Professor of Mathematics, Emeritus
University of Wisconsin-Eau Claire

Mary Walz
Mathematics Teacher
Sauk Prairie High School

Lori Williams
K-12 Mathematics Program Support
Manitowoc Public School District

DPI Facilitator
Diana Kasbaum
Mathematics Consultant
Content and Learning Team

Wisconsin Foundations for Mathematics

Wisconsin's Guiding Principles for Teaching and Learning provide important guidance for the mathematics classroom. Within the discipline of mathematics, each of the six principles has specific implications for equity, pedagogy, instruction, and assessment. Mathematics educators should consider how the six guiding principles influence their teaching.

The following foundations provide direction for the teaching and learning of mathematics in Wisconsin.

Every student must have access to and engage in meaningful, challenging, and rigorous mathematics.

Equity in mathematics education requires recognition that the standards must be kept consistent while being flexible in instructional approach and methods of assessment to accommodate the strengths and weaknesses of all students. In order to optimize student learning, the high bar that is set for all should not be moved for some students; instead, the delivery system must be varied to allow access for all. Schools and classrooms need to be organized to convey the message that all students can learn mathematics and should be expected to achieve. Effective mathematics classroom practice involves assessing students' prior knowledge, designing tasks that allow flexibility of approach, and orchestrating classroom discussions that allow every student to successfully access and learn important mathematics.

When today's students become adults, they will face new demands for mathematical proficiency that school mathematics should attempt to anticipate. Moreover, mathematics is a realm no longer restricted to a select few. All young Americans must learn to think mathematically, and they must think mathematically to learn.

(Adding It Up, National Research Council, 2001).

Mathematics should be experienced as coherent, connected, intrinsically interesting, and relevant.

The PK-12 curriculum should integrate and sequence important mathematical ideas so that students can make sense of mathematics and develop a thorough understanding of concepts. The curriculum should build from grade to grade and topic to topic so that students have experiences that are coherent. The connections of mathematical ideas in a well-designed curriculum allow students to see mathematics as important in its own right, as well as a useful subject that has relevant applications to the real world and to other disciplines.

Problem solving, understanding, reasoning, and sense-making are at the heart of mathematics teaching and learning and are central to mathematical proficiency.

Using problem solving as a vehicle for teaching mathematics not only develops knowledge and skills, but also helps students understand and make sense of mathematics. By infusing reasoning and sense-making in daily mathematics instruction, students are able to see how new concepts connect with existing knowledge and they are able to solidify their understanding. Students who are mathematically proficient see that mathematics makes sense and show a willingness to persevere. They possess both understanding of mathematical concepts and fluency with procedural skills.

Effective mathematics classroom practices include the use of collaboration, discourse, and reflection to engage students in the study of important mathematics.

Collaboration and classroom discourse can significantly deepen student understanding of mathematical concepts. In addition to teacher-student dialogue, peer collaboration and individual reflection must also be emphasized. Representing, thinking, discussing, agreeing, and disagreeing are central to what students learn about mathematics. Posing questions and tasks that elicit, engage, and challenge students' thinking, as well as asking students to clarify their thinking and justify solutions and solution paths should be evident in all mathematics classrooms.

Standards for Mathematical Practice

The Standards for Mathematical Practice are central to the teaching and learning of mathematics. These practices describe the behaviors and habits of mind that are exhibited by students who are mathematically proficient. Mathematical understanding is the intersection of these practices and mathematics content. It is critical that the *Standards for Mathematical Practice* are embedded in daily mathematics instruction.

The graphic below shows the central focus on the *Standards for Mathematical Practice* within the familiar content areas of mathematics. Some of the behaviors and dispositions exhibited by students who are mathematically proficient are elaborated in the Characteristics of Mathematically Proficient Students (see pages 29-30 of this guide).

Standards for Mathematical Content

The *Standards for Mathematical Content* describe the sequence of important mathematics content that students learn. They are a combination of procedures and understandings. These content standards are organized around domains and clusters which are specified by grade level, kindergarten through grade 8, and by conceptual category at high school. The domains at all levels are based on research-based learning progressions detailing what is known about students' mathematical knowledge, skill, and understanding. The progressions build from grade to grade and topic to topic, providing K-12 focus and coherence. Other important cross-grade themes that should be noted and investigated are concepts such as the role of units and unitizing, the properties of operations across arithmetic and algebra, operations and the problems they solve, transformational geometry, reasoning and sense-making, and modeling of and with mathematics.

The **narratives at each K-8 grade level** specify 2-4 key areas that are identified as the primary focus of instruction. These are referred to as **critical areas**. At the high school level, the narratives describe the **focus** for each conceptual category, as well as the connections to other categories and domains.

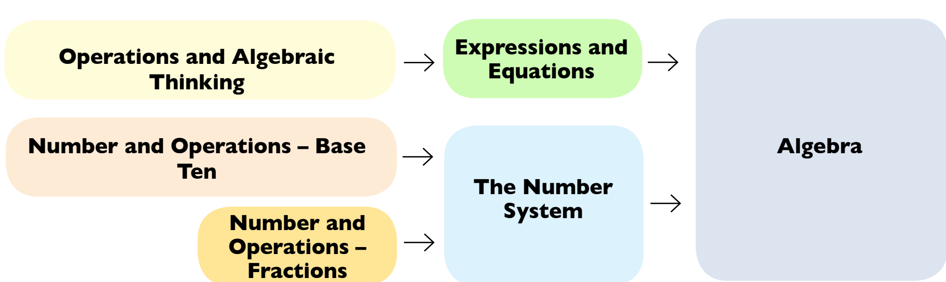
Learning mathematics with understanding is a focus of the CCSSM. Many of the *Standards for Mathematical Content* begin with the verb "understand" and are crucial for mathematical proficiency. It is generally agreed that students understand a concept in mathematics if they can use mathematical reasoning with a variety of representations and connections to explain the concept to someone else or apply the concept to another situation. This is how 'understand' should be interpreted when implementing the CCSSM.

> *One hallmark of mathematical understanding is the ability to justify, in a way appropriate to the student's mathematical maturity, why a particular mathematical statement is true or where a mathematical rule comes from... Mathematical understanding and procedural skill are equally important, and both are assessable using mathematical tasks of sufficient richness (CCSSM p. 4).*

While the *Standards for Mathematical Practice* should be addressed with all of the *Standards for Mathematical Content*, the content standards that begin with the verb "understand" are a natural intersection between the two.

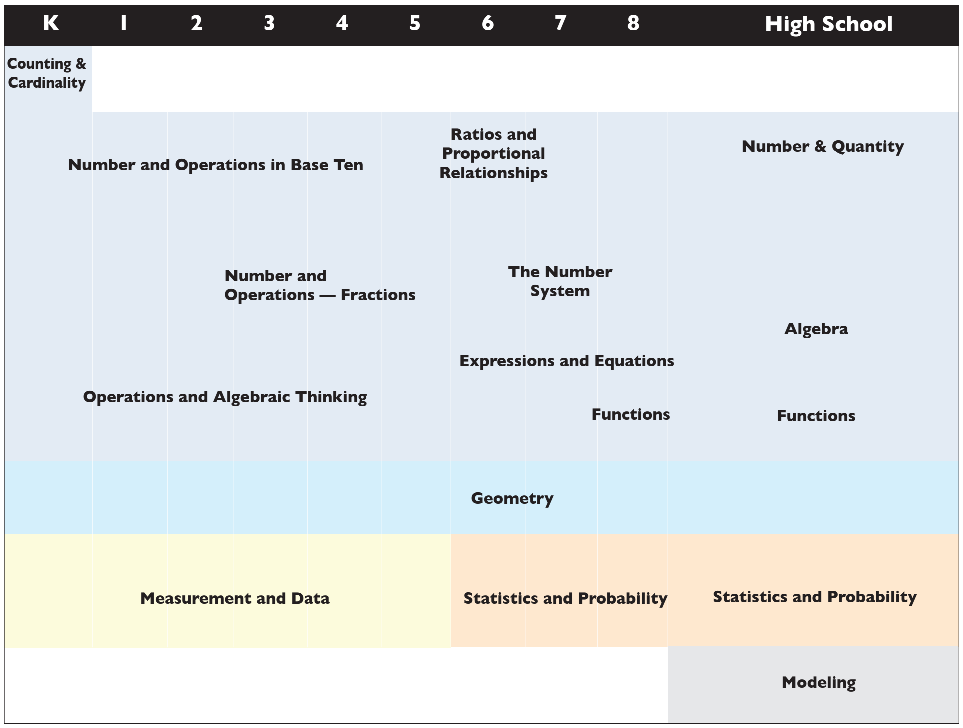
K-12 Coherence and Convergence

The *Standards for Mathematical Content* are built upon **coherence**, one of the design principles of the CCSSM. The intentional progression and sequencing of topics lays the foundation for the mathematics that is developed from kindergarten through high school. The diagram below depicts how domains at the elementary and middle school levels converge toward algebra at the high school. It is important that educators are knowledgeable about these progressions so that students learn mathematics with understanding and so that new content can build on prior learning



Focus and Organization of the Standards for Mathematical Content

The mathematics content of the CCSSM builds across grades and provides important underpinnings for the mathematics to be learned at subsequent levels. The coherence of the CCSSM lies in those connections, both within and across grade levels and topics. The graphic below illustrates the second design principle of the CCSSM – **focus**.



At the early elementary grades, the focus is largely on the areas of number and operations in base ten and algebraic thinking. This expands to a focus on fractions later in elementary school. The K-5 mathematics content provides the groundwork for the study of ratios, proportional reasoning, the number system, expressions and equations, and functions at the middle school level. By providing a focused mathematics experience in elementary and middle school, a strong foundation is developed for the content to be learned at the high school level.

Mathematical Proficiency

Mathematical proficiency is necessary for every student; therefore, understanding concepts and being fluent with procedural skills are both important. This means that educators must intentionally engage students at all levels so they are readily able to understand important concepts, use skills effectively, and apply mathematics to make sense of their changing world.

Adding it Up (National Research Council, 2001), a major research report that informed the development of the Common Core State Standards for Mathematics, emphasizes the five strands of mathematical proficiency: conceptual understanding, procedural fluency, adaptive reasoning, strategic competence, and productive disposition. These strands are not sequential, but intertwined and form the basis for the *Standards for Mathematical Content* and the *Standards for Mathematical Practice*. Together, these two sets of mathematics standards define what students should understand and be able to do in their study of K-12 mathematics.

Standards for Mathematical Practice	Characteristics of Mathematically Proficient Students*
Make sense of problems and persevere in solving them.	**Mathematically proficient students can:** Explain the meaning of a problem and restate it in their words. Analyze given information to develop possible strategies for solving the problem. Identify and execute appropriate strategies to solve the problem. Evaluate progress toward the solution and make revisions if necessary. Explain the connections among various representations of a problem or concept. Check for accuracy and reasonableness of work, strategy and solution. Understand and connect strategies used by others to solve problems.
Reason abstractly and quantitatively.	**Mathematically proficient students can:** Translate given information to create a mathematical representation for a concept. Manipulate the mathematical representation by showing the process considering the meaning of the quantities involved. Recognize the relationships between numbers/quantities within the process to evaluate a problem. Review the process for reasonableness within the original context.
Construct viable arguments and critique the reasoning of others.	**Mathematically proficient students can:** Use observations and prior knowledge (stated assumptions, definitions, and previous established results) to make conjectures and construct arguments. Compare and contrast logical arguments and identify which one makes the most sense. Justify (orally and in written form) the approach used, including how it fits in the context from which the data arose. Listen, understand, analyze, and respond to the arguments of others. Identify and explain both correct and flawed logic. Recognize and use counterexamples to refine assumptions or definitions and dispute or disprove an argument.

Standards for Mathematical Practice	Characteristics of Mathematically Proficient Students*
Model with mathematics.	**Mathematically proficient students can:** Use a variety of methods to model, represent, and solve real-world problems. Simplify a complicated problem by making assumptions and approximations. Interpret results in the context of the problem and revise the model if necessary. Choose a model that is both appropriate and efficient to arrive at one or more desired solutions.
Use appropriate tools strategically.	**Mathematically proficient students can:** Identify mathematical tools and recognize their strengths and weaknesses. Select and use appropriate tools to best model/solve problems. Use estimation to predict reasonable solutions and/or detect errors. Identify and successfully use external mathematical resources to pose or solve problems. Use a variety of technologies, including digital content, to explore, confirm, and deepen conceptual understanding.
Attend to precision.	**Mathematically proficient students can:** Understand symbols and use them consistently within the context of a problem. Calculate answers efficiently and accurately and label them appropriately. Formulate precise explanations (orally and in written form) using both mathematical representations and words. Communicate using clear mathematical definitions, vocabulary, and symbols.
Look for and make use of structure.	**Mathematically proficient students can:** Look for, identify, and accept patterns or structure within relationships. Use patterns or structure to make sense of mathematics and connect prior knowledge to similar situations and extend to novel situations. Analyze a complex problem by breaking it down into smaller parts. Reflect on the problem as a whole and shift perspective as needed.
Look for and express regularity in repeated reasoning.	**Mathematically proficient students can** Recognize similarities and patterns in repeated trials with a process. Generalize the process to create a shortcut which may lead to developing rules or creating a formula. Evaluate the reasonableness of results throughout the mathematical process while attending to the details.

** Collaborative project with Cedarburg, Franklin, Fox Point-Bayside, Grafton, Greendale, Kettle Moraine, Menomonee Falls, Oconomowoc, Pewaukee, Waukesha, and Whitefish Bay School Districts and CESA 1.*

Design Features of the Common Core State Standards for Mathematics

The design of the CCSSM has several specific features. Additional resources to support the CCSSM are available online at: http://dpi.wi.gov/standards/stds.html

- The *Standards for Mathematical Practice* must be addressed at all levels and intertwined with the *Standards for Mathematical Content*.
- K-8 grade level content standards illustrate a **coherent and rigorous curriculum** to be completed in each of these grades.
- The high school *Standards for Mathematical Content* are not by grade or course, rather they are grouped in **conceptual categories** and can be clustered in multiple ways to design courses and programs of study.
- The CCSSM are designed to provide **focus**, by identifying two to four critical areas at each K-8 grade level. These are found in the short narrative section of grades K-8, immediately before each grade level's content standards. They present the areas that should be the primary focus for instruction in that grade. Critical areas for each of the high school conceptual categories are described in the narratives.
- The CCSSM were designed to provide **coherence**, through connections and progressions both within and across grade levels. The authors of the CCSSM have developed *Progressions* documents that provide in-depth discussion of the domain progressions across grades, highlight connections across domains, elaborate on the learning expectations for students, and provide instructional suggestions.
- The CCSSM were designed to be **rigorous**, which is provided by a focus on College and Career Readiness and by emphasizing the *Standards for Mathematical Practice* across K-12. The high school CCSSM also specify additional mathematics (+ standards) that students pursuing mathematics-intensive STEM careers should accomplish.

How to use Appendix A of the Common Core State Standards for Mathematics

The *CCSSM Standards for Mathematical Content* are organized by grade level in grades K-8. A similar organization was not possible for the high school content standards, since schools and curricula do not all introduce high school content in the same order. The high school content standards are therefore organized by conceptual categories, leaving open the question of how the required content is to be distributed among high school courses. There are two commonly-used approaches: traditional/non-integrated U.S. curriculum in which content is typically divided into courses named Algebra I, Geometry, and Algebra II; and the integrated approach, more commonly used in other countries, in which the strands of mathematics are interwoven in courses which might simply be named Mathematics I, Mathematics II, and Mathematics III. The CCSSM should be fully acquired through either course sequences.

CCSSM Appendix A, *Designing High School Mathematics Courses Based on the Common Core State Standards*, provides four suggested pathways as to how this distribution might be accomplished (http://corestandards.org/assets/CCSSI_Mathematics_Appendix_A.pdf). In considering this appendix, it is important to keep in mind comments from the CCSSM authors:

> *The **pathways and courses are models, not mandates**. They illustrate possible approaches to organizing the content of the CCSS into coherent and rigorous courses that lead to college and career readiness. States and districts are not expected to adopt these courses as is; rather, they are encouraged to use these pathways and courses as a starting point for developing their own (CCSSM, Appendix A, p.2).*

SECTION 3

Common Core State Standards for Mathematics

Table of Contents

Introduction

Toward greater focus and coherence

> *Mathematics experiences in early childhood settings should concentrate on (1) number (which includes whole number, operations, and relations) and (2) geometry, spatial relations, and measurement, with more mathematics learning time devoted to number than to other topics. Mathematical process goals should be integrated in these content areas.*
>
> — Mathematics Learning in Early Childhood, National Research Council, 2009

> *The composite standards [of Hong Kong, Korea and Singapore] have a number of features that can inform an international benchmarking process for the development of K–6 mathematics standards in the U.S. First, the composite standards concentrate the early learning of mathematics on the number, measurement, and geometry strands with less emphasis on data analysis and little exposure to algebra. The Hong Kong standards for grades 1–3 devote approximately half the targeted time to numbers and almost all the time remaining to geometry and measurement.*
>
> — Ginsburg, Leinwand and Decker, 2009

> *Because the mathematics concepts in [U.S.] textbooks are often weak, the presentation becomes more mechanical than is ideal. We looked at both traditional and non-traditional textbooks used in the US and found this conceptual weakness in both.*
>
> — Ginsburg et al., 2005

> *There are many ways to organize curricula. The challenge, now rarely met, is to avoid those that distort mathematics and turn off students.*
>
> — Steen, 2007

For over a decade, research studies of mathematics education in high-performing countries have pointed to the conclusion that the mathematics curriculum in the United States must become substantially more focused and coherent in order to improve mathematics achievement in this country. To deliver on the promise of common standards, the standards must address the problem of a curriculum that is “a mile wide and an inch deep.” These Standards are a substantial answer to that challenge.

It is important to recognize that “fewer standards” are no substitute for focused standards. Achieving “fewer standards” would be easy to do by resorting to broad, general statements. Instead, these Standards aim for clarity and specificity.

Assessing the coherence of a set of standards is more difficult than assessing their focus. William Schmidt and Richard Houang (2002) have said that content standards and curricula are coherent if they are:

> *articulated over time as a sequence of topics and performances that are logical and reflect, where appropriate, the sequential or hierarchical nature of the disciplinary content from which the subject matter derives. That is, what and how students are taught should reflect not only the topics that fall within a certain academic discipline, **but also the key ideas** that determine how knowledge is organized and generated within that discipline. This implies*

that to be coherent, a set of content standards must evolve from particulars (e.g., the meaning and operations of whole numbers, including simple math facts and routine computational procedures associated with whole numbers and fractions) to deeper structures inherent in the discipline. These deeper structures then serve as a means for connecting the particulars (such as an understanding of the rational number system and its properties). (emphasis added)

These Standards endeavor to follow such a design, not only by stressing conceptual understanding of key ideas, but also by continually returning to organizing principles such as place value or the properties of operations to structure those ideas.

In addition, the "sequence of topics and performances" that is outlined in a body of mathematics standards must also respect what is known about how students learn. As Confrey (2007) points out, developing "sequenced obstacles and challenges for students…absent the insights about meaning that derive from careful study of learning, would be unfortunate and unwise." In recognition of this, the development of these Standards began with research-based learning progressions detailing what is known today about how students' mathematical knowledge, skill, and understanding develop over time.

Understanding mathematics

These Standards define what students should understand and be able to do in their study of mathematics. Asking a student to understand something means asking a teacher to assess whether the student has understood it. But what does mathematical understanding look like? One hallmark of mathematical understanding is the ability to justify, in a way appropriate to the student's mathematical maturity, *why* a particular mathematical statement is true or where a mathematical rule comes from. There is a world of difference between a student who can summon a mnemonic device to expand a product such as $(a + b)(x + y)$ and a student who can explain where the mnemonic comes from. The student who can explain the rule understands the mathematics, and may have a better chance to succeed at a less familiar task such as expanding $(a + b + c)(x + y)$. Mathematical understanding and procedural skill are equally important, and both are assessable using mathematical tasks of sufficient richness.

The Standards set grade-specific standards but do not define the intervention methods or materials necessary to support students who are well below or well above grade-level expectations. It is also beyond the scope of the Standards to define the full range of supports appropriate for English language learners and for students with special needs. At the same time, all students must have the opportunity to learn and meet the same high standards if they are to access the knowledge and skills necessary in their post-school lives. The Standards should be read as allowing for the widest possible range of students to participate fully from the outset, along with appropriate accommodations to ensure maximum participaton of students with special education needs. For example, for students with disabilities reading should allow for use of Braille, screen reader technology, or other assistive devices, while writing should include the use of a scribe, computer, or speech-to-text technology. In a similar vein, speaking and listening should be interpreted broadly to include sign language. No set of grade-specific standards can fully reflect the great variety in abilities, needs, learning rates, and achievement levels of students in any given classroom. However, the Standards do provide clear signposts along the way to the goal of college and career readiness for all students.

The Standards begin on page 6 with eight Standards for Mathematical Practice.

How to read the grade level standards

Standards define what students should understand and be able to do.

Clusters are groups of related standards. Note that standards from different clusters may sometimes be closely related, because mathematics is a connected subject.

Domains are larger groups of related standards. Standards from different domains may sometimes be closely related.

Domain

Number and Operations in Base Ten **3.NBT**

Use place value understanding and properties of operations to perform multi-digit arithmetic.

Standard

1. Use place value understanding to round whole numbers to the nearest 10 or 100.
2. Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.
3. Multiply one-digit whole numbers by multiples of 10 in the range 10-90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations.

Cluster

These Standards do not dictate curriculum or teaching methods. For example, just because topic A appears before topic B in the standards for a given grade, it does not necessarily mean that topic A must be taught before topic B. A teacher might prefer to teach topic B before topic A, or might choose to highlight connections by teaching topic A and topic B at the same time. Or, a teacher might prefer to teach a topic of his or her own choosing that leads, as a byproduct, to students reaching the standards for topics A and B.

What students can learn at any particular grade level depends upon what they have learned before. Ideally then, each standard in this document might have been phrased in the form, "Students who already know ... should next come to learn ...." But at present this approach is unrealistic—not least because existing education research cannot specify all such learning pathways. Of necessity therefore, grade placements for specific topics have been made on the basis of state and international comparisons and the collective experience and collective professional judgment of educators, researchers and mathematicians. One promise of common state standards is that over time they will allow research on learning progressions to inform and improve the design of standards to a much greater extent than is possible today. Learning opportunities will continue to vary across schools and school systems, and educators should make every effort to meet the needs of individual students based on their current understanding.

These Standards are not intended to be new names for old ways of doing business. They are a call to take the next step. It is time for states to work together to build on lessons learned from two decades of standards based reforms. It is time to recognize that standards are not just promises to our children, but promises we intend to keep.

Mathematics | Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7 Look for and make use of structure.
Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8 Look for and express regularity in repeated reasoning.
Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content
The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word “understand” are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.

In this respect, those content standards which set an expectation of understanding are potential “points of intersection” between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

Mathematics | Kindergarten

In Kindergarten, instructional time should focus on two critical areas: (1) representing, relating, and operating on whole numbers, initially with sets of objects; (2) describing shapes and space. More learning time in Kindergarten should be devoted to number than to other topics.

(1) Students use numbers, including written numerals, to represent quantities and to solve quantitative problems, such as counting objects in a set; counting out a given number of objects; comparing sets or numerals; and modeling simple joining and separating situations with sets of objects, or eventually with equations such as $5 + 2 = 7$ and $7 - 2 = 5$. (Kindergarten students should see addition and subtraction equations, and student writing of equations in kindergarten is encouraged, but it is not required.) Students choose, combine, and apply effective strategies for answering quantitative questions, including quickly recognizing the cardinalities of small sets of objects, counting and producing sets of given sizes, counting the number of objects in combined sets, or counting the number of objects that remain in a set after some are taken away.

(2) Students describe their physical world using geometric ideas (e.g., shape, orientation, spatial relations) and vocabulary. They identify, name, and describe basic two-dimensional shapes, such as squares, triangles, circles, rectangles, and hexagons, presented in a variety of ways (e.g., with different sizes and orientations), as well as three-dimensional shapes such as cubes, cones, cylinders, and spheres. They use basic shapes and spatial reasoning to model objects in their environment and to construct more complex shapes.

Grade K Overview

Counting and Cardinality

- Know number names and the count sequence.
- Count to tell the number of objects.
- Compare numbers.

Operations and Algebraic Thinking

- Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

Number and Operations in Base Ten

- Work with numbers 11–19 to gain foundations for place value.

Measurement and Data

- Describe and compare measurable attributes.
- Classify objects and count the number of objects in categories.

Geometry

- Identify and describe shapes.
- Analyze, compare, create, and compose shapes.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Counting and Cardinality K.CC

Know number names and the count sequence.

1. Count to 100 by ones and by tens.
2. Count forward beginning from a given number within the known sequence (instead of having to begin at 1).
3. Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects).

Count to tell the number of objects.

4. Understand the relationship between numbers and quantities; connect counting to cardinality.
    a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object.
    b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.
    c. Understand that each successive number name refers to a quantity that is one larger.
5. Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects.

Compare numbers.

6. Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies.[1]
7. Compare two numbers between 1 and 10 presented as written numerals.

Operations and Algebraic Thinking K.OA

Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

1. Represent addition and subtraction with objects, fingers, mental images, drawings[2], sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.
2. Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.
3. Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).
4. For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.
5. Fluently add and subtract within 5.

---

[1]Include groups with up to ten objects.
[2]Drawings need not show details, but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.)

Number and Operations in Base Ten K.NBT

Work with numbers 11–19 to gain foundations for place value.

1. Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (e.g., 18 = 10 + 8); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.

Measurement and Data K.MD

Describe and compare measurable attributes.

1. Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object.
2. Directly compare two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute, and describe the difference. *For example, directly compare the heights of two children and describe one child as taller/shorter.*

Classify objects and count the number of objects in each category.

3. Classify objects into given categories; count the numbers of objects in each category and sort the categories by count.[3]

Geometry K.G

Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).

1. Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as *above*, *below*, *beside*, *in front of*, *behind*, and *next to*.
2. Correctly name shapes regardless of their orientations or overall size.
3. Identify shapes as two-dimensional (lying in a plane, "flat") or three-dimensional ("solid").

Analyze, compare, create, and compose shapes.

4. Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/"corners") and other attributes (e.g., having sides of equal length).
5. Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes.
6. Compose simple shapes to form larger shapes. *For example, "Can you join these two triangles with full sides touching to make a rectangle?"*

[3]Limit category counts to be less than or equal to 10.

Mathematics | Grade 1

In Grade 1, instructional time should focus on four critical areas: (1) developing understanding of addition, subtraction, and strategies for addition and subtraction within 20; (2) developing understanding of whole number relationships and place value, including grouping in tens and ones; (3) developing understanding of linear measurement and measuring lengths as iterating length units; and (4) reasoning about attributes of, and composing and decomposing geometric shapes.

(1) Students develop strategies for adding and subtracting whole numbers based on their prior work with small numbers. They use a variety of models, including discrete objects and length-based models (e.g., cubes connected to form lengths), to model add-to, take-from, put-together, take-apart, and compare situations to develop meaning for the operations of addition and subtraction, and to develop strategies to solve arithmetic problems with these operations. Students understand connections between counting and addition and subtraction (e.g., adding two is the same as counting on two). They use properties of addition to add whole numbers and to create and use increasingly sophisticated strategies based on these properties (e.g., "making tens") to solve addition and subtraction problems within 20. By comparing a variety of solution strategies, children build their understanding of the relationship between addition and subtraction.

(2) Students develop, discuss, and use efficient, accurate, and generalizable methods to add within 100 and subtract multiples of 10. They compare whole numbers (at least to 100) to develop understanding of and solve problems involving their relative sizes. They think of whole numbers between 10 and 100 in terms of tens and ones (especially recognizing the numbers 11 to 19 as composed of a ten and some ones). Through activities that build number sense, they understand the order of the counting numbers and their relative magnitudes.

(3) Students develop an understanding of the meaning and processes of measurement, including underlying concepts such as iterating (the mental activity of building up the length of an object with equal-sized units) and the transitivity principle for indirect measurement.[1]

(4) Students compose and decompose plane or solid figures (e.g., put two triangles together to make a quadrilateral) and build understanding of part-whole relationships as well as the properties of the original and composite shapes. As they combine shapes, they recognize them from different perspectives and orientations, describe their geometric attributes, and determine how they are alike and different, to develop the background for measurement and for initial understandings of properties such as congruence and symmetry.

[1]Students should apply the principle of transitivity of measurement to make indirect comparisons, but they need not use this technical term.

Grade 1 Overview

Operations and Algebraic Thinking

- **Represent and solve problems involving addition and subtraction.**
- **Understand and apply properties of operations and the relationship between addition and subtraction.**
- **Add and subtract within 20.**
- **Work with addition and subtraction equations.**

Number and Operations in Base Ten

- **Extend the counting sequence.**
- **Understand place value.**
- **Use place value understanding and properties of operations to add and subtract.**

Measurement and Data

- **Measure lengths indirectly and by iterating length units.**
- **Tell and write time.**
- **Represent and interpret data.**

Geometry

- **Reason with shapes and their attributes.**

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Operations and Algebraic Thinking 1.OA

Represent and solve problems involving addition and subtraction.

1. Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.[2]

2. Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.

Understand and apply properties of operations and the relationship between addition and subtraction.

3. Apply properties of operations as strategies to add and subtract.[3] *Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known. (Commutative property of addition.) To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12. (Associative property of addition.)*

4. Understand subtraction as an unknown-addend problem. *For example, subtract 10 – 8 by finding the number that makes 10 when added to 8.*

Add and subtract within 20.

5. Relate counting to addition and subtraction (e.g., by counting on 2 to add 2).

6. Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).

Work with addition and subtraction equations.

7. Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. *For example, which of the following equations are true and which are false? 6 = 6, 7 = 8 – 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2.*

8. Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 + ? = 11, 5 = $\square$ – 3, 6 + 6 = $\square$.*

Number and Operations in Base Ten 1.NBT

Extend the counting sequence.

1. Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral.

Understand place value.

2. Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases:

   a. 10 can be thought of as a bundle of ten ones — called a “ten.”

   b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.

   c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).

[2]See Glossary, Table 1.
[3]Students need not use formal terms for these properties.

3. Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <.

Use place value understanding and properties of operations to add and subtract.

4. Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.
5. Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.
6. Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Measurement and Data 1.MD

Measure lengths indirectly and by iterating length units.

1. Order three objects by length; compare the lengths of two objects indirectly by using a third object.
2. Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. *Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.*

Tell and write time.

3. Tell and write time in hours and half-hours using analog and digital clocks.

Represent and interpret data.

4. Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.

Geometry 1.G

Reason with shapes and their attributes.

1. Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes.
2. Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape.[4]
3. Partition circles and rectangles into two and four equal shares, describe the shares using the words *halves*, *fourths*, and *quarters*, and use the phrases *half of*, *fourth of*, and *quarter of*. Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares.

[4]Students do not need to learn formal names such as “right rectangular prism.”

Mathematics | Grade 2

In Grade 2, instructional time should focus on four critical areas: (1) extending understanding of base-ten notation; (2) building fluency with addition and subtraction; (3) using standard units of measure; and (4) describing and analyzing shapes.

(1) Students extend their understanding of the base-ten system. This includes ideas of counting in fives, tens, and multiples of hundreds, tens, and ones, as well as number relationships involving these units, including comparing. Students understand multi-digit numbers (up to 1000) written in base-ten notation, recognizing that the digits in each place represent amounts of thousands, hundreds, tens, or ones (e.g., 853 is 8 hundreds + 5 tens + 3 ones).

(2) Students use their understanding of addition to develop fluency with addition and subtraction within 100. They solve problems within 1000 by applying their understanding of models for addition and subtraction, and they develop, discuss, and use efficient, accurate, and generalizable methods to compute sums and differences of whole numbers in base-ten notation, using their understanding of place value and the properties of operations. They select and accurately apply methods that are appropriate for the context and the numbers involved to mentally calculate sums and differences for numbers with only tens or only hundreds.

(3) Students recognize the need for standard units of measure (centimeter and inch) and they use rulers and other measurement tools with the understanding that linear measure involves an iteration of units. They recognize that the smaller the unit, the more iterations they need to cover a given length.

(4) Students describe and analyze shapes by examining their sides and angles. Students investigate, describe, and reason about decomposing and combining shapes to make other shapes. Through building, drawing, and analyzing two- and three-dimensional shapes, students develop a foundation for understanding area, volume, congruence, similarity, and symmetry in later grades.

Grade 2 Overview

Operations and Algebraic Thinking

- **Represent and solve problems involving addition and subtraction.**
- **Add and subtract within 20.**
- **Work with equal groups of objects to gain foundations for multiplication.**

Number and Operations in Base Ten

- **Understand place value.**
- **Use place value understanding and properties of operations to add and subtract.**

Measurement and Data

- **Measure and estimate lengths in standard units.**
- **Relate addition and subtraction to length.**
- **Work with time and money.**
- **Represent and interpret data.**

Geometry

- **Reason with shapes and their attributes.**

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Operations and Algebraic Thinking — 2.OA

Represent and solve problems involving addition and subtraction.

1. Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.[1]

Add and subtract within 20.

2. Fluently add and subtract within 20 using mental strategies.[2] By end of Grade 2, know from memory all sums of two one-digit numbers.

Work with equal groups of objects to gain foundations for multiplication.

3. Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.
4. Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

Number and Operations in Base Ten — 2.NBT

Understand place value.

1. Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases:
   a. 100 can be thought of as a bundle of ten tens — called a "hundred."
   b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).
2. Count within 1000; skip-count by 5s, 10s, and 100s.
3. Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.
4. Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.

Use place value understanding and properties of operations to add and subtract.

5. Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.
6. Add up to four two-digit numbers using strategies based on place value and properties of operations.
7. Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.
8. Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900.
9. Explain why addition and subtraction strategies work, using place value and the properties of operations.[3]

---

[1]See Glossary, Table 1.
[2]See standard 1.OA.6 for a list of mental strategies.
[3]Explanations may be supported by drawings or objects.

Measurement and Data 2.MD

Measure and estimate lengths in standard units.

1. Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.
2. Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.
3. Estimate lengths using units of inches, feet, centimeters, and meters.
4. Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.

Relate addition and subtraction to length.

5. Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem.
6. Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number line diagram.

Work with time and money.

7. Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m.
8. Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately. *Example: If you have 2 dimes and 3 pennies, how many cents do you have?*

Represent and interpret data.

9. Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units.
10. Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems[4] using information presented in a bar graph.

Geometry 2.G

Reason with shapes and their attributes.

1. Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces.[5] Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.
2. Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.
3. Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words *halves*, *thirds*, *half of*, *a third of*, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.

[4]See Glossary, Table 1.
[5]Sizes are compared directly or visually, not compared by measuring.

Mathematics | Grade 3

In Grade 3, instructional time should focus on four critical areas: (1) developing understanding of multiplication and division and strategies for multiplication and division within 100; (2) developing understanding of fractions, especially unit fractions (fractions with numerator 1); (3) developing understanding of the structure of rectangular arrays and of area; and (4) describing and analyzing two-dimensional shapes.

(1) Students develop an understanding of the meanings of multiplication and division of whole numbers through activities and problems involving equal-sized groups, arrays, and area models; multiplication is finding an unknown product, and division is finding an unknown factor in these situations. For equal-sized group situations, division can require finding the unknown number of groups or the unknown group size. Students use properties of operations to calculate products of whole numbers, using increasingly sophisticated strategies based on these properties to solve multiplication and division problems involving single-digit factors. By comparing a variety of solution strategies, students learn the relationship between multiplication and division.

(2) Students develop an understanding of fractions, beginning with unit fractions. Students view fractions in general as being built out of unit fractions, and they use fractions along with visual fraction models to represent parts of a whole. Students understand that the size of a fractional part is relative to the size of the whole. For example, 1/2 of the paint in a small bucket could be less paint than 1/3 of the paint in a larger bucket, but 1/3 of a ribbon is longer than 1/5 of the same ribbon because when the ribbon is divided into 3 equal parts, the parts are longer than when the ribbon is divided into 5 equal parts. Students are able to use fractions to represent numbers equal to, less than, and greater than one. They solve problems that involve comparing fractions by using visual fraction models and strategies based on noticing equal numerators or denominators.

(3) Students recognize area as an attribute of two-dimensional regions. They measure the area of a shape by finding the total number of same-size units of area required to cover the shape without gaps or overlaps, a square with sides of unit length being the standard unit for measuring area. Students understand that rectangular arrays can be decomposed into identical rows or into identical columns. By decomposing rectangles into rectangular arrays of squares, students connect area to multiplication, and justify using multiplication to determine the area of a rectangle.

(4) Students describe, analyze, and compare properties of two-dimensional shapes. They compare and classify shapes by their sides and angles, and connect these with definitions of shapes. Students also relate their fraction work to geometry by expressing the area of part of a shape as a unit fraction of the whole.

Grade 3 Overview

Operations and Algebraic Thinking

- **Represent and solve problems involving multiplication and division.**
- **Understand properties of multiplication and the relationship between multiplication and division.**
- **Multiply and divide within 100.**
- **Solve problems involving the four operations, and identify and explain patterns in arithmetic.**

Number and Operations in Base Ten

- **Use place value understanding and properties of operations to perform multi-digit arithmetic.**

Number and Operations—Fractions

- **Develop understanding of fractions as numbers.**

Measurement and Data

- **Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.**
- **Represent and interpret data.**
- **Geometric measurement: understand concepts of area and relate area to multiplication and to addition.**
- **Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.**

Geometry

- **Reason with shapes and their attributes.**

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Operations and Algebraic Thinking 3.OA

Represent and solve problems involving multiplication and division.

1. Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5 × 7.*
2. Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.*
3. Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.[1]
4. Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = ▯ ÷ 3, 6 × 6 = ?.*

Understand properties of multiplication and the relationship between multiplication and division.

5. Apply properties of operations as strategies to multiply and divide.[2] *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)*
6. Understand division as an unknown-factor problem. *For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.*

Multiply and divide within 100.

7. Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.

Solve problems involving the four operations, and identify and explain patterns in arithmetic.

8. Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.[3]
9. Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.*

---

[1]See Glossary, Table 2.
[2]Students need not use formal terms for these properties.
[3]This standard is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order (Order of Operations).

Number and Operations in Base Ten 3.NBT

Use place value understanding and properties of operations to perform multi-digit arithmetic.[4]

1. Use place value understanding to round whole numbers to the nearest 10 or 100.
2. Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.
3. Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations.

Number and Operations—Fractions[5] 3.NF

Develop understanding of fractions as numbers.

1. Understand a fraction 1/*b* as the quantity formed by 1 part when a whole is partitioned into *b* equal parts; understand a fraction *a*/*b* as the quantity formed by *a* parts of size 1/*b*.
2. Understand a fraction as a number on the number line; represent fractions on a number line diagram.
   a. Represent a fraction 1/*b* on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size 1/*b* and that the endpoint of the part based at 0 locates the number 1/*b* on the number line.
   b. Represent a fraction *a*/*b* on a number line diagram by marking off *a* lengths 1/*b* from 0. Recognize that the resulting interval has size *a*/*b* and that its endpoint locates the number *a*/*b* on the number line.
3. Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.
   a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.
   b. Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3). Explain why the fractions are equivalent, e.g., by using a visual fraction model.
   c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.*
   d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Measurement and Data 3.MD

Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.

1. Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.

---

[4]A range of algorithms may be used.
[5]Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.

2. Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l).[6] Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.[7]

Represent and interpret data.

3. Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. *For example, draw a bar graph in which each square in the bar graph might represent 5 pets.*

4. Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units— whole numbers, halves, or quarters.

Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

5. Recognize area as an attribute of plane figures and understand concepts of area measurement.
   a. A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area.
   b. A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units.

6. Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).

7. Relate area to the operations of multiplication and addition.
   a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.
   b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.
   c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning.
   d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.

Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.

8. Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

[6]Excludes compound units such as cm^3 and finding the geometric volume of a container.
[7]Excludes multiplicative comparison problems (problems involving notions of "times as much"; see Glossary, Table 2).

Geometry 3.G

Reason with shapes and their attributes.

1. Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.
2. Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. *For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape.*

Mathematics | Grade 4

In Grade 4, instructional time should focus on three critical areas: (1) developing understanding and fluency with multi-digit multiplication, and developing understanding of dividing to find quotients involving multi-digit dividends; (2) developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers; (3) understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, particular angle measures, and symmetry.

(1) Students generalize their understanding of place value to 1,000,000, understanding the relative sizes of numbers in each place. They apply their understanding of models for multiplication (equal-sized groups, arrays, area models), place value, and properties of operations, in particular the distributive property, as they develop, discuss, and use efficient, accurate, and generalizable methods to compute products of multi-digit whole numbers. Depending on the numbers and the context, they select and accurately apply appropriate methods to estimate or mentally calculate products. They develop fluency with efficient procedures for multiplying whole numbers; understand and explain why the procedures work based on place value and properties of operations; and use them to solve problems. Students apply their understanding of models for division, place value, properties of operations, and the relationship of division to multiplication as they develop, discuss, and use efficient, accurate, and generalizable procedures to find quotients involving multi-digit dividends. They select and accurately apply appropriate methods to estimate and mentally calculate quotients, and interpret remainders based upon the context.

(2) Students develop understanding of fraction equivalence and operations with fractions. They recognize that two different fractions can be equal (e.g., 15/9 = 5/3), and they develop methods for generating and recognizing equivalent fractions. Students extend previous understandings about how fractions are built from unit fractions, composing fractions from unit fractions, decomposing fractions into unit fractions, and using the meaning of fractions and the meaning of multiplication to multiply a fraction by a whole number.

(3) Students describe, analyze, compare, and classify two-dimensional shapes. Through building, drawing, and analyzing two-dimensional shapes, students deepen their understanding of properties of two-dimensional objects and the use of them to solve problems involving symmetry.

Grade 4 Overview

Operations and Algebraic Thinking

- **Use the four operations with whole numbers to solve problems.**
- **Gain familiarity with factors and multiples.**
- **Generate and analyze patterns.**

Number and Operations in Base Ten

- **Generalize place value understanding for multi-digit whole numbers.**
- **Use place value understanding and properties of operations to perform multi-digit arithmetic.**

Number and Operations—Fractions

- **Extend understanding of fraction equivalence and ordering.**
- **Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.**
- **Understand decimal notation for fractions, and compare decimal fractions.**

Measurement and Data

- **Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.**
- **Represent and interpret data.**
- **Geometric measurement: understand concepts of angle and measure angles.**

Geometry

- **Draw and identify lines and angles, and classify shapes by properties of their lines and angles.**

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Operations and Algebraic Thinking 4.OA

Use the four operations with whole numbers to solve problems.

1. Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.
2. Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.[1]
3. Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Gain familiarity with factors and multiples.

4. Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.

Generate and analyze patterns.

5. Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.*

Number and Operations in Base Ten[2] 4.NBT

Generalize place value understanding for multi-digit whole numbers.

1. Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.*
2. Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.
3. Use place value understanding to round multi-digit whole numbers to any place.

Use place value understanding and properties of operations to perform multi-digit arithmetic.

4. Fluently add and subtract multi-digit whole numbers using the standard algorithm.
5. Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

---

[1]See Glossary, Table 2.
[2]Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.

6. Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Number and Operations—Fractions[3] 4.NF

Extend understanding of fraction equivalence and ordering.

1. Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.
2. Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

3. Understand a fraction a/b with $a > 1$ as a sum of fractions $1/b$.
   a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
   b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples: 3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.*
   c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.
   d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.
4. Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.
   a. Understand a fraction a/b as a multiple of $1/b$. *For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).*
   b. Understand a multiple of a/b as a multiple of $1/b$, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)*
   c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*

[3] Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

Understand decimal notation for fractions, and compare decimal fractions.

5. Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.[4] *For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.*

6. Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.*

7. Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.

Measurement and Data 4.MD

Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

1. Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. *For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...*

2. Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.

3. Apply the area and perimeter formulas for rectangles in real world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.*

Represent and interpret data.

4. Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. *For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.*

Geometric measurement: understand concepts of angle and measure angles.

5. Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:

   a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles.

   b. An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees.

[4]Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade.

6. Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.

7. Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.

Geometry 4.G

Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

1. Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.

2. Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.

3. Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.

Mathematics | Grade 5

In Grade 5, instructional time should focus on three critical areas: (1) developing fluency with addition and subtraction of fractions, and developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions); (2) extending division to 2-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations; and (3) developing understanding of volume.

(1) Students apply their understanding of fractions and fraction models to represent the addition and subtraction of fractions with unlike denominators as equivalent calculations with like denominators. They develop fluency in calculating sums and differences of fractions, and make reasonable estimates of them. Students also use the meaning of fractions, of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for multiplying and dividing fractions make sense. (Note: this is limited to the case of dividing unit fractions by whole numbers and whole numbers by unit fractions.)

(2) Students develop understanding of why division procedures work based on the meaning of base-ten numerals and properties of operations. They finalize fluency with multi-digit addition, subtraction, multiplication, and division. They apply their understandings of models for decimals, decimal notation, and properties of operations to add and subtract decimals to hundredths. They develop fluency in these computations, and make reasonable estimates of their results. Students use the relationship between decimals and fractions, as well as the relationship between finite decimals and whole numbers (i.e., a finite decimal multiplied by an appropriate power of 10 is a whole number), to understand and explain why the procedures for multiplying and dividing finite decimals make sense. They compute products and quotients of decimals to hundredths efficiently and accurately.

(3) Students recognize volume as an attribute of three-dimensional space. They understand that volume can be measured by finding the total number of same-size units of volume required to fill the space without gaps or overlaps. They understand that a 1-unit by 1-unit by 1-unit cube is the standard unit for measuring volume. They select appropriate units, strategies, and tools for solving problems that involve estimating and measuring volume. They decompose three-dimensional shapes and find volumes of right rectangular prisms by viewing them as decomposed into layers of arrays of cubes. They measure necessary attributes of shapes in order to determine volumes to solve real world and mathematical problems.

Grade 5 Overview

Operations and Algebraic Thinking

- **Write and interpret numerical expressions.**
- **Analyze patterns and relationships.**

Number and Operations in Base Ten

- **Understand the place value system.**
- **Perform operations with multi-digit whole numbers and with decimals to hundredths.**

Number and Operations—Fractions

- **Use equivalent fractions as a strategy to add and subtract fractions.**
- **Apply and extend previous understandings of multiplication and division to multiply and divide fractions.**

Measurement and Data

- **Convert like measurement units within a given measurement system.**
- **Represent and interpret data.**
- **Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.**

Geometry

- **Graph points on the coordinate plane to solve real-world and mathematical problems.**
- **Classify two-dimensional figures into categories based on their properties.**

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Operations and Algebraic Thinking 5.OA

Write and interpret numerical expressions.

1. Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.
2. Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. *For example, express the calculation "add 8 and 7, then multiply by 2" as 2 × (8 + 7). Recognize that 3 × (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product.*

Analyze patterns and relationships.

3. Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. *For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.*

Number and Operations in Base Ten 5.NBT

Understand the place value system.

1. Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.
2. Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.
3. Read, write, and compare decimals to thousandths.
   a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., 347.392 = 3 × 100 + 4 × 10 + 7 × 1 + 3 × (1/10) + 9 × (1/100) + 2 × (1/1000).
   b. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.
4. Use place value understanding to round decimals to any place.

Perform operations with multi-digit whole numbers and with decimals to hundredths.

5. Fluently multiply multi-digit whole numbers using the standard algorithm.
6. Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
7. Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Number and Operations—Fractions 5.NF

Use equivalent fractions as a strategy to add and subtract fractions.

1. Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)*

2. Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2.*

Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

3. Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?*

4. Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

   a. Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. *For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = ac/bd.)*

   b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

5. Interpret multiplication as scaling (resizing), by:

   a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.

   b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n\times a)/(n\times b)$ to the effect of multiplying a/b by 1.

6. Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

7. Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.[1]

   a. Interpret division of a unit fraction by a non-zero whole number,

[1]Students able to multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between multiplication and division. But division of a fraction by a fraction is not a requirement at this grade.

and compute such quotients. *For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3.*

b. Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4.*

c. Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?*

Measurement and Data 5.MD

Convert like measurement units within a given measurement system.

1. Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.

Represent and interpret data.

2. Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.*

Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.

3. Recognize volume as an attribute of solid figures and understand concepts of volume measurement.

   a. A cube with side length 1 unit, called a “unit cube,” is said to have “one cubic unit” of volume, and can be used to measure volume.

   b. A solid figure which can be packed without gaps or overlaps using n unit cubes is said to have a volume of n cubic units.

4. Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.

5. Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.

   a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.

   b. Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems.

   c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.

Geometry 5.G

Graph points on the coordinate plane to solve real-world and mathematical problems.

1. Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate).
2. Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.

Classify two-dimensional figures into categories based on their properties.

3. Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. *For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.*
4. Classify two-dimensional figures in a hierarchy based on properties.

Mathematics | Grade 6

In Grade 6, instructional time should focus on four critical areas: (1) connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems; (2) completing understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers; (3) writing, interpreting, and using expressions and equations; and (4) developing understanding of statistical thinking.

(1) Students use reasoning about multiplication and division to solve ratio and rate problems about quantities. By viewing equivalent ratios and rates as deriving from, and extending, pairs of rows (or columns) in the multiplication table, and by analyzing simple drawings that indicate the relative size of quantities, students connect their understanding of multiplication and division with ratios and rates. Thus students expand the scope of problems for which they can use multiplication and division to solve problems, and they connect ratios and fractions. Students solve a wide variety of problems involving ratios and rates.

(2) Students use the meaning of fractions, the meanings of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for dividing fractions make sense. Students use these operations to solve problems. Students extend their previous understandings of number and the ordering of numbers to the full system of rational numbers, which includes negative rational numbers, and in particular negative integers. They reason about the order and absolute value of rational numbers and about the location of points in all four quadrants of the coordinate plane.

(3) Students understand the use of variables in mathematical expressions. They write expressions and equations that correspond to given situations, evaluate expressions, and use expressions and formulas to solve problems. Students understand that expressions in different forms can be equivalent, and they use the properties of operations to rewrite expressions in equivalent forms. Students know that the solutions of an equation are the values of the variables that make the equation true. Students use properties of operations and the idea of maintaining the equality of both sides of an equation to solve simple one-step equations. Students construct and analyze tables, such as tables of quantities that are in equivalent ratios, and they use equations (such as $3x = y$) to describe relationships between quantities.

(4) Building on and reinforcing their understanding of number, students begin to develop their ability to think statistically. Students recognize that a data distribution may not have a definite center and that different ways to measure center yield different values. The median measures center in the sense that it is roughly the middle value. The mean measures center in the sense that it is the value that each data point would take on if the total of the data values were redistributed equally, and also in the sense that it is a balance point. Students recognize that a measure of variability (interquartile range or mean absolute deviation) can also be useful for summarizing data because two very different sets of data can have the same mean and

median yet be distinguished by their variability. Students learn to describe and summarize numerical data sets, identifying clusters, peaks, gaps, and symmetry, considering the context in which the data were collected.

Students in Grade 6 also build on their work with area in elementary school by reasoning about relationships among shapes to determine area, surface area, and volume. They find areas of right triangles, other triangles, and special quadrilaterals by decomposing these shapes, rearranging or removing pieces, and relating the shapes to rectangles. Using these methods, students discuss, develop, and justify formulas for areas of triangles and parallelograms. Students find areas of polygons and surface areas of prisms and pyramids by decomposing them into pieces whose area they can determine. They reason about right rectangular prisms with fractional side lengths to extend formulas for the volume of a right rectangular prism to fractional side lengths. They prepare for work on scale drawings and constructions in Grade 7 by drawing polygons in the coordinate plane.

Grade 6 Overview

Ratios and Proportional Relationships

- **Understand ratio concepts and use ratio reasoning to solve problems.**

The Number System

- **Apply and extend previous understandings of multiplication and division to divide fractions by fractions.**
- **Compute fluently with multi-digit numbers and find common factors and multiples.**
- **Apply and extend previous understandings of numbers to the system of rational numbers.**

Expressions and Equations

- **Apply and extend previous understandings of arithmetic to algebraic expressions.**
- **Reason about and solve one-variable equations and inequalities.**
- **Represent and analyze quantitative relationships between dependent and independent variables.**

Geometry

- **Solve real-world and mathematical problems involving area, surface area, and volume.**

Statistics and Probability

- **Develop understanding of statistical variability.**
- **Summarize and describe distributions.**

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Ratios and Proportional Relationships 6.RP

Understand ratio concepts and use ratio reasoning to solve problems.

1. Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."*
2. Understand the concept of a unit rate *a/b* associated with a ratio *a:b* with $b \neq 0$, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."*[1]
3. Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
   a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
   b. Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?*
   c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.
   d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

The Number System 6.NS

Apply and extend previous understandings of multiplication and division to divide fractions by fractions.

1. Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a story context for $(2/3) \div (3/4)$ and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that $(2/3) \div (3/4) = 8/9$ because 3/4 of 8/9 is 2/3. (In general, $(a/b) \div (c/d) = ad/bc$.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?*

Compute fluently with multi-digit numbers and find common factors and multiples.

2. Fluently divide multi-digit numbers using the standard algorithm.
3. Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.
4. Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express 36 + 8 as 4 (9 + 2).*

[1]Expectations for unit rates in this grade are limited to non-complex fractions.

Apply and extend previous understandings of numbers to the system of rational numbers.

5. Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.

6. Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.

   a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., $-(-3) = 3$, and that 0 is its own opposite.

   b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.

   c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.

7. Understand ordering and absolute value of rational numbers.

   a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. *For example, interpret $-3 > -7$ as a statement that -3 is located to the right of -7 on a number line oriented from left to right.*

   b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write $-3\ °C > -7\ °C$ to express the fact that $-3\ °C$ is warmer than $-7\ °C$.*

   c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of -30 dollars, write $|-30| = 30$ to describe the size of the debt in dollars.*

   d. Distinguish comparisons of absolute value from statements about order. *For example, recognize that an account balance less than -30 dollars represents a debt greater than 30 dollars.*

8. Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

Expressions and Equations 6.EE

Apply and extend previous understandings of arithmetic to algebraic expressions.

1. Write and evaluate numerical expressions involving whole-number exponents.

2. Write, read, and evaluate expressions in which letters stand for numbers.

   a. Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as $5 - y$.*

b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression 2 (8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.*

c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas* $V = s^3$ *and* $A = 6 s^2$ *to find the volume and surface area of a cube with sides of length* $s = 1/2$.

3. Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.*

4. Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.*

Reason about and solve one-variable equations and inequalities.

5. Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.

6. Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.

7. Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.

8. Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.

Represent and analyze quantitative relationships between dependent and independent variables.

9. Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.*

Geometry 6.G

Solve real-world and mathematical problems involving area, surface area, and volume.

1. Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

2. Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = l\ w\ h$ and $V = b\ h$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.
3. Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.
4. Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

Statistics and Probability 6.SP

Develop understanding of statistical variability.

1. Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. *For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.*
2. Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.
3. Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.

Summarize and describe distributions.

4. Display numerical data in plots on a number line, including dot plots, histograms, and box plots.
5. Summarize numerical data sets in relation to their context, such as by:
    a. Reporting the number of observations.
    b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.
    c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.
    d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.

Mathematics | Grade 7

In Grade 7, instructional time should focus on four critical areas: (1) developing understanding of and applying proportional relationships; (2) developing understanding of operations with rational numbers and working with expressions and linear equations; (3) solving problems involving scale drawings and informal geometric constructions, and working with two- and three-dimensional shapes to solve problems involving area, surface area, and volume; and (4) drawing inferences about populations based on samples.

(1) Students extend their understanding of ratios and develop understanding of proportionality to solve single- and multi-step problems. Students use their understanding of ratios and proportionality to solve a wide variety of percent problems, including those involving discounts, interest, taxes, tips, and percent increase or decrease. Students solve problems about scale drawings by relating corresponding lengths between the objects or by using the fact that relationships of lengths within an object are preserved in similar objects. Students graph proportional relationships and understand the unit rate informally as a measure of the steepness of the related line, called the slope. They distinguish proportional relationships from other relationships.

(2) Students develop a unified understanding of number, recognizing fractions, decimals (that have a finite or a repeating decimal representation), and percents as different representations of rational numbers. Students extend addition, subtraction, multiplication, and division to all rational numbers, maintaining the properties of operations and the relationships between addition and subtraction, and multiplication and division. By applying these properties, and by viewing negative numbers in terms of everyday contexts (e.g., amounts owed or temperatures below zero), students explain and interpret the rules for adding, subtracting, multiplying, and dividing with negative numbers. They use the arithmetic of rational numbers as they formulate expressions and equations in one variable and use these equations to solve problems.

(3) Students continue their work with area from Grade 6, solving problems involving the area and circumference of a circle and surface area of three-dimensional objects. In preparation for work on congruence and similarity in Grade 8 they reason about relationships among two-dimensional figures using scale drawings and informal geometric constructions, and they gain familiarity with the relationships between angles formed by intersecting lines. Students work with three-dimensional figures, relating them to two-dimensional figures by examining cross-sections. They solve real-world and mathematical problems involving area, surface area, and volume of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes and right prisms.

(4) Students build on their previous work with single data distributions to compare two data distributions and address questions about differences between populations. They begin informal work with random sampling to generate data sets and learn about the importance of representative samples for drawing inferences.

Grade 7 Overview

Ratios and Proportional Relationships

- **Analyze proportional relationships and use them to solve real-world and mathematical problems.**

The Number System

- **Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.**

Expressions and Equations

- **Use properties of operations to generate equivalent expressions.**
- **Solve real-life and mathematical problems using numerical and algebraic expressions and equations.**

Geometry

- **Draw, construct and describe geometrical figures and describe the relationships between them.**
- **Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.**

Statistics and Probability

- **Use random sampling to draw inferences about a population.**
- **Draw informal comparative inferences about two populations.**
- **Investigate chance processes and develop, use, and evaluate probability models.**

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Ratios and Proportional Relationships 7.RP

Analyze proportional relationships and use them to solve real-world and mathematical problems.

1. Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction* $\frac{1/2}{1/4}$ *miles per hour, equivalently 2 miles per hour.*

2. Recognize and represent proportional relationships between quantities.

   a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

   b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

   c. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn.*

   d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and $(1, r)$ where r is the unit rate.

3. Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*

The Number System 7.NS

Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

1. Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

   a. Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.*

   b. Understand $p + q$ as the number located a distance $|q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.

   c. Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.

   d. Apply properties of operations as strategies to add and subtract rational numbers.

2. Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.

   a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.

b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts.

c. Apply properties of operations as strategies to multiply and divide rational numbers.

d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.

3. Solve real-world and mathematical problems involving the four operations with rational numbers.[1]

Expressions and Equations — 7.EE

Use properties of operations to generate equivalent expressions.

1. Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.
2. Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, $a + 0.05a = 1.05a$ means that "increase by 5%" is the same as "multiply by 1.05."*

Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

3. Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making \$25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or \$2.50, for a new salary of \$27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.*
4. Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

   a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*

   b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid \$50 per week plus \$3 per sale. This week you want your pay to be at least \$100. Write an inequality for the number of sales you need to make, and describe the solutions.*

Geometry — 7.G

Draw, construct, and describe geometrical figures and describe the relationships between them.

1. Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

[1]Computations with rational numbers extend the rules for manipulating fractions to complex fractions.

2. Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.
3. Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.

Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

4. Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.
5. Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.
6. Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Statistics and Probability 7.SP

Use random sampling to draw inferences about a population.

1. Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.
2. Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. *For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.*

Draw informal comparative inferences about two populations.

3. Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. *For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.*
4. Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.*

Investigate chance processes and develop, use, and evaluate probability models.

5. Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.

6. Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.*

7. Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.

    a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. *For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.*

    b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. *For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?*

8. Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.

    a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.

    b. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., “rolling double sixes”), identify the outcomes in the sample space which compose the event.

    c. Design and use a simulation to generate frequencies for compound events. *For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?*

Mathematics | Grade 8

In Grade 8, instructional time should focus on three critical areas: (1) formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations; (2) grasping the concept of a function and using functions to describe quantitative relationships; (3) analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem.

(1) Students use linear equations and systems of linear equations to represent, analyze, and solve a variety of problems. Students recognize equations for proportions ($y/x = m$ or $y = mx$) as special linear equations ($y = mx + b$), understanding that the constant of proportionality (m) is the slope, and the graphs are lines through the origin. They understand that the slope (m) of a line is a constant rate of change, so that if the input or x-coordinate changes by an amount A, the output or y-coordinate changes by the amount $m \cdot A$. Students also use a linear equation to describe the association between two quantities in bivariate data (such as arm span vs. height for students in a classroom). At this grade, fitting the model, and assessing its fit to the data are done informally. Interpreting the model in the context of the data requires students to express a relationship between the two quantities in question and to interpret components of the relationship (such as slope and y-intercept) in terms of the situation.

Students strategically choose and efficiently implement procedures to solve linear equations in one variable, understanding that when they use the properties of equality and the concept of logical equivalence, they maintain the solutions of the original equation. Students solve systems of two linear equations in two variables and relate the systems to pairs of lines in the plane; these intersect, are parallel, or are the same line. Students use linear equations, systems of linear equations, linear functions, and their understanding of slope of a line to analyze situations and solve problems.

(2) Students grasp the concept of a function as a rule that assigns to each input exactly one output. They understand that functions describe situations where one quantity determines another. They can translate among representations and partial representations of functions (noting that tabular and graphical representations may be partial representations), and they describe how aspects of the function are reflected in the different representations.

(3) Students use ideas about distance and angles, how they behave under translations, rotations, reflections, and dilations, and ideas about congruence and similarity to describe and analyze two-dimensional figures and to solve problems. Students show that the sum of the angles in a triangle is the angle formed by a straight line, and that various configurations of lines give rise to similar triangles because of the angles created when a transversal cuts parallel lines. Students understand the statement of the Pythagorean Theorem and its converse, and can explain why the Pythagorean Theorem holds, for example, by decomposing a square in two different ways. They apply the Pythagorean Theorem to find distances between points on the coordinate plane, to find lengths, and to analyze polygons. Students complete their work on volume by solving problems involving cones, cylinders, and spheres.

Grade 8 Overview

The Number System

- Know that there are numbers that are not rational, and approximate them by rational numbers.

Expressions and Equations

- Work with radicals and integer exponents.
- Understand the connections between proportional relationships, lines, and linear equations.
- Analyze and solve linear equations and pairs of simultaneous linear equations.

Functions

- Define, evaluate, and compare functions.
- Use functions to model relationships between quantities.

Geometry

- Understand congruence and similarity using physical models, transparencies, or geometry software.
- Understand and apply the Pythagorean Theorem.
- Solve real-world and mathematical problems involving volume of cylinders, cones and spheres.

Statistics and Probability

- Investigate patterns of association in bivariate data.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

The Number System 8.NS

Know that there are numbers that are not rational, and approximate them by rational numbers.

1. Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.

2. Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2). *For example, by truncating the decimal expansion of* $\sqrt{2}$*, show that* $\sqrt{2}$ *is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.*

Expressions and Equations 8.EE

Work with radicals and integer exponents.

1. Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example,* $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$*.*

2. Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.

3. Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as* 3×10^8 *and the population of the world as* 7×10^9*, and determine that the world population is more than 20 times larger.*

4. Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

Understand the connections between proportional relationships, lines, and linear equations.

5. Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*

6. Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.

Analyze and solve linear equations and pairs of simultaneous linear equations.

7. Solve linear equations in one variable.

   a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).

   b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

8. Analyze and solve pairs of simultaneous linear equations.
   a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.
   b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.*
   c. Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*

Functions 8.F

Define, evaluate, and compare functions.

1. Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.[1]
2. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*
3. Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.*

Use functions to model relationships between quantities.

4. Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.
5. Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Geometry 8.G

Understand congruence and similarity using physical models, transparencies, or geometry software.

1. Verify experimentally the properties of rotations, reflections, and translations:
   a. Lines are taken to lines, and line segments to line segments of the same length.
   b. Angles are taken to angles of the same measure.
   c. Parallel lines are taken to parallel lines.
2. Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.

[1]Function notation is not required in Grade 8.

3. Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.
4. Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.
5. Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*

Understand and apply the Pythagorean Theorem.

6. Explain a proof of the Pythagorean Theorem and its converse.
7. Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.
8. Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.

9. Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

Statistics and Probability — 8.SP

Investigate patterns of association in bivariate data.

1. Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.
2. Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.
3. Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.*
4. Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*

Mathematics Standards for High School

The high school standards specify the mathematics that all students should study in order to be college and career ready. Additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics is indicated by (+), as in this example:

> (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers).

All standards without a (+) symbol should be in the common mathematics curriculum for all college and career ready students. Standards with a (+) symbol may also appear in courses intended for all students.

The high school standards are listed in conceptual categories:

- Number and Quantity
- Algebra
- Functions
- Modeling
- Geometry
- Statistics and Probability

Conceptual categories portray a coherent view of high school mathematics; a student's work with functions, for example, crosses a number of traditional course boundaries, potentially up through and including calculus.

Modeling is best interpreted not as a collection of isolated topics but in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (★). The star symbol sometimes appears on the heading for a group of standards; in that case, it should be understood to apply to all standards in that group.

Mathematics | High School—Number and Quantity

Numbers and Number Systems. During the years from kindergarten to eighth grade, students must repeatedly extend their conception of number. At first, "number" means "counting number": 1, 2, 3... Soon after that, 0 is used to represent "none" and the whole numbers are formed by the counting numbers together with zero. The next extension is fractions. At first, fractions are barely numbers and tied strongly to pictorial representations. Yet by the time students understand division of fractions, they have a strong concept of fractions as numbers and have connected them, via their decimal representations, with the base-ten system used to represent the whole numbers. During middle school, fractions are augmented by negative fractions to form the rational numbers. In Grade 8, students extend this system once more, augmenting the rational numbers with the irrational numbers to form the real numbers. In high school, students will be exposed to yet another extension of number, when the real numbers are augmented by the imaginary numbers to form the complex numbers.

With each extension of number, the meanings of addition, subtraction, multiplication, and division are extended. In each new number system—integers, rational numbers, real numbers, and complex numbers—the four operations stay the same in two important ways: They have the commutative, associative, and distributive properties and their new meanings are consistent with their previous meanings.

Extending the properties of whole-number exponents leads to new and productive notation. For example, properties of whole-number exponents suggest that $(5^{1/3})^3$ should be $5^{(1/3)3} = 5^1 = 5$ and that $5^{1/3}$ should be the cube root of 5.

Calculators, spreadsheets, and computer algebra systems can provide ways for students to become better acquainted with these new number systems and their notation. They can be used to generate data for numerical experiments, to help understand the workings of matrix, vector, and complex number algebra, and to experiment with non-integer exponents.

Quantities. In real world problems, the answers are usually not numbers but quantities: numbers with units, which involves measurement. In their work in measurement up through Grade 8, students primarily measure commonly used attributes such as length, area, and volume. In high school, students encounter a wider variety of units in modeling, e.g., acceleration, currency conversions, derived quantities such as person-hours and heating degree days, social science rates such as per-capita income, and rates in everyday life such as points scored per game or batting averages. They also encounter novel situations in which they themselves must conceive the attributes of interest. For example, to find a good measure of overall highway safety, they might propose measures such as fatalities per year, fatalities per year per driver, or fatalities per vehicle-mile traveled. Such a conceptual process is sometimes called quantification. Quantification is important for science, as when surface area suddenly "stands out" as an important variable in evaporation. Quantification is also important for companies, which must conceptualize relevant attributes and create or choose suitable measures for them.

Number and Quantity Overview

The Real Number System

- Extend the properties of exponents to rational exponents
- Use properties of rational and irrational numbers.

Quantities

- Reason quantitatively and use units to solve problems

The Complex Number System

- Perform arithmetic operations with complex numbers
- Represent complex numbers and their operations on the complex plane
- Use complex numbers in polynomial identities and equations

Vector and Matrix Quantities

- Represent and model with vector quantities.
- Perform operations on vectors.
- Perform operations on matrices and use matrices in applications.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

The Real Number System N-RN

Extend the properties of exponents to rational exponents.

1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.*

2. Rewrite expressions involving radicals and rational exponents using the properties of exponents.

Use properties of rational and irrational numbers.

3. Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

Quantities* N-Q

Reason quantitatively and use units to solve problems.

1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

2. Define appropriate quantities for the purpose of descriptive modeling.

3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

The Complex Number System N-CN

Perform arithmetic operations with complex numbers.

1. Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.

2. Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

3. (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.

Represent complex numbers and their operations on the complex plane.

4. (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.

5. (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example,* $(-1 + \sqrt{3}\,i)^3 = 8$ *because* $(-1 + \sqrt{3}\,i)$ *has modulus* 2 *and argument* 120°.

6. (+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.

Use complex numbers in polynomial identities and equations.

7. Solve quadratic equations with real coefficients that have complex solutions.

8. (+) Extend polynomial identities to the complex numbers. *For example, rewrite* $x^2 + 4$ *as* $(x + 2i)(x – 2i)$.

9. (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

Vector and Matrix Quantities N-VM

Represent and model with vector quantities.

1. (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., $\boldsymbol{v}$, $|\boldsymbol{v}|$, $||\boldsymbol{v}||$, v).
2. (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.
3. (+) Solve problems involving velocity and other quantities that can be represented by vectors.

Perform operations on vectors.

4. (+) Add and subtract vectors.
   a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.
   b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.
   c. Understand vector subtraction $\boldsymbol{v} - \boldsymbol{w}$ as $\boldsymbol{v} + (-\boldsymbol{w})$, where $-\boldsymbol{w}$ is the additive inverse of $\boldsymbol{w}$, with the same magnitude as $\boldsymbol{w}$ and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.
5. (+) Multiply a vector by a scalar.
   a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$.
   b. Compute the magnitude of a scalar multiple $c\boldsymbol{v}$ using $||c\boldsymbol{v}|| = |c|v$. Compute the direction of $c\boldsymbol{v}$ knowing that when $|c|v \neq 0$, the direction of $c\boldsymbol{v}$ is either along $\boldsymbol{v}$ (for $c > 0$) or against $\boldsymbol{v}$ (for $c < 0$).

Perform operations on matrices and use matrices in applications.

6. (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.
7. (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.
8. (+) Add, subtract, and multiply matrices of appropriate dimensions.
9. (+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.
10. (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.
11. (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.
12. (+) Work with 2×2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.

Mathematics | High School—Algebra

Expressions. An expression is a record of a computation with numbers, symbols that represent numbers, arithmetic operations, exponentiation, and, at more advanced levels, the operation of evaluating a function. Conventions about the use of parentheses and the order of operations assure that each expression is unambiguous. Creating an expression that describes a computation involving a general quantity requires the ability to express the computation in general terms, abstracting from specific instances.

Reading an expression with comprehension involves analysis of its underlying structure. This may suggest a different but equivalent way of writing the expression that exhibits some different aspect of its meaning. For example, $p + 0.05p$ can be interpreted as the addition of a 5% tax to a price p. Rewriting $p + 0.05p$ as $1.05p$ shows that adding a tax is the same as multiplying the price by a constant factor.

Algebraic manipulations are governed by the properties of operations and exponents, and the conventions of algebraic notation. At times, an expression is the result of applying operations to simpler expressions. For example, $p + 0.05p$ is the sum of the simpler expressions p and $0.05p$. Viewing an expression as the result of operation on simpler expressions can sometimes clarify its underlying structure.

A spreadsheet or a computer algebra system (CAS) can be used to experiment with algebraic expressions, perform complicated algebraic manipulations, and understand how algebraic manipulations behave.

Equations and inequalities. An equation is a statement of equality between two expressions, often viewed as a question asking for which values of the variables the expressions on either side are in fact equal. These values are the solutions to the equation. An identity, in contrast, is true for all values of the variables; identities are often developed by rewriting an expression in an equivalent form.

The solutions of an equation in one variable form a set of numbers; the solutions of an equation in two variables form a set of ordered pairs of numbers, which can be plotted in the coordinate plane. Two or more equations and/or inequalities form a system. A solution for such a system must satisfy every equation and inequality in the system.

An equation can often be solved by successively deducing from it one or more simpler equations. For example, one can add the same constant to both sides without changing the solutions, but squaring both sides might lead to extraneous solutions. Strategic competence in solving includes looking ahead for productive manipulations and anticipating the nature and number of solutions.

Some equations have no solutions in a given number system, but have a solution in a larger system. For example, the solution of $x + 1 = 0$ is an integer, not a whole number; the solution of $2x + 1 = 0$ is a rational number, not an integer; the solutions of $x^2 - 2 = 0$ are real numbers, not rational numbers; and the solutions of $x^2 + 2 = 0$ are complex numbers, not real numbers.

The same solution techniques used to solve equations can be used to rearrange formulas. For example, the formula for the area of a trapezoid, $A = ((b_1+b_2)/2)h$, can be solved for h using the same deductive process.

Inequalities can be solved by reasoning about the properties of inequality. Many, but not all, of the properties of equality continue to hold for inequalities and can be useful in solving them.

Connections to Functions and Modeling. Expressions can define functions, and equivalent expressions define the same function. Asking when two functions have the same value for the same input leads to an equation; graphing the two functions allows for finding approximate solutions of the equation. Converting a verbal description to an equation, inequality, or system of these is an essential skill in modeling.

Algebra Overview

Seeing Structure in Expressions

- Interpret the structure of expressions
- Write expressions in equivalent forms to solve problems

Arithmetic with Polynomials and Rational Expressions

- Perform arithmetic operations on polynomials
- Understand the relationship between zeros and factors of polynomials
- Use polynomial identities to solve problems
- Rewrite rational expressions

Creating Equations

- Create equations that describe numbers or relationships

Reasoning with Equations and Inequalities

- Understand solving equations as a process of reasoning and explain the reasoning
- Solve equations and inequalities in one variable
- Solve systems of equations
- Represent and solve equations and inequalities graphically

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Seeing Structure in Expressions — A-SSE

Interpret the structure of expressions

1. Interpret expressions that represent a quantity in terms of its context.★
   a. Interpret parts of an expression, such as terms, factors, and coefficients.
   b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret* $P(1+r)^n$ *as the product of P and a factor not depending on P.*
2. Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$*, thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$.

Write expressions in equivalent forms to solve problems

3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.★
   a. Factor a quadratic expression to reveal the zeros of the function it defines.
   b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
   c. Use the properties of exponents to transform expressions for exponential functions. *For example the expression* 1.15^t *can be rewritten as* $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ *to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*
4. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.*★

Arithmetic with Polynomials and Rational Expressions — A-APR

Perform arithmetic operations on polynomials

1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Understand the relationship between zeros and factors of polynomials

2. Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.
3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

Use polynomial identities to solve problems

4. Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity* $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ *can be used to generate Pythagorean triples.*
5. (+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.[1]

[1]The Binomial Theorem can be proved by mathematical induction or by a combinatorial argument.

Rewrite rational expressions

6. Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.

7. (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

Creating Equations* — A-CED

Create equations that describe numbers or relationships

1. Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*

2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*

4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law $V = IR$ to highlight resistance R.*

Reasoning with Equations and Inequalities — A-REI

Understand solving equations as a process of reasoning and explain the reasoning

1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

Solve equations and inequalities in one variable

3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

4. Solve quadratic equations in one variable.

    a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.

    b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

Solve systems of equations

5. Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
7. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line* $y = -3x$ and the circle $x^2 + y^2 = 3$.
8. (+) Represent a system of linear equations as a single matrix equation in a vector variable.
9. (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater).

Represent and solve equations and inequalities graphically

10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
11. Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★
12. Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Mathematics | High School—Functions

Functions describe situations where one quantity determines another. For example, the return on $10,000 invested at an annualized percentage rate of 4.25% is a function of the length of time the money is invested. Because we continually make theories about dependencies between quantities in nature and society, functions are important tools in the construction of mathematical models.

In school mathematics, functions usually have numerical inputs and outputs and are often defined by an algebraic expression. For example, the time in hours it takes for a car to drive 100 miles is a function of the car's speed in miles per hour, v; the rule $T(v) = 100/v$ expresses this relationship algebraically and defines a function whose name is T.

The set of inputs to a function is called its domain. We often infer the domain to be all inputs for which the expression defining a function has a value, or for which the function makes sense in a given context.

A function can be described in various ways, such as by a graph (e.g., the trace of a seismograph); by a verbal rule, as in, "I'll give you a state, you give me the capital city;" by an algebraic expression like $f(x) = a + bx$; or by a recursive rule. The graph of a function is often a useful way of visualizing the relationship of the function models, and manipulating a mathematical expression for a function can throw light on the function's properties.

Functions presented as expressions can model many important phenomena. Two important families of functions characterized by laws of growth are linear functions, which grow at a constant rate, and exponential functions, which grow at a constant percent rate. Linear functions with a constant term of zero describe proportional relationships.

A graphing utility or a computer algebra system can be used to experiment with properties of these functions and their graphs and to build computational models of functions, including recursively defined functions.

Connections to Expressions, Equations, Modeling, and Coordinates.

Determining an output value for a particular input involves evaluating an expression; finding inputs that yield a given output involves solving an equation. Questions about when two functions have the same value for the same input lead to equations, whose solutions can be visualized from the intersection of their graphs. Because functions describe relationships between quantities, they are frequently used in modeling. Sometimes functions are defined by a recursive process, which can be displayed effectively using a spreadsheet or other technology.

Functions Overview

Interpreting Functions

- **Understand the concept of a function and use function notation**
- **Interpret functions that arise in applications in terms of the context**
- **Analyze functions using different representations**

Building Functions

- **Build a function that models a relationship between two quantities**
- **Build new functions from existing functions**

Linear, Quadratic, and Exponential Models

- **Construct and compare linear, quadratic, and exponential models and solve problems**
- **Interpret expressions for functions in terms of the situation they model**

Trigonometric Functions

- **Extend the domain of trigonometric functions using the unit circle**
- **Model periodic phenomena with trigonometric functions**
- **Prove and apply trigonometric identities**

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Interpreting Functions F-IF

Understand the concept of a function and use function notation

1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.

2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by* $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ *for* $n \geq 1$.

Interpret functions that arise in applications in terms of the context

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.★*

5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.★*

6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★

Analyze functions using different representations

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★

   a. Graph linear and quadratic functions and show intercepts, maxima, and minima.

   b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

   c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

   d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.

   e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

   a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

   b. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as* $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, *and classify them as representing exponential growth or decay.*

9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

Building Functions — F-BF

Build a function that models a relationship between two quantities

1. Write a function that describes a relationship between two quantities.*
   a. Determine an explicit expression, a recursive process, or steps for calculation from a context.
   b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*
   c. (+) Compose functions. *For example, if $T(y)$ is the temperature in the atmosphere as a function of height, and $h(t)$ is the height of a weather balloon as a function of time, then $T(h(t))$ is the temperature at the location of the weather balloon as a function of time.*
2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.*

Build new functions from existing functions

3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*
4. Find inverse functions.
   a. Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse. *For example, $f(x) = 2x^3$ or $f(x) = (x+1)/(x-1)$ for $x \neq 1$.*
   b. (+) Verify by composition that one function is the inverse of another.
   c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.
   d. (+) Produce an invertible function from a non-invertible function by restricting the domain.
5. (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Linear, Quadratic, and Exponential Models* — F-LE

Construct and compare linear, quadratic, and exponential models and solve problems

1. Distinguish between situations that can be modeled with linear functions and with exponential functions.
   a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
   b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
   c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
4. For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.

Interpret expressions for functions in terms of the situation they model

5. Interpret the parameters in a linear or exponential function in terms of a context.

Trigonometric Functions F-TF

Extend the domain of trigonometric functions using the unit circle

1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
3. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi-x$, $\pi+x$, and $2\pi-x$ in terms of their values for x, where x is any real number.
4. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

Model periodic phenomena with trigonometric functions

5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.★
6. (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
7. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.★

Prove and apply trigonometric identities

8. Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.
9. (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

Mathematics | High School—Modeling

Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data.

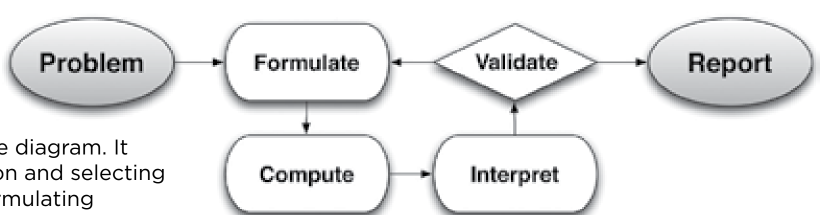
A model can be very simple, such as writing total cost as a product of unit price and number bought, or using a geometric shape to describe a physical object like a coin. Even such simple models involve making choices. It is up to us whether to model a coin as a three-dimensional cylinder, or whether a two-dimensional disk works well enough for our purposes. Other situations—modeling a delivery route, a production schedule, or a comparison of loan amortizations—need more elaborate models that use other tools from the mathematical sciences. Real-world situations are not organized and labeled for analysis; formulating tractable models, representing such models, and analyzing them is appropriately a creative process. Like every such process, this depends on acquired expertise as well as creativity.

Some examples of such situations might include:

- Estimating how much water and food is needed for emergency relief in a devastated city of 3 million people, and how it might be distributed.
- Planning a table tennis tournament for 7 players at a club with 4 tables, where each player plays against each other player.
- Designing the layout of the stalls in a school fair so as to raise as much money as possible.
- Analyzing stopping distance for a car.
- Modeling savings account balance, bacterial colony growth, or investment growth.
- Engaging in critical path analysis, e.g., applied to turnaround of an aircraft at an airport.
- Analyzing risk in situations such as extreme sports, pandemics, and terrorism.
- Relating population statistics to individual predictions.

In situations like these, the models devised depend on a number of factors: How precise an answer do we want or need? What aspects of the situation do we most need to understand, control, or optimize? What resources of time and tools do we have? The range of models that we can create and analyze is also constrained by the limitations of our mathematical, statistical, and technical skills, and our ability to recognize significant variables and relationships among them. Diagrams of various kinds, spreadsheets and other technology, and algebra are powerful tools for understanding and solving problems drawn from different types of real-world situations.

One of the insights provided by mathematical modeling is that essentially the same mathematical or statistical structure can sometimes model seemingly different situations. Models can also shed light on the mathematical structures themselves, for example, as when a model of bacterial growth makes more vivid the explosive growth of the exponential function.



The basic modeling cycle is summarized in the diagram. It involves (1) identifying variables in the situation and selecting those that represent essential features, (2) formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables, (3) analyzing and performing operations on these relationships to draw conclusions, (4) interpreting the results of the mathematics in terms of the original situation, (5) validating the conclusions by comparing them with the situation, and then either improving the model or, if it is acceptable, (6) reporting on the conclusions and the reasoning behind them. Choices, assumptions, and approximations are present throughout this cycle.

In descriptive modeling, a model simply describes the phenomena or summarizes them in a compact form. Graphs of observations are a familiar descriptive model—for example, graphs of global temperature and atmospheric CO_2 over time.

Analytic modeling seeks to explain data on the basis of deeper theoretical ideas, albeit with parameters that are empirically based; for example, exponential growth of bacterial colonies (until cut-off mechanisms such as pollution or starvation intervene) follows from a constant reproduction rate. Functions are an important tool for analyzing such problems.

Graphing utilities, spreadsheets, computer algebra systems, and dynamic geometry software are powerful tools that can be used to model purely mathematical phenomena (e.g., the behavior of polynomials) as well as physical phenomena.

Modeling Standards *Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol* (★).

Mathematics | High School—Geometry

An understanding of the attributes and relationships of geometric objects can be applied in diverse contexts—interpreting a schematic drawing, estimating the amount of wood needed to frame a sloping roof, rendering computer graphics, or designing a sewing pattern for the most efficient use of material.

Although there are many types of geometry, school mathematics is devoted primarily to plane Euclidean geometry, studied both synthetically (without coordinates) and analytically (with coordinates). Euclidean geometry is characterized most importantly by the Parallel Postulate, that through a point not on a given line there is exactly one parallel line. (Spherical geometry, in contrast, has no parallel lines.)

During high school, students begin to formalize their geometry experiences from elementary and middle school, using more precise definitions and developing careful proofs. Later in college some students develop Euclidean and other geometries carefully from a small set of axioms.

The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation. Fundamental are the rigid motions: translations, rotations, reflections, and combinations of these, all of which are here assumed to preserve distance and angles (and therefore shapes generally). Reflections and rotations each explain a particular type of symmetry, and the symmetries of an object offer insight into its attributes—as when the reflective symmetry of an isosceles triangle assures that its base angles are congruent.

In the approach taken here, two geometric figures are defined to be congruent if there is a sequence of rigid motions that carries one onto the other. This is the principle of superposition. For triangles, congruence means the equality of all corresponding pairs of sides and all corresponding pairs of angles. During the middle grades, through experiences drawing triangles from given conditions, students notice ways to specify enough measures in a triangle to ensure that all triangles drawn with those measures are congruent. Once these triangle congruence criteria (ASA, SAS, and SSS) are established using rigid motions, they can be used to prove theorems about triangles, quadrilaterals, and other geometric figures.

Similarity transformations (rigid motions followed by dilations) define similarity in the same way that rigid motions define congruence, thereby formalizing the similarity ideas of "same shape" and "scale factor" developed in the middle grades. These transformations lead to the criterion for triangle similarity that two pairs of corresponding angles are congruent.

The definitions of sine, cosine, and tangent for acute angles are founded on right triangles and similarity, and, with the Pythagorean Theorem, are fundamental in many real-world and theoretical situations. The Pythagorean Theorem is generalized to non-right triangles by the Law of Cosines. Together, the Laws of Sines and Cosines embody the triangle congruence criteria for the cases where three pieces of information suffice to completely solve a triangle. Furthermore, these laws yield two possible solutions in the ambiguous case, illustrating that Side-Side-Angle is not a congruence criterion.

Analytic geometry connects algebra and geometry, resulting in powerful methods of analysis and problem solving. Just as the number line associates numbers with locations in one dimension, a pair of perpendicular axes associates pairs of numbers with locations in two dimensions. This correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof. Geometric transformations of the graphs of equations correspond to algebraic changes in their equations.

Dynamic geometry environments provide students with experimental and modeling tools that allow them to investigate geometric phenomena in much the same way as computer algebra systems allow them to experiment with algebraic phenomena.

Connections to Equations. The correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof.

Geometry Overview

Congruence

- Experiment with transformations in the plane
- Understand congruence in terms of rigid motions
- Prove geometric theorems
- Make geometric constructions

Similarity, Right Triangles, and Trigonometry

- Understand similarity in terms of similarity transformations
- Prove theorems involving similarity
- Define trigonometric ratios and solve problems involving right triangles
- Apply trigonometry to general triangles

Circles

- Understand and apply theorems about circles
- Find arc lengths and areas of sectors of circles

Expressing Geometric Properties with Equations

- Translate between the geometric description and the equation for a conic section
- Use coordinates to prove simple geometric theorems algebraically

Geometric Measurement and Dimension

- Explain volume formulas and use them to solve problems
- Visualize relationships between two-dimensional and three-dimensional objects

Modeling with Geometry

- Apply geometric concepts in modeling situations

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Congruence G-CO

Experiment with transformations in the plane

1. Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
2. Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).
3. Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.
4. Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.
5. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

Understand congruence in terms of rigid motions

6. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
7. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
8. Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

Prove geometric theorems

9. Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*
10. Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*
11. Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.*

Make geometric constructions

12. Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.*
13. Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

Similarity, Right Triangles, and Trigonometry G-SRT

Understand similarity in terms of similarity transformations

1. Verify experimentally the properties of dilations given by a center and a scale factor:
   a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
   b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.
2. Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.
3. Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

Prove theorems involving similarity

4. Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.*
5. Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

Define trigonometric ratios and solve problems involving right triangles

6. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
7. Explain and use the relationship between the sine and cosine of complementary angles.
8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.★

Apply trigonometry to general triangles

9. (+) Derive the formula $A = 1/2\ ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
10. (+) Prove the Laws of Sines and Cosines and use them to solve problems.
11. (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

Circles G-C

Understand and apply theorems about circles

1. Prove that all circles are similar.
2. Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*
3. Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.
4. (+) Construct a tangent line from a point outside a given circle to the circle.

Find arc lengths and areas of sectors of circles

5. Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

Expressing Geometric Properties with Equations G-GPE

Translate between the geometric description and the equation for a conic section

1. Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.
2. Derive the equation of a parabola given a focus and directrix.
3. (+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.

Use coordinates to prove simple geometric theorems algebraically

4. Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, $\sqrt{3}$) lies on the circle centered at the origin and containing the point (0, 2).*
5. Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).
6. Find the point on a directed line segment between two given points that partitions the segment in a given ratio.
7. Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.★

Geometric Measurement and Dimension G-GMD

Explain volume formulas and use them to solve problems

1. Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.*
2. (+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.
3. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.★

Visualize relationships between two-dimensional and three-dimensional objects

4. Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

Modeling with Geometry G-MG

Apply geometric concepts in modeling situations

1. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).★
2. Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).★
3. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).★

Mathematics | High School—Statistics and Probability★

Decisions or predictions are often based on data—numbers in context. These decisions or predictions would be easy if the data always sent a clear message, but the message is often obscured by variability. Statistics provides tools for describing variability in data and for making informed decisions that take it into account.

Data are gathered, displayed, summarized, examined, and interpreted to discover patterns and deviations from patterns. Quantitative data can be described in terms of key characteristics: measures of shape, center, and spread. The shape of a data distribution might be described as symmetric, skewed, flat, or bell shaped, and it might be summarized by a statistic measuring center (such as mean or median) and a statistic measuring spread (such as standard deviation or interquartile range). Different distributions can be compared numerically using these statistics or compared visually using plots. Knowledge of center and spread are not enough to describe a distribution. Which statistics to compare, which plots to use, and what the results of a comparison might mean, depend on the question to be investigated and the real-life actions to be taken.

Randomization has two important uses in drawing statistical conclusions. First, collecting data from a random sample of a population makes it possible to draw valid conclusions about the whole population, taking variability into account. Second, randomly assigning individuals to different treatments allows a fair comparison of the effectiveness of those treatments. A statistically significant outcome is one that is unlikely to be due to chance alone, and this can be evaluated only under the condition of randomness. The conditions under which data are collected are important in drawing conclusions from the data; in critically reviewing uses of statistics in public media and other reports, it is important to consider the study design, how the data were gathered, and the analyses employed as well as the data summaries and the conclusions drawn.

Random processes can be described mathematically by using a probability model: a list or description of the possible outcomes (the sample space), each of which is assigned a probability. In situations such as flipping a coin, rolling a number cube, or drawing a card, it might be reasonable to assume various outcomes are equally likely. In a probability model, sample points represent outcomes and combine to make up events; probabilities of events can be computed by applying the Addition and Multiplication Rules. Interpreting these probabilities relies on an understanding of independence and conditional probability, which can be approached through the analysis of two-way tables.

Technology plays an important role in statistics and probability by making it possible to generate plots, regression functions, and correlation coefficients, and to simulate many possible outcomes in a short amount of time.

Connections to Functions and Modeling. Functions may be used to describe data; if the data suggest a linear relationship, the relationship can be modeled with a regression line, and its strength and direction can be expressed through a correlation coefficient.

Statistics and Probability Overview

Interpreting Categorical and Quantitative Data

- **Summarize, represent, and interpret data on a single count or measurement variable**
- **Summarize, represent, and interpret data on two categorical and quantitative variables**
- **Interpret linear models**

Making Inferences and Justifying Conclusions

- **Understand and evaluate random processes underlying statistical experiments**
- **Make inferences and justify conclusions from sample surveys, experiments and observational studies**

Conditional Probability and the Rules of Probability

- **Understand independence and conditional probability and use them to interpret data**
- **Use the rules of probability to compute probabilities of compound events in a uniform probability model**

Using Probability to Make Decisions

- **Calculate expected values and use them to solve problems**
- **Use probability to evaluate outcomes of decisions**

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Interpreting Categorical and Quantitative Data — S-ID

Summarize, represent, and interpret data on a single count or measurement variable

1. Represent data with plots on the real number line (dot plots, histograms, and box plots).
2. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
3. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).
4. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

Summarize, represent, and interpret data on two categorical and quantitative variables

5. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.
6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.
   a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.*
   b. Informally assess the fit of a function by plotting and analyzing residuals.
   c. Fit a linear function for a scatter plot that suggests a linear association.

Interpret linear models

7. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
8. Compute (using technology) and interpret the correlation coefficient of a linear fit.
9. Distinguish between correlation and causation.

Making Inferences and Justifying Conclusions — S-IC

Understand and evaluate random processes underlying statistical experiments

1. Understand statistics as a process for making inferences about population parameters based on a random sample from that population.
2. Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?*

Make inferences and justify conclusions from sample surveys, experiments, and observational studies

3. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.

4. Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.

5. Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.

6. Evaluate reports based on data.

Conditional Probability and the Rules of Probability S-CP

Understand independence and conditional probability and use them to interpret data

1. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").

2. Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.

3. Understand the conditional probability of *A* given *B* as *P*(*A* and *B*)/*P*(*B*), and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*.

4. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.*

5. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.*

Use the rules of probability to compute probabilities of compound events in a uniform probability model

6. Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A*, and interpret the answer in terms of the model.

7. Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model.

8. (+) Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model.

9. (+) Use permutations and combinations to compute probabilities of compound events and solve problems.

Using Probability to Make Decisions S-MD

Calculate expected values and use them to solve problems

1. (+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.

2. (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.

3. (+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. *For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.*

4. (+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. *For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?*

Use probability to evaluate outcomes of decisions

5. (+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.

   a. Find the expected payoff for a game of chance. *For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant.*

   b. Evaluate and compare strategies on the basis of expected values. *For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.*

6. (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).

7. (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).

Note on courses and transitions

The high school portion of the Standards for Mathematical Content specifies the mathematics all students should study for college and career readiness. These standards do not mandate the sequence of high school courses. However, the organization of high school courses is a critical component to implementation of the standards. To that end, sample high school pathways for mathematics – in both a traditional course sequence (Algebra I, Geometry, and Algebra II) as well as an integrated course sequence (Mathematics 1, Mathematics 2, Mathematics 3) – will be made available shortly after the release of the final Common Core State Standards. It is expected that additional model pathways based on these standards will become available as well.

The standards themselves do not dictate curriculum, pedagogy, or delivery of content. In particular, states may handle the transition to high school in different ways. For example, many students in the U.S. today take Algebra I in the 8th grade, and in some states this is a requirement. The K-7 standards contain the prerequisites to prepare students for Algebra I by 8th grade, and the standards are designed to permit states to continue existing policies concerning Algebra I in 8th grade.

A second major transition is the transition from high school to post-secondary education for college and careers. The evidence concerning college and career readiness shows clearly that the knowledge, skills, and practices important for readiness include a great deal of mathematics prior to the boundary defined by (+) symbols in these standards. Indeed, some of the highest priority content for college and career readiness comes from Grades 6-8. This body of material includes powerfully useful proficiencies such as applying ratio reasoning in real-world and mathematical problems, computing fluently with positive and negative fractions and decimals, and solving real-world and mathematical problems involving angle measure, area, surface area, and volume. Because important standards for college and career readiness are distributed across grades and courses, systems for evaluating college and career readiness should reach as far back in the standards as Grades 6-8. It is important to note as well that cut scores or other information generated by assessment systems for college and career readiness should be developed in collaboration with representatives from higher education and workforce development programs, and should be validated by subsequent performance of students in college and the workforce.

Glossary

Addition and subtraction within 5, 10, 20, 100, or 1000. Addition or subtraction of two whole numbers with whole number answers, and with sum or minuend in the range 0-5, 0-10, 0-20, or 0-100, respectively. Example: 8 + 2 = 10 is an addition within 10, 14 – 5 = 9 is a subtraction within 20, and 55 – 18 = 37 is a subtraction within 100.

Additive inverses. Two numbers whose sum is 0 are additive inverses of one another. Example: $^3/4$ and – $^3/4$ are additive inverses of one another because $^3/4 + (- ^3/4) = (- ^3/4) + ^3/4 = 0$.

Associative property of addition. See Table 3 in this Glossary.

Associative property of multiplication. See Table 3 in this Glossary.

Bivariate data. Pairs of linked numerical observations. Example: a list of heights and weights for each player on a football team.

Box plot. A method of visually displaying a distribution of data values by using the median, quartiles, and extremes of the data set. A box shows the middle 50% of the data.[1]

Commutative property. See Table 3 in this Glossary.

Complex fraction. A fraction $^A/_B$ where *A* and/or *B* are fractions (*B* nonzero).

Computation algorithm. A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly. *See also:* computation strategy.

Computation strategy. Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another. *See also:* computation algorithm.

Congruent. Two plane or solid figures are congruent if one can be obtained from the other by rigid motion (a sequence of rotations, reflections, and translations).

Counting on. A strategy for finding the number of objects in a group without having to count every member of the group. For example, if a stack of books is known to have 8 books and 3 more books are added to the top, it is not necessary to count the stack all over again. One can find the total by *counting on*—pointing to the top book and saying "eight," following this with "nine, ten, eleven. There are eleven books now."

Dot plot. *See:* line plot.

Dilation. A transformation that moves each point along the ray through the point emanating from a fixed center, and multiplies distances from the center by a common scale factor.

Expanded form. A multi-digit number is expressed in expanded form when it is written as a sum of single-digit multiples of powers of ten. For example, 643 = 600 + 40 + 3.

Expected value. For a random variable, the weighted average of its possible values, with weights given by their respective probabilities.

First quartile. For a data set with median *M*, the first quartile is the median of the data values less than *M*. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the first quartile is 6.[2] *See also:* median, third quartile, interquartile range.

Fraction. A number expressible in the form $^a/_b$ where *a* is a whole number and *b* is a positive whole number. (The word *fraction* in these standards always refers to a non-negative number.) *See also:* rational number.

Identity property of 0. See Table 3 in this Glossary.

Independently combined probability models. Two probability models are said to be combined independently if the probability of each ordered pair in the combined model equals the product of the original probabilities of the two individual outcomes in the ordered pair.

---

[1]Adapted from Wisconsin Department of Public Instruction, http://dpi.wi.gov/standards/mathglos.html, accessed March 2, 2010.
[2]Many different methods for computing quartiles are in use. The method defined here is sometimes called the Moore and McCabe method. See Langford, E., "Quartiles in Elementary Statistics," *Journal of Statistics Education* Volume 14, Number 3 (2006).

Integer. A number expressible in the form a or $-a$ for some whole number a.

Interquartile Range. A measure of variation in a set of numerical data, the interquartile range is the distance between the first and third quartiles of the data set. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the interquartile range is 15 – 6 = 9. *See also:* first quartile, third quartile.

Line plot. A method of visually displaying a distribution of data values where each data value is shown as a dot or mark above a number line. Also known as a dot plot.[3]

Mean. A measure of center in a set of numerical data, computed by adding the values in a list and then dividing by the number of values in the list.[4] Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean is 21.

Mean absolute deviation. A measure of variation in a set of numerical data, computed by adding the distances between each data value and the mean, then dividing by the number of data values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean absolute deviation is 20.

Median. A measure of center in a set of numerical data. The median of a list of values is the value appearing at the center of a sorted version of the list—or the mean of the two central values, if the list contains an even number of values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 90}, the median is 11.

Midline. In the graph of a trigonometric function, the horizontal line halfway between its maximum and minimum values.

Multiplication and division within 100. Multiplication or division of two whole numbers with whole number answers, and with product or dividend in the range 0-100. Example: 72 ÷ 8 = 9.

Multiplicative inverses. Two numbers whose product is 1 are multiplicative inverses of one another. Example: 3/4 and 4/3 are multiplicative inverses of one another because 3/4 × 4/3 = 4/3 × 3/4 = 1.

Number line diagram. A diagram of the number line used to represent numbers and support reasoning about them. In a number line diagram for measurement quantities, the interval from 0 to 1 on the diagram represents the unit of measure for the quantity.

Percent rate of change. A rate of change expressed as a percent. Example: if a population grows from 50 to 55 in a year, it grows by 5/50 = 10% per year.

Probability distribution. The set of possible values of a random variable with a probability assigned to each.

Properties of operations. See Table 3 in this Glossary.

Properties of equality. See Table 4 in this Glossary.

Properties of inequality. See Table 5 in this Glossary.

Properties of operations. See Table 3 in this Glossary.

Probability. A number between 0 and 1 used to quantify likelihood for processes that have uncertain outcomes (such as tossing a coin, selecting a person at random from a group of people, tossing a ball at a target, or testing for a medical condition).

Probability model. A probability model is used to assign probabilities to outcomes of a chance process by examining the nature of the process. The set of all outcomes is called the sample space, and their probabilities sum to 1. *See also:* uniform probability model.

Random variable. An assignment of a numerical value to each outcome in a sample space.

Rational expression. A quotient of two polynomials with a non-zero denominator.

Rational number. A number expressible in the form a/b or $-a/b$ for some fraction a/b. The rational numbers include the integers.

Rectilinear figure. A polygon all angles of which are right angles.

Rigid motion. A transformation of points in space consisting of a sequence of

[3]Adapted from Wisconsin Department of Public Instruction, *op. cit*.
[4]To be more precise, this defines the *arithmetic mean*.

one or more translations, reflections, and/or rotations. Rigid motions are here assumed to preserve distances and angle measures.

Repeating decimal. The decimal form of a rational number. *See also:* terminating decimal.

Sample space. In a probability model for a random process, a list of the individual outcomes that are to be considered.

Scatter plot. A graph in the coordinate plane representing a set of bivariate data. For example, the heights and weights of a group of people could be displayed on a scatter plot.[5]

Similarity transformation. A rigid motion followed by a dilation.

Tape diagram. A drawing that looks like a segment of tape, used to illustrate number relationships. Also known as a strip diagram, bar model, fraction strip, or length model.

Terminating decimal. A decimal is called terminating if its repeating digit is 0.

Third quartile. For a data set with median *M*, the third quartile is the median of the data values greater than *M*. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the third quartile is 15. *See also:* median, first quartile, interquartile range.

Transitivity principle for indirect measurement. If the length of object A is greater than the length of object B, and the length of object B is greater than the length of object C, then the length of object A is greater than the length of object C. This principle applies to measurement of other quantities as well.

Uniform probability model. A probability model which assigns equal probability to all outcomes. *See also:* probability model.

Vector. A quantity with magnitude and direction in the plane or in space, defined by an ordered pair or triple of real numbers.

Visual fraction model. A tape diagram, number line diagram, or area model.

Whole numbers. The numbers 0, 1, 2, 3, ….

[5]Adapted from Wisconsin Department of Public Instruction, *op. cit*.

Table 1. Common addition and subtraction situations.[6]

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ?	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 = 5
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? 5 – 2 = ?	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? 5 – ? = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? ? – 2 = 3
	Total Unknown	**Addend Unknown**	**Both Addends Unknown[1]**
Put Together/ Take Apart[2]	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5, 5 – 3 = ?	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? 5 = 0 + 5, 5 = 5 + 0 5 = 1 + 4, 5 = 4 + 1 5 = 2 + 3, 5 = 3 + 2
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare[3]	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? ("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? 2 + ? = 5, 5 – 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 – 3 = ?, ? + 3 = 5

[1]These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean makes or results in but always does mean is the same number as.
[2]Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation, especially for small numbers less than or equal to 10.
[3]For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.

[6]Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).

TABLE 2. Common multiplication and division situations.[7]

	Unknown Product	Group Size Unknown ("How many in each group?" Division)	Number of Groups Unknown ("How many groups?" Division)
	3 × 6 = ?	**3 × ? = 18, and 18 ÷ 3 = ?**	**? × 6 = 18, and 18 ÷ 6 = ?**
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example*. You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example*. You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example*. You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays,[4] Area[5]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example*. What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example*. A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example*. A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example*. A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example*. A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat? *Measurement example*. A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	$a \times b = ?$	$a \times ? = p$, and $p \div a = ?$	$? \times b = p$, and $p \div b = ?$

[4]The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.
[5]Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

[7]The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

Table 3. The properties of operations. Here a, b and c stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number system.

Associative property of addition	$(a + b) + c = a + (b + c)$
Commutative property of addition	$a + b = b + a$
Additive identity property of 0	$a + 0 = 0 + a = a$
Existence of additive inverses	For every a there exists $-a$ so that $a + (-a) = (-a) + a = 0$.
Associative property of multiplication	$(a \times b) \times c = a \times (b \times c)$
Commutative property of multiplication	$a \times b = b \times a$
Multiplicative identity property of 1	$a \times 1 = 1 \times a = a$
Existence of multiplicative inverses	For every $a \neq 0$ there exists $1/a$ so that $a \times 1/a = 1/a \times a = 1$.
Distributive property of multiplication over addition	$a \times (b + c) = a \times b + a \times c$

Table 4. The properties of equality. Here a, b and c stand for arbitrary numbers in the rational, real, or complex number systems.

Reflexive property of equality	$a = a$
Symmetric property of equality	If $a = b$, then $b = a$.
Transitive property of equality	If $a = b$ and $b = c$, then $a = c$.
Addition property of equality	If $a = b$, then $a + c = b + c$.
Subtraction property of equality	If $a = b$, then $a - c = b - c$.
Multiplication property of equality	If $a = b$, then $a \times c = b \times c$.
Division property of equality	If $a = b$ and $c \neq 0$, then $a \div c = b \div c$.
Substitution property of equality	If $a = b$, then b may be substituted for a in any expression containing a.

Table 5. The properties of inequality. Here a, b and c stand for arbitrary numbers in the rational or real number systems.

Exactly one of the following is true: $a < b$, $a = b$, $a > b$.
If $a > b$ and $b > c$ then $a > c$.
If $a > b$, then $b < a$.
If $a > b$, then $-a < -b$.
If $a > b$, then $a \pm c > b \pm c$.
If $a > b$ and $c > 0$, then $a \times c > b \times c$.
If $a > b$ and $c < 0$, then $a \times c < b \times c$.
If $a > b$ and $c > 0$, then $a \div c > b \div c$.
If $a > b$ and $c < 0$, then $a \div c < b \div c$.

Sample of Works Consulted

Existing state standards documents.

Research summaries and briefs provided to the Working Group by researchers.

National Assessment Governing Board, *Mathematics Framework for the 2009 National Assessment of Educational Progress*. U.S. Department of Education, 2008.

NAEP Validity Studies Panel, Validity Study of the NAEP Mathematics Assessment: Grades 4 and 8. Daro et al., 2007.

Mathematics documents from: Alberta, Canada; Belgium; China; Chinese Taipei; Denmark; England; Finland; Hong Kong; India; Ireland; Japan; Korea; New Zealand; Singapore; Victoria (British Columbia).

Adding it Up: Helping Children Learn Mathematics. National Research Council, Mathematics Learning Study Committee, 2001.

Benchmarking for Success: Ensuring U.S. Students Receive a World-Class Education. National Governors Association, Council of Chief State School Officers, and Achieve, Inc., 2008.

Crossroads in Mathematics (1995) and *Beyond Crossroads* (2006). American Mathematical Association of Two-Year Colleges (AMATYC).

Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A Quest for Coherence. National Council of Teachers of Mathematics, 2006.

Focus in High School Mathematics: *Reasoning and Sense Making*. National Council of Teachers of Mathematics. Reston, VA: NCTM.

Foundations for Success: The Final Report of the National Mathematics Advisory Panel. U.S. Department of Education: Washington, DC, 2008.

Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report: A PreK-12 Curriculum Framework.

How People Learn: Brain, Mind, Experience, and School. Bransford, J.D., Brown, A.L., and Cocking, R.R., eds. Committee on Developments in the Science of Learning, Commission on Behavioral and Social Sciences and Education, National Research Council, 1999.

Mathematics and Democracy, The Case for Quantitative Literacy, Steen, L.A. (ed.). National Council on Education and the Disciplines, 2001.

Mathematics Learning in Early Childhood: Paths Toward Excellence and Equity. Cross, C.T., Woods, T.A., and Schweingruber, S., eds. Committee on Early Childhood Mathematics, National Research Council, 2009.

The Opportunity Equation: Transforming Mathematics and Science Education for Citizenship and the Global Economy. The Carnegie Corporation of New York and the Institute for Advanced Study, 2009. Online: http://www.opportunityequation.org/

Principles and Standards for School Mathematics. National Council of Teachers of Mathematics, 2000.

The Proficiency Illusion. Cronin, J., Dahlin, M., Adkins, D., and Kingsbury, G.G.; foreword by C.E. Finn, Jr., and M. J. Petrilli. Thomas B. Fordham Institute, 2007.

Ready or Not: Creating a High School Diploma That Counts. American Diploma Project, 2004.

A Research Companion to Principles and Standards for School Mathematics. National Council of Teachers of Mathematics, 2003.

Sizing Up State Standards 2008. American Federation of Teachers, 2008.

A Splintered Vision: An Investigation of U.S. Science and Mathematics Education. Schmidt, W.H., McKnight, C.C., Raizen, S.A., et al. U.S. National Research Center for the Third International Mathematics and Science Study, Michigan State University, 1997.

Stars By Which to Navigate? Scanning National and International Education Standards in 2009. Carmichael, S.B., Wilson. W.S, Finn, Jr., C.E., Winkler, A.M., and Palmieri, S. Thomas B. Fordham Institute, 2009.

Askey, R., "Knowing and Teaching Elementary Mathematics," *American Educator*, Fall 1999.

Aydogan, C., Plummer, C., Kang, S. J., Bilbrey, C., Farran, D. C., & Lipsey, M. W. (2005). An investigation of prekindergarten curricula: Influences on classroom characteristics and child engagement. Paper presented at the NAEYC.

Blum, W., Galbraith, P. L., Henn, H-W. and Niss, M. (Eds) *Applications and Modeling in Mathematics Education*, ICMI Study 14. Amsterdam: Springer.

Brosterman, N. (1997). *Inventing kindergarten*. New York: Harry N. Abrams.

Clements, D. H., & Sarama, J. (2009). *Learning and teaching early math: The learning trajectories approach*. New York: Routledge.

Clements, D. H., Sarama, J., & DiBiase, A.-M. (2004). Clements, D. H., Sarama, J., & DiBiase, A.-M. (2004). *Engaging young children in mathematics: Standards for early childhood mathematics education*. Mahwah, NJ: Lawrence Erlbaum Associates.

Cobb and Moore, "Mathematics, Statistics, and Teaching," *Amer. Math. Monthly* 104(9), pp. 801-823, 1997.

Confrey, J., "Tracing the Evolution of Mathematics Content Standards in the United States: Looking Back and Projecting Forward." K12 Mathematics Curriculum Standards conference proceedings, February 5-6, 2007.

Conley, D.T. *Knowledge and Skills for University Success*, 2008.

Conley, D.T. *Toward a More Comprehensive Conception of College Readiness*, 2007.

Cuoco, A., Goldenberg, E. P., and Mark, J., "Habits of Mind: An Organizing Principle for a Mathematics Curriculum," *Journal of Mathematical Behavior*, 15(4), 375-402, 1996.

Carpenter, T. P., Fennema, E., Franke, M. L., Levi, L., & Empson, S. B. (1999). *Children's Mathematics: Cognitively Guided Instruction*. Portsmouth, NH: Heinemann.

Van de Walle, J. A., Karp, K., & Bay-Williams, J. M. (2010). *Elementary and Middle School Mathematics: Teaching Developmentally* (Seventh ed.). Boston: Allyn and Bacon.

Ginsburg, A., Leinwand, S., and Decker, K., "Informing Grades 1-6 Standards Development: What Can Be Learned from High-Performing Hong Kong, Korea, and Singapore?" American Institutes for Research, 2009.

Ginsburg et al., "What the United States Can Learn From Singapore's World-Class Mathematics System (and what Singapore can learn from the United States)," American Institutes for Research, 2005.

Ginsburg et al., "Reassessing U.S. International Mathematics Performance: New Findings from the 2003 TIMMS and PISA," American Institutes for Research, 2005.

Ginsburg, H. P., Lee, J. S., & Stevenson-Boyd, J. (2008). Mathematics education for young children: What it is and how to promote it. *Social Policy Report*, 22(1), 1-24.

Harel, G., "What is Mathematics? A Pedagogical Answer to a Philosophical Question," in R. B. Gold and R. Simons (eds.), *Current Issues in the Philosophy of Mathematics from the Perspective of Mathematicians*. Mathematical Association of America, 2008.

Henry, V. J., & Brown, R. S. (2008). First-grade basic facts: An investigation into teaching and learning of an accelerated, high-demand memorization standard. *Journal for Research in Mathematics Education*, 39, 153-183.

Howe, R., "From Arithmetic to Algebra."

Howe, R., "Starting Off Right in Arithmetic," http://math.arizona.edu/~ime/2008-09/MIME/BegArith.pdf.

Jordan, N. C., Kaplan, D., Ramineni, C., and Locuniak, M. N., "Early math matters: kindergarten number competence and later mathematics outcomes," *Dev. Psychol.* 45, 850–867, 2009.

Kader, G., "Means and MADS," *Mathematics Teaching in the Middle School*, 4(6), 1999, pp. 398-403.

Kilpatrick, J., Mesa, V., and Sloane, F., "U.S. Algebra Performance in an International Context," in Loveless (ed.), *Lessons Learned: What International Assessments Tell Us About Math Achievement*. Washington, D.C.: Brookings Institution Press, 2007.

Leinwand, S., and Ginsburg, A., "Measuring Up: How the Highest Performing State (Massachusetts) Compares to the Highest Performing Country (Hong Kong) in Grade 3 Mathematics," American Institutes for Research, 2009.

Niss, M., "Quantitative Literacy and Mathematical Competencies," in *Quantitative Literacy: Why Numeracy Matters for Schools and Colleges*, Madison, B. L., and Steen, L.A. (eds.), National Council on Education and the Disciplines. Proceedings of the National Forum on Quantitative Literacy held at the National Academy of Sciences in Washington, D.C., December 1-2, 2001.

Pratt, C. (1948). I learn from children. New York: Simon and Schuster.

Reys, B. (ed.), *The Intended Mathematics Curriculum as Represented in State-Level Curriculum Standards: Consensus or Confusion?* IAP-Information Age Publishing, 2006.

Sarama, J., & Clements, D. H. (2009). *Early childhood mathematics education research: Learning trajectories for young children*. New York: Routledge.

Schmidt, W., Houang, R., and Cogan, L., "A Coherent Curriculum: The Case of Mathematics," *American Educator*, Summer 2002, p. 4.

Schmidt, W.H., and Houang, R.T., "Lack of Focus in the Intended Mathematics Curriculum: Symptom or Cause?" in Loveless (ed.), *Lessons Learned: What International Assessments Tell Us About Math Achievement*. Washington, D.C.: Brookings Institution Press, 2007.

Steen, L.A., "Facing Facts: Achieving Balance in High School Mathematics." *Mathematics Teacher*, Vol. 100. Special Issue.

Wu, H., "Fractions, decimals, and rational numbers," 2007, http://math.berkeley.edu/~wu/ (March 19, 2008).

Wu, H., "Lecture Notes for the 2009 Pre-Algebra Institute," September 15, 2009.

Wu, H., "Preservice professional development of mathematics teachers," http://math.berkeley.edu/~wu/pspd2.pdf.

Massachusetts Department of Education. Progress Report of the Mathematics Curriculum Framework Revision Panel, Massachusetts Department of Elementary and Secondary Education, 2009.

www.doe.mass.edu/boe/docs/0509/item5_report.pdf.

ACT College Readiness Benchmarks™

ACT College Readiness Standards™

ACT National Curriculum Survey™

Adelman, C., *The Toolbox Revisited: Paths to Degree Completion From High School Through College*, 2006.

Advanced Placement Calculus, Statistics and Computer Science Course Descriptions. May 2009, May 2010. College Board, 2008.

Aligning Postsecondary Expectations and High School Practice: The Gap Defined (ACT: Policy Implications of the ACT National Curriculum Survey Results 2005-2006).

Condition of Education, 2004: Indicator 30, Top 30 Postsecondary Courses, U.S. Department of Education, 2004.

Condition of Education, 2007: High School Course-Taking. U.S. Department of Education, 2007.

Crisis at the Core: Preparing All Students for College and Work, ACT.

Achieve, Inc., Florida Postsecondary Survey, 2008.

Golfin, Peggy, et al. CNA Corporation. *Strengthening Mathematics at the Postsecondary Level: Literature Review and Analysis*, 2005.

Camara, W.J., Shaw, E., and Patterson, B. (June 13, 2009). First Year English and Math College Coursework. College Board: New York, NY (Available from authors).

CLEP Precalculus Curriculum Survey: Summary of Results. The College Board, 2005.

College Board Standards for College Success: Mathematics and Statistics. College Board, 2006.

Miller, G.E., Twing, J., and Meyers, J. "Higher Education Readiness Component (HERC) Correlation Study." Austin, TX: Pearson.

On Course for Success: A Close Look at Selected High School Courses That Prepare All Students for College and Work, ACT.

Out of Many, One: Towards Rigorous Common Core Standards from the Ground Up. Achieve, 2008.

Ready for College and Ready for Work: Same or Different? ACT.

Rigor at Risk: Reaffirming Quality in the High School Core Curriculum, ACT.

The Forgotten Middle: Ensuring that All Students Are on Target for College and Career Readiness before High School, ACT.

Achieve, Inc., Virginia Postsecondary Survey, 2004.

ACT Job Skill Comparison Charts.

Achieve, Mathematics at Work, 2008.

The American Diploma Project Workplace Study. National Alliance of Business Study, 2002.

Carnevale, Anthony and Desrochers, Donna. *Connecting Education Standards and Employment: Course-taking Patterns of Young Workers,* 2002.

Colorado Business Leaders' Top Skills, 2006.

Hawai'i Career Ready Study: access to living wage careers from high school, 2007.

States' Career Cluster Initiative. *Essential Knowledge and Skill Statements*, 2008.

ACT WorkKeys Occupational Profiles™.

Program for International Student Assessment (PISA), 2006.

Trends in International Mathematics and Science Study (TIMSS), 2007.

International Baccalaureate, Mathematics Standard Level, 2006.

University of Cambridge International Examinations: General Certificate of Secondary Education in Mathematics, 2009.

EdExcel, General Certificate of Secondary Education, Mathematics, 2009.

Blachowicz, Camille, and Fisher, Peter. "Vocabulary Instruction." In *Handbook of Reading Research*, Volume III, edited by Michael Kamil, Peter Mosenthal, P. David Pearson, and Rebecca Barr, pp. 503-523. Mahwah, NJ: Lawrence Erlbaum Associates, 2000.

Gándara, Patricia, and Contreras, Frances. *The Latino Education Crisis: The Consequences of Failed Social Policies.* Cambridge, Ma: Harvard University Press, 2009.

Moschkovich, Judit N. "Supporting the Participation of English Language Learners in Mathematical Discussions." *For the Learning of Mathematics* 19 (March 1999): 11-19.

Moschkovich, J. N. (in press). Language, culture, and equity in secondary mathematics classrooms. To appear in F. Lester & J. Lobato (ed.), Teaching and Learning Mathematics: Translating Research to the Secondary Classroom, Reston, VA: NCTM.

Moschkovich, Judit N. "Examining Mathematical Discourse Practices," *For the Learning of Mathematics* 27 (March 2007): 24-30.

Moschkovich, Judit N. "Using Two Languages when Learning Mathematics: How Can Research Help Us Understand Mathematics Learners Who Use Two Languages?" *Research Brief and Clip*, National Council of Teachers of Mathematics, 2009 http://www.nctm.org/uploadedFiles/Research_News_and_Advocacy/Research/Clips_and_Briefs/Research_brief_12_Using_2.pdf. (accessed November 25, 2009).

Moschkovich, J.N. (2007) Bilingual Mathematics Learners: How views of language, bilingual learners, and mathematical communication impact instruction. In Nasir, N. and Cobb, P. (eds.), *Diversity, Equity, and Access to Mathematical Ideas*. New York: Teachers College Press, 89-104.

Schleppegrell, M.J. (2007). The linguistic challenges of mathematics teaching and learning: A research review. *Reading & Writing Quarterly,* 23:139-159.

Individuals with Disabilities Education Act (IDEA), 34 CFR §300.34 (a). (2004).

Individuals with Disabilities Education Act (IDEA), 34 CFR §300.39 (b)(3). (2004).

Office of Special Education Programs, U.S. Department of Education. "IDEA Regulations: Identification of Students with Specific Learning Disabilities," 2006.

Thompson, S. J., Morse, A.B., Sharpe, M., and Hall, S., "Accommodations Manual: How to Select, Administer and Evaluate Use of Accommodations and Assessment for Students with Disabilities," 2nd Edition. Council of Chief State School Officers, 2005.

SECTION 4

Wisconsin's Approach to Literacy in All Subjects

Acknowledgements

Disciplinary Literacy Leadership Team

Angela Arneson
Technology Education Instructor
Denmark Middle/High School

Doug Buehl
Disciplinary Literacy Consultant
Retired Reading Specialist
Madison Metropolitan School District

Margaret Foss
Science/Mathematics Teacher
Ladysmith Middle School

Jessica Gallo
Undergraduate Instructor
UW-Madison

Paul Gilbertson
Principal
Ashland Middle School

Jane Gustafson
Physical Education Teacher
Chequamegon High School
Park Falls, Wisconsin

Melissa Hedges
Curriculum Director
PK-6/8 Mathematics and Bilingual Education
Mequon-Thiensville School District

Pam Hilleshiem-Setz
Curriculum and Instruction
School to Work & Youth Apprenticeship
CESA 5
Portage, Wisconsin

Julie Kodl
Business Education and Information Technology Teacher
Owen-Withee High School

Sara Kreibich
Social Studies Teacher
Osceola High School

JoAnn Lens
Environmental Education Teacher
Hawley Environmental School
Milwaukee, Wisconsin

Sheila Marmorstone
ASC and ABE Instructor
Southwest Technical College
Fennimore, Wisconsin

Lindsay Matuszewski
Marketing Education Teacher
Bay Port High School
Green Bay, Wisconsin

Sally Michalko
Retired Social Studies Teacher
Waukesha, Wisconsin

Barb Novak
Literacy Coach
Carl Traeger Middle School
Oshkosh, Wisconsin

Christina Peters
German Teacher
Northland Pines High School
Eagle River, Wisconsin

Jerry Redman
Instructional Services Coordinator
CESA 3
Fennimore, Wisconsin

Rachel Sauvola
Agriscience Instructor
New Richmond High School

Jody Schneider
French Teacher
Woodlands School
Milwaukee, Wisconsin

Aaron Steffes
Art Teacher
Delavan-Darien High School

Nola Starling-Ratliff
Principal
Roosevelt Elementary School
Kenosha, Wisconsin

Amy Thiel
Music Teacher
Oconto Falls High School

Peg Vogel
Director, Instructional Improvement
CESA 4
West Salem, Wisconsin

Becky Walker
Mathematics/Science/Health Curriculum Director
Appleton Area School District

Dottie Winger
Health Science Education/ Family and Consumer Education Teacher
Madison East High School

Wisconsin Department of Public Instruction Facilitators

Emilie Amundson
Assistant Director
Content and Learning Team

Janice Atkinson
Health Science Education Consultant
Career and Technical Education Team

Sara Baird
Marketing Education Consultant
Career and Technical Education Team

Barbara Bitters
Assistant Director
Career and Technical Education Team

Sheila Briggs
Assistant State Superintendent
Division for Academic Excellence

Sue Grady
Executive Assistant
Office of the State Superintendent

Jeff Hicken
Agriculture and Natural Resources Education Consultant
Career and Technical Education Team

Eric Larsen
Career Pathways Consultant
Career and Technical Education Team

Shelley Lee
Science Education Consultant
Content and Learning Team

Diana Kasbaum
Mathematics Education Consultant
Content and Learning Team

Kris McDaniel
Social Studies Education Consultant
Content and Learning Team

Diane Ryberg
Family and Consumer Education Consultant
Career and Technical Education Team

Paul Sandrock
Former Assistant Director
Content and Learning Team

Rebecca Vail
Director
Content and Learning Team

Jennifer Wegner
Business and Information Technology Education Consultant
Career and Technical Education Team

Sharon Wendt
Director
Career and Technical Education Team

Mary Jo Ziegler
Reading Education Consultant
Content and Learning Team

What is Disciplinary Literacy?

Literacy, the ability to read, write, listen, speak, think critically and perform in different ways and for different purposes, begins to develop early and becomes increasingly important as students pursue specialized fields of study in high school and beyond. The Common Core State Standards (CCSS) for Literacy in Science, Social Studies, History, and the Technical Subjects are connected to College and Career Readiness Standards that guide educators as they strive to help students meet the literacy challenges within each particular field of study. This national effort is referred to as disciplinary literacy.

In Wisconsin, disciplinary literacy is defined as the confluence of content knowledge, experiences, and skills merged with the ability to read, write, listen, speak, think critically and perform in a way that is meaningful within the context of a given field.

These abilities are important in ALL courses and subjects. While the Common Core State Standards (CCSS) for Literacy in Science, Social Studies, History, and the Technical Subjects provide standards for cross-discipline reading and writing in grades 6-12, Wisconsin recognizes the need to broaden this effort and include **all disciplines and every educator in every grade level K-12.** This literacy focus must begin as soon as children have access to formal education and continue intentionally as college and career readiness goals advance for all children in Wisconsin.

To address this expanded definition and approach to disciplinary literacy, excerpts from the K-5 Common Core State Standards for English Language Arts are included in this document. Elementary classroom teachers build the foundational literacy skills necessary for students to access all learning. Additionally, they develop content specific to deep literary study, oratory tradition and linguistic analysis; skills specific to English language arts. Literacy reaches beyond this knowledge in one content area to include reading, writing, listening, speaking and thinking critically in each discipline beginning at an early age. The applicable K-5 standards help educators in Wisconsin build a ladder of skills and dispositions that lead to accelerated achievement across disciplines and will be included in every content-specific standards document into the future.

Why is disciplinary literacy important?

The modern global society, of which our students are a part, requires postsecondary learning. An analysis of workforce trends by Georgetown University economist Anthony Carnevale and his colleagues found that nearly 60 percent of all job openings in 2007 required some postsecondary education; postsecondary success depends on students' ability to comprehend and produce the kinds of complex texts found in all disciplines. Therefore, the economic future of our state, as well as our students and their success as productive citizens and critical thinkers link to disciplinary literacy.

Textbooks, articles, manuals and historical primary source documents create specialized challenges for learners. These texts often include abstracts, figures, tables, diagrams and specialized vocabulary. The ideas are complex and build across a number of paragraphs requiring focus and strategic processing. To comprehend and produce this type of text, students must be immersed in the language and thinking processes of that discipline and they must be supported by an expert guide, their teacher (Carnegie Report, 2010).

A focus at the elementary level on foundational reading, when expanded to include engaging experiences connected to informational texts, vocabulary, and writing for content-specific purposes builds background knowledge and skills in each discipline. This increases opportunities for success as students approach more rigorous content in those disciplines (Alliance for Excellent Education, 2011).

Reading, writing, speaking, listening and critical thinking must be integrated into each discipline across all grades so that all students gradually build knowledge and skills toward college and career readiness. Collaboration among institutes of higher education, CESA Statewide Network, districts, schools, teachers and family and community will guide the implementation of the Common Core State Standards in Wisconsin.

The message is that literacy is integral to attainment of content knowledge and content is essential background knowledge for literacy development.

This interdependent relationship exists in all disciplines.

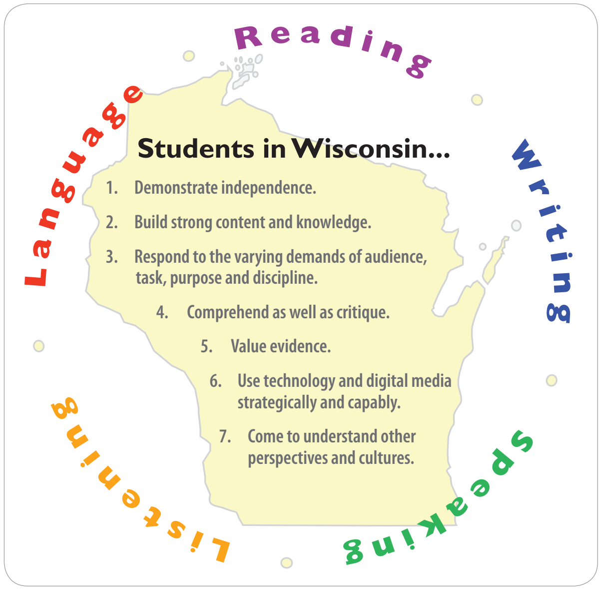
The Common Core State Standards require educators to support literacy in each classroom across the state. Since the impact of this effort is significant, it is essential that resources and supports be accessible to all educators. To build consistent understanding, DPI convened a statewide Disciplinary Literacy Leadership Team in 2011 comprised of educators from many content areas and educational backgrounds. This team was charged with examining the CCSS for Disciplinary Literacy, identifying the needs in the field for support, and gathering materials and resources to address those needs. Resources are available at www.dpi.wi.gov/standards

Wisconsin Foundations for Disciplinary Literacy

To guide understanding and professional learning, a set of foundations, developed in concert with Wisconsin's *Guiding Principles for Teaching and Learning,* directs Wisconsin's approach to disciplinary literacy.

Academic learning begins in early childhood and develops across all disciplines.

Each discipline has its own specific vocabulary, text types, and ways of communicating. Children begin learning these context- and content-specific differences early in life and continue through high school and beyond. While gardening, small children observe and learn the form and function of a root, stem, leaf and soil; or measure, mix and blend while baking a cake. School offers all students opportunities to develop the ability to, for example, think like a scientist, write like a historian, critique like an artist, problem-solve like an auto mechanic, or analyze technological advances like a health care technician. As literacy skills develop, educators gradually shift the responsibility for reading, writing, listening, speaking and critical thinking to students through guided supports in both individual and collaborative learning experiences.



Content knowledge is strengthened when educators integrate discipline-specific literacy into teaching and learning.

Educators help students recognize and understand the nuances of a discipline by using strategies that "make their thinking visible." They promote classroom reading, writing, listening, speaking and critical thinking using authentic materials that support the development of content-specific knowledge. They guide students through these complex texts by using strategies that develop conceptual understanding of language and set expectations for relevant application of skills. These literacy practices deepen students' content knowledge, strategies and skills so that their learning transfers to real world situations.

The literacy skills of reading, writing, listening, speaking and critical thinking improve when content-rich learning experiences motivate and engage students.

Educators who foster disciplinary literacy develop experiences that integrate rigorous content with relevant collaborative and creative literacy processes to motivate and engage students. Setting high expectations, they structure routines and supports that empower students to take charge of their own learning. When students work in teams to research science and mathematics concepts in the development of an invention or a graphic arts design; when they collaboratively build a blog that explains their recent marketing venture, they use specific literacy skills and strategies to solidify learning. Students need these opportunities over time to develop the precise and complex reading, writing, listening, speaking and critical thinking skills demanded in today's careers.

Students demonstrate their content knowledge through reading, writing, listening, and speaking as part of a content-literate community.

Students who are literate in a particular discipline are able to successfully read, write, and speak about that discipline and can listen to and think critically as others communicate in that community. Performance tasks that allow students to present the complexity of a content area in a way that is meaningful to the field become authentic approaches to assessing mastery within a discipline. Such tasks empower students to discover the real world connections across disciplines and to actively participate in communities of discipline-literate peers. As Wisconsin moves to the SMARTER Balanced Assessment System these performance tasks will be integral to assessment of student learning.

What research and resources are available to support educators' use of the Common Core State Standards for Literacy in All Subjects?

The Common Core State Standards for Literacy in All Subjects reflect the importance of literacy in both the oral and written language and in both productive (speaking and writing) and receptive (listening and reading) discourse. Clearly, critical and precise thinking are required to develop all of these specific strategies and skills. The standards also address the learning and functioning of language in a technological, media-driven world because the language that we use is selective depending upon the context of the conversation.

The following section will offer relevant research and resources to support professional learning in reading, writing, speaking, listening and language across disciplines. Collegial conversation and learning, both cross-discipline and within-discipline will help make the Common Core State Standards more applicable to schools and districts, and will address the needs of unique programs within those contexts. A collection of online resources will continue to develop as support materials emerge.

Reading Connections

While early reading focuses on learning that letters make sounds, and that words carry meaning, reading quickly develops to a point where the message taken from text depends on what the reader brings to it. The Carnegie Report, *Reading in the Disciplines* (2010) describes this phenomenon:

"The ability to comprehend written texts is not a static or fixed ability, but rather one that involves a dynamic relationship between the demands of texts and prior knowledge and goals of the reader."

Therefore, a musician reading a journal article that describes concepts in music theory will take more information away from the text than a music novice because of their knowledge and experience in music. As well, an individual who spends a significant amount of time reading automotive manuals will more easily navigate a cell phone manual because of familiarity with that type of text.

A chart excerpted from the Carnegie Report (2010) details a few of the generic and more discipline-specific strategies that support students as they attempt to comprehend complex text. While the generic strategies pertain across content areas, discipline-specific ones must be tailored to match the demands of the content area.

Both generic and discipline focused strategies and knowledge must be applied to the comprehension and evaluation of:

- Textbooks
- Journal and magazine articles
- Historically situated primary documents
- Full Length Books
- Newspaper Articles
- Book Chapters
- Multimedia and Digital Texts

Generic Reading Strategies	Discipline-Specific Reading Strategies
Monitor comprehension	Build prior knowledge
Pre-read	Build specialized vocabulary
Set goals	Learn to deconstruct complex sentences
Think about what one already knows	Use knowledge of text structures and genres to predict main and subordinate ideas
Ask questions	Map graphic (and mathematical) representations against explanations in the text
Make predictions	Pose discipline relevant questions
Test predictions against the text	Compare claims and propositions across texts
Re-read	Use norms for reasoning within the discipline (i.e. what counts as evidence) to evaluate claims
Summarize	
	Source: Carnegie Report, (2010)

Additional resources that support reading in specific subjects include *Content Counts! Developing Disciplinary Literacy Skills, K–6* by Jennifer L. Altieri (2011). This guide for discipline-specific literacy at the elementary level offers strategies to balance the demands of literacy while continuing to make content count and help students meet the reading, writing, speaking and listening demands of the content areas as they advance in school.

A resource by Doug Buehl (2011) entitled *Developing Readers in the Academic Disciplines* describes what it means to read, write, and think through a disciplinary lens in the adolescent years. This teacher-friendly guide helps connect literacy with disciplinary understandings to bridge academic knowledge gaps, frontload instruction, and build critical thinking through questioning.

Note on range and content of student reading

To become college and career ready, students must grapple with works of exceptional craft and thought whose range extends across genres, cultures, and centuries. Such works offer profound insights into the human condition and serve as models for students' own thinking and writing. Along with high-quality contemporary works, these texts should be chosen from seminal U.S. documents, the classics of American literature, and the timeless dramas of Shakespeare. Through wide and deep reading of literature and literary nonfiction of steadily increasing sophistication, students gain a reservoir of literary and cultural knowledge, references, and images; the ability to evaluate intricate arguments; and the capacity to surmount the challenges posed by complex texts. (*CCSS p. 35 http://www.corestandards.org/assets/CCSSI_ELA%20Standards.pdf*)

The Common Core State Standards require that all students "be able to comprehend texts of steadily increasing complexity as they progress through school" (Appendix A: Research Supporting Key Elements of the Standards, p. 2). More detailed definitions of complex text and examples of complex texts across disciplines are available in Appendix B of the English Language Arts CCSS at: www.dpi.wi.gov/standards.

Writing Connections

The Common Core State Standards call for emphasis on three types of writing: narrative, informational and logical argument. Writing that presents a logical argument is especially appropriate to discipline-specific work since credible evidence differs across content areas. The ability to consider multiple perspectives, assess the validity of claims and present a point of view is required in argumentative writing. These thinking and communication skills are "critical to college and career readiness" (Appendix A: p. 24).

A 2007 report entitled *Writing Next: Effective Strategies to Improve Writing of Adolescents in Middle and High Schools* detailed research on writing to learn, rather than only for assessment, as having a significant impact on content learning.

The study found writing to learn was equally effective for all content areas in the study (social studies, math and science) and at every grade (4-12).

Note on range and content of student writing

For students, writing is a key means of asserting and defending claims, showing what they know about a subject, and conveying what they have experienced, imagined, thought, and felt. To be college- and career-ready writers, students must take task, purpose, and audience into careful consideration, choosing words, information, structures, and formats deliberately. They need to know how to combine elements of different kinds of writing—for example, to use narrative strategies within an argument and explanation within narrative—to produce complex and nuanced writing. They need to be able to use technology strategically when creating, refining, and collaborating on writing. They have to become adept at gathering information, evaluating sources, and citing material accurately, reporting findings from their research and analysis of sources in a clear and cogent manner. They must have flexibility, concentration, and fluency to produce high quality first draft text under a tight deadline as well as the capacity to revisit and make improvements to a piece of writing over multiple drafts when circumstances encourage or require it. *(CCSS p.41 http://www.corestandards.org/assets/CCSSI_ELA%20Standards.pdf)*

When a social studies teacher guides students in taking on the perspective of a person from a specific historical era, she might ask students to write a first person narrative from that perspective. Research into that era leads students to discover personal beliefs of that historical person. They may dig into the personal experiences, ideas, and events involved in the era to visualize life in that period. They then develop a rich understanding of the era and embed language from that era into the texts that they create. (Samples of discipline-specific writing across grades and content areas are available in Appendix C of the English Language Arts CCSS at: www.dpi.wi.gov/standards.

Speaking, Listening and Language Connections

The ability to share ideas and orally communicate with credibility in a specific academic discourse empowers students and allows access to specialized groups. In *Situated Language and Learning: A Critique of Traditional Schooling*, James Paul Gee (2004) describes the need to prioritize these skills so that students are at ease as they enter situations connected to a specific content area and are more likely to continue their learning in that discipline.

As expertise develops, students feel more and more comfortable applying knowledge and skills while speaking and listening in a specific discipline.

- A media course may teach students appropriate expression, tone and rate of speech when addressing a large audience.
- Listening carefully to questions posed is a specialized skill that debate facilitators develop.
- Scientists learn to listen for bias in the perspectives presented by peers to determine the reliability of scientific outcomes.
- Artists have very specialized and specific ways of speaking about the many aspects of a piece.

A policy brief from the Alliance for Excellent Education called, *Engineering Solutions to the National Crisis in Literacy: How to Make Good on the Promise of the Common Core State Standards* describes "a staircase of literacy demands" and emphasizes the importance of a progressive development of language and literacy over time.

The conceptual understanding of "functions" in mathematics may begin to develop in elementary school in its simplest form. As the concept develops over the years, students will use the word "function" in a meaningful way when speaking and writing to describe the mathematical concept they apply. When educators explicitly connect a mathematical term to its application and repeatedly expose students to the concept connected to the term, a specialized language becomes second nature to the mathematics classroom.

Students must have extensive vocabularies, built through reading and explicit instruction embedded in the context of content learning. This enables them to comprehend complex texts, engage in purposeful writing and communicate effectively within a discipline.

Skills in determining or clarifying the meaning of words and phrases encountered, choosing flexibly from an array of strategies, and seeing an individual word as part of a network of other words that, for example, have similar denotations but different connotations allow students to access information and support their own learning.

Literacy in Multiple Languages

Increasing economic, security, cross-cultural and global demands underscore the value of literacy in more than one language. Students who think, read, write, and communicate in multiple languages are an asset to our own country and can more easily interact and compete in the world at large.

English language learners (ELL) in our classrooms face significant challenges as they add a new language and work to grasp content at the same rate as their English-speaking peers. In a report to the Carnegie Corporation entitled *Double the Work: Challenges and Solutions to Acquiring Academic Literacy for Adolescent English Language Learners (2007)* researchers found that a focus on academic literacy is crucial for ELL's success in school. In their description of academic literacy they include reading, writing and oral discourse that:

- Varies from subject to subject.
- Requires knowledge of multiple genres of text, purposes for text use and text media.
- Is influenced by students' literacies in context outside of school.
- Is influenced by students' personal, social, and cultural experiences.

The needs of our English language learners are addressed when we embed disciplinary literacy strategies into our subject area teaching. These high impact strategies and skills allow English language learners and all students to more readily access content knowledge and connect it to the prior knowledge they bring to the classroom. When educators take the initiative to understand and embed these strategies and skills, they offer additional opportunities for success to all of our students.

Who Should Use the Common Core State Standards for Literacy in All Subjects?

The term "disciplinary literacy" may be new to many Wisconsin teachers. The Common Core State Standards for Literacy in All Subjects as excerpted from the Common Core Standards for English Language Arts, are intended for all K-12 educators. Each standard is written broadly in content-neutral language, breaking down the complex skills that comprise reading, writing, speaking, listening, and language. These standards serve as a complement to the specific content-related standards of each individual discipline. Administrators and communities may also find the disciplinary literacy standards helpful in charting a clear and consistent school or district-wide approach to literacy that moves Wisconsin forward toward the goal of every student career and college ready.

References:

Altieri, Jennifer (2011). *Content Counts! Developing Disciplinary Literacy Skills, K–6*. International Reading Association. ISBN 13: 978-0-87207-838-3

Buehl, Doug. (2011). *Developing Readers in the Academic Disciplines*. International Reading Association. ISBN 13: 978-0-87207-845-1

Carnevale, A. (2010) *Center on Education and the Workforce Forecasts of Education Demand to 2018*

College and Career Readiness Standards; http://www.nc4ea.org/files/appropriate_college-readiness_standards_for_all_students-05-03-06.pdf

Common Core Standards for English Language Arts; www.corestandards.org

Washington, DC: Georgetown Center on Education and the Workforce, 2010, available at: http://www9.georgetown.edu/grad/gppi/hpi/cew/pdfs/CEW_press_conference_ppt.pdf (accessed June 7, 2011)

Double the work: Challenges and Solutions to Acquiring Academic Literacy for Adolescent English Language Learners. Carnegie Corporation. New York: 2007.

Engineering Solutions to the National Crisis in Literacy: How to Make Good on the Promise of the Common Core State Standards. Alliance for Excellent Education. Washington D.C. 2011

Gee, James Paul (2004) *Situated Language and Learning: A Critique of Traditional Schooling*

Reading in the Disciplines: The Challenges of Adolescent Literacy. Carnegie Corporation. New York: 2010

State Superintendent's Adolescent Literacy Plan (2008) Wisconsin Department of Public Instruction, Madison, WI

Vygotsky, (1978) *Mind in Society: The Development of Higher Psychological Processes* Harvard University Press; 14th edition

Writing Next: Effective Strategies to Improve Writing of Adolescents in Middle and High Schools (2007)

SECTION 5

Common Core State Standards for Literacy in All Subjects

Table of Contents

Key Design Considerations

CCR and grade-specific standards

The CCR standards anchor the document and define general, cross-disciplinary literacy expectations that must be met for students to be prepared to enter college and workforce training programs ready to succeed. The K–12 grade-specific standards define end-of-year expectations and a cumulative progression designed to enable students to meet college and career readiness expectations no later than the end of high school. The CCR and high school (grades 9–12) standards work in tandem to define the college and career readiness line—the former providing broad standards, the latter providing additional specificity. Hence, both should be considered when developing college and career readiness assessments.

Students advancing through the grades are expected to meet each year's grade-specific standards, retain or further develop skills and understandings mastered in preceding grades, and work steadily toward meeting the more general expectations described by the CCR standards.

Grade levels for K–8; grade bands for 9–10 and 11–12

The Standards use individual grade levels in kindergarten through grade 8 to provide useful specificity; the Standards use two-year bands in grades 9–12 to allow schools, districts, and states flexibility in high school course design.

A focus on results rather than means

By emphasizing required achievements, the Standards leave room for teachers, curriculum developers, and states to determine how those goals should be reached and what additional topics should be addressed. Thus, the Standards do not mandate such things as a particular writing process or the full range of metacognitive strategies that students may need to monitor and direct their thinking and learning. Teachers are thus free to provide students with whatever tools and knowledge their professional judgment and experience identify as most helpful for meeting the goals set out in the Standards.

An integrated model of literacy

Although the Standards are divided into Reading, Writing, Speaking and Listening, and Language strands for conceptual clarity, the processes of communication are closely connected, as reflected throughout this document. For example, Writing standard 9 requires that students be able to write about what they read. Likewise, Speaking and Listening standard 4 sets the expectation that students will share findings from their research.

Research and media skills blended into the Standards as a whole

To be ready for college, workforce training, and life in a technological society, students need the ability to gather, comprehend, evaluate, synthesize, and report on information and ideas, to conduct original research in order to answer questions or solve problems, and to analyze and create a high volume and extensive range of print and nonprint texts in media forms old and new. The need to conduct research and to produce and consume media is embedded into every aspect of today's curriculum. In like fashion, research and media skills and understandings are embedded throughout the Standards rather than treated in a separate section.

Shared responsibility for students' literacy development

The Standards insist that instruction in reading, writing, speaking, listening, and language be a shared responsibility within the school. The K–5 standards include expectations for reading, writing, speaking, listening, and language applicable to a range of subjects, including but not limited to ELA. The grades 6–12 standards are divided into two sections, one for ELA and the other for history/social studies, science, and technical subjects. This division reflects the unique, time-honored place of ELA teachers in developing students' literacy skills while at the same time recognizing that teachers in other areas must have a role in this development as well.

Part of the motivation behind the interdisciplinary approach to literacy promulgated by the Standards is extensive research establishing the need for college and career ready students to be proficient in reading complex informational text independently in a variety of content areas. Most of the required reading in college and workforce training programs is informational in structure and challenging in content; postsecondary education programs typically provide students with both a higher volume of such reading than is generally required in K–12 schools and comparatively little scaffolding.

The Standards are not alone in calling for a special emphasis on informational text. The 2009 reading framework of the National Assessment of Educational Progress (NAEP) requires a high and increasing proportion of informational text on its assessment as students advance through the grades.

Distribution of Literary and Informational Passages by Grade in the 2009 NAEP Reading Framework

Grade	Literary	Informational
4	50%	50%
8	45%	55%
12	30%	70%

Source: National Assessment Governing Board. (2008). *Reading framework for the 2009 National Assessment of Educational Progress.* Washington, DC: U.S. Government Printing Office.

The Standards aim to align instruction with this framework so that many more students than at present can meet the requirements of college and career readiness. In K–5, the Standards follow NAEP's lead in balancing the reading of literature with the reading of informational texts, including texts in history/social studies, science, and technical subjects. In accord with NAEP's growing emphasis on informational texts in the higher grades, the Standards demand that a significant amount of reading of informational texts take place in and outside the ELA classroom. Fulfilling the Standards for 6–12 ELA requires much greater attention to a specific category of informational text—literary nonfiction—than has been traditional. Because the ELA classroom must focus on literature (stories, drama, and poetry) as well as literary nonfiction, a great deal of informational reading in grades 6–12 must take place in other classes if the NAEP assessment framework is to be matched instructionally.[1] To measure students' growth toward college and career readiness, assessments aligned with the Standards should adhere to the distribution of texts across grades cited in the NAEP framework.

NAEP likewise outlines a distribution across the grades of the core purposes and types of student writing. The 2011 NAEP framework, like the Standards, cultivates the development of three mutually reinforcing writing capacities: writing to persuade, to explain, and to convey real or imagined experience. Evidence concerning the demands of college and career readiness gathered during development of the Standards concurs with NAEP's shifting emphases: standards for grades 9–12 describe writing in all three forms, but, consistent with NAEP, the overwhelming focus of writing throughout high school should be on arguments and informative/explanatory texts.[2]

Distribution of Communicative Purposes by Grade in the 2011 NAEP Writing Framework

Grade	To Persuade	To Explain	To Convey Experience
4	30%	35%	35%
8	35%	35%	30%
12	40%	40%	20%

Source: National Assessment Governing Board. (2007). *Writing framework for the 2011 National Assessment of Educational Progress, pre-publication edition.* Iowa City, IA: ACT, Inc.

It follows that writing assessments aligned with the Standards should adhere to the distribution of writing purposes across grades outlined by NAEP.

Focus and coherence in instruction and assessment

While the Standards delineate specific expectations in reading, writing, speaking, listening, and language, each standard need not be a separate focus for instruction and assessment. Often, several standards can be addressed by a single rich task. For example, when editing writing, students address Writing standard 5 ("Develop and strengthen writing as needed by planning, revising, editing, rewriting, or trying a new approach") as well as Language standards 1–3 (which deal with conventions of standard English and knowledge of language). When drawing evidence from literary and informational texts per Writing standard 9, students are also demonstrating their comprehension skill in relation to specific standards in Reading. When discussing something they have read or written, students are also demonstrating their speaking and listening skills. The CCR anchor standards themselves provide another source of focus and coherence.

The same ten CCR anchor standards for Reading apply to both literary and informational texts, including texts in history/social studies, science, and technical subjects. The ten CCR anchor standards for Writing cover numerous text types and subject areas. This means that students can develop mutually reinforcing skills and exhibit mastery of standards for reading and writing across a range of texts and classrooms.

[1]The percentages on the table reflect the sum of student reading, not just reading in ELA settings. Teachers of senior English classes, for example, are not required to devote 70 percent of reading to informational texts. Rather, 70 percent of student reading across the grade should be informational.
[2]As with reading, the percentages in the table reflect the sum of student writing, not just writing in ELA settings.

What is Not Covered by the Standards

The Standards should be recognized for what they are not as well as what they are. The most important intentional design limitations are as follows:

1. The Standards define what all students are expected to know and be able to do, not how teachers should teach. For instance, the use of play with young children is not specified by the Standards, but it is welcome as a valuable activity in its own right and as a way to help students meet the expectations in this document. Furthermore, while the Standards make references to some particular forms of content, including mythology, foundational U.S. documents, and Shakespeare, they do not—indeed, cannot—enumerate all or even most of the content that students should learn. The Standards must therefore be complemented by a well-developed, content-rich curriculum consistent with the expectations laid out in this document.

2. While the Standards focus on what is most essential, they do not describe all that can or should be taught. A great deal is left to the discretion of teachers and curriculum developers. The aim of the Standards is to articulate the fundamentals, not to set out an exhaustive list or a set of restrictions that limits what can be taught beyond what is specified herein.

3. The Standards do not define the nature of advanced work for students who meet the Standards prior to the end of high school. For those students, advanced work in such areas as literature, composition, language, and journalism should be available. This work should provide the next logical step up from the college and career readiness baseline established here.

4. The Standards set grade-specific standards but do not define the intervention methods or materials necessary to support students who are well below or well above grade-level expectations. No set of grade-specific standards can fully reflect the great variety in abilities, needs, learning rates, and achievement levels of students in any given classroom. However, the Standards do provide clear signposts along the way to the goal of college and career readiness for all students.

5. It is also beyond the scope of the Standards to define the full range of supports appropriate for English language learners and for students with special needs. At the same time, all students must have the opportunity to learn and meet the same high standards if they are to access the knowledge and skills necessary in their post–high school lives.

   Each grade will include students who are still acquiring English. For those students, it is possible to meet the standards in reading, writing, speaking, and listening without displaying native-like control of conventions and vocabulary.

   The Standards should also be read as allowing for the widest possible range of students to participate fully from the outset and as permitting appropriate accommodations to ensure maximum participation of students with special education needs. For example, for students with disabilities *reading* should allow for the use of Braille, screen-reader technology, or other assistive devices, while *writing* should include the use of a scribe, computer, or speech-to-text technology. In a similar vein, *speaking* and *listening* should be interpreted broadly to include sign language.

6. While the ELA and content area literacy components described herein are critical to college and career readiness, they do not define the whole of such readiness. Students require a wide-ranging, rigorous academic preparation and, particularly in the early grades, attention to such matters as social, emotional, and physical development and approaches to learning. Similarly, the Standards define literacy expectations in history/social studies, science, and technical subjects, but literacy standards in other areas, such as mathematics and health education, modeled on those in this document are strongly encouraged to facilitate a comprehensive, schoolwide literacy program.

Students Who are College and Career Ready in Reading, Writing, Speaking, Listening, and Language

The descriptions that follow are not standards themselves but instead offer a portrait of students who meet the standards set out in this document. As students advance through the grades and master the standards in reading, writing, speaking, listening, and language, they are able to exhibit with increasing fullness and regularity these capacities of the literate individual.

They demonstrate independence.

Students can, without significant scaffolding, comprehend and evaluate complex texts across a range of types and disciplines, and they can construct effective arguments and convey intricate or multifaceted information. Likewise, students are able independently to discern a speaker's key points, request clarification, and ask relevant questions. They build on others' ideas, articulate their own ideas, and confirm they have been understood. Without prompting, they demonstrate command of standard English and acquire and use a wide-ranging vocabulary. More broadly, they become self-directed learners, effectively seeking out and using resources to assist them, including teachers, peers, and print and digital reference materials.

They build strong content knowledge.

Students establish a base of knowledge across a wide range of subject matter by engaging with works of quality and substance. They become proficient in new areas through research and study. They read purposefully and listen attentively to gain both general knowledge and discipline-specific expertise. They refine and share their knowledge through writing and speaking.

They respond to the varying demands of audience, task, purpose, and discipline.

Students adapt their communication in relation to audience, task, purpose, and discipline. They set and adjust purpose for reading, writing, speaking, listening, and language use as warranted by the task. They appreciate nuances, such as how the composition of an audience should affect tone when speaking and how the connotations of words affect meaning. They also know that different disciplines call for different types of evidence (e.g., documentary evidence in history, experimental evidence in science).

They comprehend as well as critique.

Students are engaged and open-minded—but discerning—readers and listeners. They work diligently to understand precisely what an author or speaker is saying, but they also question an author's or speaker's assumptions and premises and assess the veracity of claims and the soundness of reasoning.

They value evidence.

Students cite specific evidence when offering an oral or written interpretation of a text. They use relevant evidence when supporting their own points in writing and speaking, making their reasoning clear to the reader or listener, and they constructively evaluate others' use of evidence.

They use technology and digital media strategically and capably.

Students employ technology thoughtfully to enhance their reading, writing, speaking, listening, and language use. They tailor their searches online to acquire useful information efficiently, and they integrate what they learn using technology with what they learn offline. They are familiar with the strengths and limitations of various technological tools and mediums and can select and use those best suited to their communication goals.

They come to understand other perspectives and cultures.

Students appreciate that the twenty-first-century classroom and workplace are settings in which people from often widely divergent cultures and who represent diverse experiences and perspectives must learn and work together. Students actively seek to understand other perspectives and cultures through reading and listening, and they are able to communicate effectively with people of varied backgrounds. They evaluate other points of view critically and constructively. Through reading great classic and contemporary works of literature representative of a variety of periods, cultures, and worldviews, students can vicariously inhabit worlds and have experiences much different than their own.

How to Read This Document

Overall Document Organization

The Standards comprise three main sections: a comprehensive K–5 section and two content area–specific sections for grades 6–12, one for ELA and one for history/social studies, science, and technical subjects. Three appendices accompany the main document.

Each section is divided into strands. K–5 and 6–12 ELA have Reading, Writing, Speaking and Listening, and Language strands; the 6–12 history/ social studies, science, and technical subjects section focuses on Reading and Writing. Each strand is headed by a strand-specific set of College and Career Readiness Anchor Standards that is identical across all grades and content areas.

Standards for each grade within K–8 and for grades 9–10 and 11–12 follow the CCR anchor standards in each strand. Each grade-specific standard (as these standards are collectively referred to) corresponds to the same-numbered CCR anchor standard. Put another way, each CCR anchor standard has an accompanying grade-specific standard translating the broader CCR statement into grade-appropriate end-of-year expectations.

Individual CCR anchor standards can be identified by their strand, CCR status, and number (R.CCR.6, for example). Individual grade-specific standards can be identified by their strand, grade, and number (or number and letter, where applicable), so that RI.4.3, for example, stands for Reading, Informational Text, grade 4, standard 3 and W.5.1a stands for Writing, grade 5, standard 1a. Strand designations can be found in brackets alongside the full strand title.

Who is responsible for which portion of the Standards

A single K–5 section lists standards for reading, writing, speaking, listening, and language across the curriculum, reflecting the fact that most or all of the instruction students in these grades receive comes from one teacher. Grades 6–12 are covered in two content area–specific sections, the first for the English language arts teacher and the second for teachers of history/social studies, science, and technical subjects. Each section uses the same CCR anchor standards but also includes grade-specific standards tuned to the literacy requirements of the particular discipline(s).

Key Features of the Standards

Reading: Text complexity and the growth of comprehension

The Reading standards place equal emphasis on the sophistication of what students read and the skill with which they read. Standard 10 defines a grade-by-grade "staircase" of increasing text complexity that rises from beginning reading to the college and career readiness level. Whatever they are reading, students must also show a steadily growing ability to discern more from and make fuller use of text, including making an increasing number of connections among ideas and between texts, considering a wider range of textual evidence, and becoming more sensitive to inconsistencies, ambiguities, and poor reasoning in texts.

Writing: Text types, responding to reading, and research

The Standards acknowledge the fact that whereas some writing skills, such as the ability to plan, revise, edit, and publish, are applicable to many types of writing, other skills are more properly defined in terms of specific writing types: arguments, informative/explanatory texts, and narratives. Standard 9 stresses the importance of the writing-reading connection by requiring students to draw upon and write about evidence from literary and informational texts. Because of the centrality of writing to most forms of inquiry, research standards are prominently included in this strand, though skills important to research are infused throughout the document.

Speaking and Listening: Flexible communication and collaboration

Including but not limited to skills necessary for formal presentations, the Speaking and Listening standards require students to develop a range of broadly useful oral communication and interpersonal skills. Students must learn to work together, express and listen carefully to ideas, integrate information from oral, visual, quantitative, and media sources, evaluate what they hear, use media and visual displays strategically to help achieve communicative purposes, and adapt speech to context and task.

Language: Conventions, effective use, and vocabulary

The Language standards include the essential "rules" of standard written and spoken English, but they also approach language as a matter of craft and informed choice among alternatives. The vocabulary standards focus on understanding words and phrases, their relationships, and their nuances and on acquiring new vocabulary, particularly general academic and domain-specific words and phrases.

STANDARDS FOR

Literacy in All Subjects

K-5

College and Career Readiness Anchor Standards for Reading

The K–5 standards on the following pages define what students should understand and be able to do by the end of each grade. They correspond to the College and Career Readiness (CCR) anchor standards below by number. The CCR and grade-specific standards are necessary complements—the former providing broad standards, the latter providing additional specificity—that together define the skills and understandings that all students must demonstrate.

Key Ideas and Details

1. Read closely to determine what the text says explicitly and to make logical inferences from it; cite specific textual evidence when writing or speaking to support conclusions drawn from the text.
2. Determine central ideas or themes of a text and analyze their development; summarize the key supporting details and ideas.
3. Analyze how and why individuals, events, and ideas develop and interact over the course of a text.

Craft and Structure

4. Interpret words and phrases as they are used in a text, including determining technical, connotative, and figurative meanings, and analyze how specific word choices shape meaning or tone.
5. Analyze the structure of texts, including how specific sentences, paragraphs, and larger portions of the text (e.g., a section, chapter, scene, or stanza) relate to each other and the whole.
6. Assess how point of view or purpose shapes the content and style of a text.

Integration of Knowledge and Ideas

7. Integrate and evaluate content presented in diverse media and formats, including visually and quantitatively, as well as in words.*
8. Delineate and evaluate the argument and specific claims in a text, including the validity of the reasoning as well as the relevance and sufficiency of the evidence.
9. Analyze how two or more texts address similar themes or topics in order to build knowledge or to compare the approaches the authors take.

Range of Reading and Level of Text Complexity

10. Read and comprehend complex literary and informational texts independently and proficiently.

*Please see “Research to Build and Present Knowledge” in Writing and “Comprehension and Collaboration” in Speaking and Listening for additional standards relevant to gathering, assessing, and applying information from print and digital sources.

Note on range and content of student reading

To build a foundation for college and career readiness, students must read widely and deeply from among a broad range of high-quality, increasingly challenging literary and informational texts. Through extensive reading of stories, dramas, poems, and myths from diverse cultures and different time periods, students gain literary and cultural knowledge as well as familiarity with various text structures and elements. By reading texts in history/social studies, science, and other disciplines, students build a foundation of knowledge in these fields that will also give them the background to be better readers in all content areas. Students can only gain this foundation when the curriculum is intentionally and coherently structured to develop rich content knowledge within and across grades. Students also acquire the habits of reading independently and closely, which are essential to their future success.

Reading Standards for Informational Text K–5

RI

Kindergartners:	Grade 1 students:	Grade 2 students:
Key Ideas and Details		
1. With prompting and support, ask and answer questions about key details in a text.	1. Ask and answer questions about key details in a text.	1. Ask and answer such questions as *who, what, where, when, why,* and *how* to demonstrate understanding of key details in a text.
2. With prompting and support, identify the main topic and retell key details of a text.	2. Identify the main topic and retell key details of a text.	2. Identify the main topic of a multiparagraph text as well as the focus of specific paragraphs within the text.
3. With prompting and support, describe the connection between two individuals, events, ideas, or pieces of information in a text.	3. Describe the connection between two individuals, events, ideas, or pieces of information in a text.	3. Describe the connection between a series of historical events, scientific ideas or concepts, or steps in technical procedures in a text.
Craft and Structure		
4. With prompting and support, ask and answer questions about unknown words in a text.	4. Ask and answer questions to help determine or clarify the meaning of words and phrases in a text.	4. Determine the meaning of words and phrases in a text relevant to a *grade 2 topic or subject area*.
5. Identify the front cover, back cover, and title page of a book.	5. Know and use various text features (e.g., headings, tables of contents, glossaries, electronic menus, icons) to locate key facts or information in a text.	5. Know and use various text features (e.g., captions, bold print, subheadings, glossaries, indexes, electronic menus, icons) to locate key facts or information in a text efficiently.
6. Name the author and illustrator of a text and define the role of each in presenting the ideas or information in a text.	6. Distinguish between information provided by pictures or other illustrations and information provided by the words in a text.	6. Identify the main purpose of a text, including what the author wants to answer, explain, or describe.
Integration of Knowledge and Ideas		
7. With prompting and support, describe the relationship between illustrations and the text in which they appear (e.g., what person, place, thing, or idea in the text an illustration depicts).	7. Use the illustrations and details in a text to describe its key ideas.	7. Explain how specific images (e.g., a diagram showing how a machine works) contribute to and clarify a text.
8. With prompting and support, identify the reasons an author gives to support points in a text.	8. Identify the reasons an author gives to support points in a text.	8. Describe how reasons support specific points the author makes in a text.
9. With prompting and support, identify basic similarities in and differences between two texts on the same topic (e.g., in illustrations, descriptions, or procedures).	9. Identify basic similarities in and differences between two texts on the same topic (e.g., in illustrations, descriptions, or procedures).	9. Compare and contrast the most important points presented by two texts on the same topic.
Range of Reading and Level of Text Complexity		
10. Actively engage in group reading activities with purpose and understanding.	10. With prompting and support, read informational texts appropriately complex for grade 1.	10. By the end of year, read and comprehend informational texts, including history/social studies, science, and technical texts, in the grades 2–3 text complexity band proficiently, with scaffolding as needed at the high end of the range.

Reading Standards for Informational Text K–5

RI

Grade 3 students:	Grade 4 students:	Grade 5 students:
Key Ideas and Details		
1. Ask and answer questions to demonstrate understanding of a text, referring explicitly to the text as the basis for the answers.	1. Refer to details and examples in a text when explaining what the text says explicitly and when drawing inferences from the text.	1. Quote accurately from a text when explaining what the text says explicitly and when drawing inferences from the text.
2. Determine the main idea of a text; recount the key details and explain how they support the main idea.	2. Determine the main idea of a text and explain how it is supported by key details; summarize the text.	2. Determine two or more main ideas of a text and explain how they are supported by key details; summarize the text.
3. Describe the relationship between a series of historical events, scientific ideas or concepts, or steps in technical procedures in a text, using language that pertains to time, sequence, and cause/effect.	3. Explain events, procedures, ideas, or concepts in a historical, scientific, or technical text, including what happened and why, based on specific information in the text.	3. Explain the relationships or interactions between two or more individuals, events, ideas, or concepts in a historical, scientific, or technical text based on specific information in the text.
Craft and Structure		
4. Determine the meaning of general academic and domain-specific words and phrases in a text relevant to a *grade 3 topic or subject area*.	4. Determine the meaning of general academic and domain-specific words or phrases in a text relevant to a *grade 4 topic or subject area*.	4. Determine the meaning of general academic and domain-specific words and phrases in a text relevant to a *grade 5 topic or subject area*.
5. Use text features and search tools (e.g., key words, sidebars, hyperlinks) to locate information relevant to a given topic efficiently.	5. Describe the overall structure (e.g., chronology, comparison, cause/effect, problem/solution) of events, ideas, concepts, or information in a text or part of a text.	5. Compare and contrast the overall structure (e.g., chronology, comparison, cause/effect, problem/solution) of events, ideas, concepts, or information in two or more texts.
6. Distinguish their own point of view from that of the author of a text.	6. Compare and contrast a firsthand and secondhand account of the same event or topic; describe the differences in focus and the information provided.	6. Analyze multiple accounts of the same event or topic, noting important similarities and differences in the point of view they represent.
Integration of Knowledge and Ideas		
7. Use information gained from illustrations (e.g., maps, photographs) and the words in a text to demonstrate understanding of the text (e.g., where, when, why, and how key events occur).	7. Interpret information presented visually, orally, or quantitatively (e.g., in charts, graphs, diagrams, time lines, animations, or interactive elements on Web pages) and explain how the information contributes to an understanding of the text in which it appears.	7. Draw on information from multiple print or digital sources, demonstrating the ability to locate an answer to a question quickly or to solve a problem efficiently.
8. Describe the logical connection between particular sentences and paragraphs in a text (e.g., comparison, cause/effect, first/second/third in a sequence).	8. Explain how an author uses reasons and evidence to support particular points in a text.	8. Explain how an author uses reasons and evidence to support particular points in a text, identifying which reasons and evidence support which point(s).
9. Compare and contrast the most important points and key details presented in two texts on the same topic.	9. Integrate information from two texts on the same topic in order to write or speak about the subject knowledgeably.	9. Integrate information from several texts on the same topic in order to write or speak about the subject knowledgeably.
Range of Reading and Level of Text Complexity		
10. By the end of the year, read and comprehend informational texts, including history/social studies, science, and technical texts, at the high end of the grades 2–3 text complexity band independently and proficiently.	10. By the end of year, read and comprehend informational texts, including history/social studies, science, and technical texts, in the grades 4–5 text complexity band proficiently, with scaffolding as needed at the high end of the range.	10. By the end of the year, read and comprehend informational texts, including history/social studies, science, and technical texts, at the high end of the grades 4–5 text complexity band independently and proficiently.

College and Career Readiness Anchor Standards for Writing

The K–5 standards on the following pages define what students should understand and be able to do by the end of each grade. They correspond to the College and Career Readiness (CCR) anchor standards below by number. The CCR and grade-specific standards are necessary complements—the former providing broad standards, the latter providing additional specificity—that together define the skills and understandings that all students must demonstrate.

Text Types and Purposes*

1. Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient evidence.
2. Write informative/explanatory texts to examine and convey complex ideas and information clearly and accurately through the effective selection, organization, and analysis of content.
3. Write narratives to develop real or imagined experiences or events using effective technique, well-chosen details, and well-structured event sequences.

Production and Distribution of Writing

4. Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.
5. Develop and strengthen writing as needed by planning, revising, editing, rewriting, or trying a new approach.
6. Use technology, including the Internet, to produce and publish writing and to interact and collaborate with others.

Research to Build and Present Knowledge

7. Conduct short as well as more sustained research projects based on focused questions, demonstrating understanding of the subject under investigation.
8. Gather relevant information from multiple print and digital sources, assess the credibility and accuracy of each source, and integrate the information while avoiding plagiarism.
9. Draw evidence from literary or informational texts to support analysis, reflection, and research.

Range of Writing

10. Write routinely over extended time frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two) for a range of tasks, purposes, and audiences.

*These broad types of writing include many subgenres.

Note on range and content of student writing

To build a foundation for college and career readiness, students need to learn to use writing as a way of offering and supporting opinions, demonstrating understanding of the subjects they are studying, and conveying real and imagined experiences and events. They learn to appreciate that a key purpose of writing is to communicate clearly to an external, sometimes unfamiliar audience, and they begin to adapt the form and content of their writing to accomplish a particular task and purpose. They develop the capacity to build knowledge on a subject through research projects and to respond analytically to literary and informational sources. To meet these goals, students must devote significant time and effort to writing, producing numerous pieces over short and extended time frames throughout the year.

Writing Standards K–5

W

The following standards for K–5 offer a focus for instruction each year to help ensure that students gain adequate mastery of a range of skills and applications. Each year in their writing, students should demonstrate increasing sophistication in all aspects of language use, from vocabulary and syntax to the development and organization of ideas, and they should address increasingly demanding content and sources. *Students advancing through the grades are expected to meet each year's grade-specific standards and retain or further develop skills and understandings mastered in preceding grades.*

Kindergartners:	Grade 1 students:	Grade 2 students:
Text Types and Purposes		
1. Use a combination of drawing, dictating, and writing to compose opinion pieces in which they tell a reader the topic or the name of the book they are writing about and state an opinion or preference about the topic or book (e.g., *My favorite book is . . .*).	1. Write opinion pieces in which they introduce the topic or name the book they are writing about, state an opinion, supply a reason for the opinion, and provide some sense of closure.	1. Write opinion pieces in which they introduce the topic or book they are writing about, state an opinion, supply reasons that support the opinion, use linking words (e.g., *because*, *and*, *also*) to connect opinion and reasons, and provide a concluding statement or section.
2. Use a combination of drawing, dictating, and writing to compose informative/explanatory texts in which they name what they are writing about and supply some information about the topic.	2. Write informative/explanatory texts in which they name a topic, supply some facts about the topic, and provide some sense of closure.	2. Write informative/explanatory texts in which they introduce a topic, use facts and definitions to develop points, and provide a concluding statement or section.
3. Use a combination of drawing, dictating, and writing to narrate a single event or several loosely linked events, tell about the events in the order in which they occurred, and provide a reaction to what happened.	3. Write narratives in which they recount two or more appropriately sequenced events, include some details regarding what happened, use temporal words to signal event order, and provide some sense of closure.	3. Write narratives in which they recount a well-elaborated event or short sequence of events, include details to describe actions, thoughts, and feelings, use temporal words to signal event order, and provide a sense of closure.
Production and Distribution of Writing		
4. (Begins in grade 3)	4. (Begins in grade 3)	4. (Begins in grade 3)
5. With guidance and support from adults, respond to questions and suggestions from peers and add details to strengthen writing as needed.	5. With guidance and support from adults, focus on a topic, respond to questions and suggestions from peers, and add details to strengthen writing as needed.	5. With guidance and support from adults and peers, focus on a topic and strengthen writing as needed by revising and editing.
6. With guidance and support from adults, explore a variety of digital tools to produce and publish writing, including in collaboration with peers.	6. With guidance and support from adults, use a variety of digital tools to produce and publish writing, including in collaboration with peers.	6. With guidance and support from adults, use a variety of digital tools to produce and publish writing, including in collaboration with peers.
Research to Build and Present Knowledge		
7. Participate in shared research and writing projects (e.g., explore a number of books by a favorite author and express opinions about them).	7. Participate in shared research and writing projects (e.g., explore a number of "how-to" books on a given topic and use them to write a sequence of instructions).	7. Participate in shared research and writing projects (e.g., read a number of books on a single topic to produce a report; record science observations).
8. With guidance and support from adults, recall information from experiences or gather information from provided sources to answer a question.	8. With guidance and support from adults, recall information from experiences or gather information from provided sources to answer a question.	8. Recall information from experiences or gather information from provided sources to answer a question.
9. (Begins in grade 4)	9. (Begins in grade 4)	9. (Begins in grade 4)
Range of Writing		
10. (Begins in grade 3)	10. (Begins in grade 3)	10. (Begins in grade 3)

Writing Standards K–5

W

Grade 3 students:	Grade 4 students:	Grade 5 students:
Text Types and Purposes		
1. Write opinion pieces on topics or texts, supporting a point of view with reasons. a. Introduce the topic or text they are writing about, state an opinion, and create an organizational structure that lists reasons. b. Provide reasons that support the opinion. c. Use linking words and phrases (e.g., *because*, *therefore*, *since*, *for example*) to connect opinion and reasons. d. Provide a concluding statement or section.	1. Write opinion pieces on topics or texts, supporting a point of view with reasons and information. a. Introduce a topic or text clearly, state an opinion, and create an organizational structure in which related ideas are grouped to support the writer's purpose. b. Provide reasons that are supported by facts and details. c. Link opinion and reasons using words and phrases (e.g., *for instance*, *in order to*, *in addition*). d. Provide a concluding statement or section related to the opinion presented.	1. Write opinion pieces on topics or texts, supporting a point of view with reasons and information. a. Introduce a topic or text clearly, state an opinion, and create an organizational structure in which ideas are logically grouped to support the writer's purpose. b. Provide logically ordered reasons that are supported by facts and details. c. Link opinion and reasons using words, phrases, and clauses (e.g., *consequently*, *specifically*). d. Provide a concluding statement or section related to the opinion presented.
2. Write informative/explanatory texts to examine a topic and convey ideas and information clearly. a. Introduce a topic and group related information together; include illustrations when useful to aiding comprehension. b. Develop the topic with facts, definitions, and details. c. Use linking words and phrases (e.g., *also*, *another*, *and*, *more*, *but*) to connect ideas within categories of information. d. Provide a concluding statement or section.	2. Write informative/explanatory texts to examine a topic and convey ideas and information clearly. a. Introduce a topic clearly and group related information in paragraphs and sections; include formatting (e.g., headings), illustrations, and multimedia when useful to aiding comprehension. b. Develop the topic with facts, definitions, concrete details, quotations, or other information and examples related to the topic. c. Link ideas within categories of information using words and phrases (e.g., *another*, *for example*, *also*, *because*). d. Use precise language and domain-specific vocabulary to inform about or explain the topic. e. Provide a concluding statement or section related to the information or explanation presented.	2. Write informative/explanatory texts to examine a topic and convey ideas and information clearly. a. Introduce a topic clearly, provide a general observation and focus, and group related information logically; include formatting (e.g., headings), illustrations, and multimedia when useful to aiding comprehension. b. Develop the topic with facts, definitions, concrete details, quotations, or other information and examples related to the topic. c. Link ideas within and across categories of information using words, phrases, and clauses (e.g., *in contrast*, *especially*). d. Use precise language and domain-specific vocabulary to inform about or explain the topic. e. Provide a concluding statement or section related to the information or explanation presented.
3. Write narratives to develop real or imagined experiences or events using effective technique, descriptive details, and clear event sequences. a. Establish a situation and introduce a narrator and/or characters; organize an event sequence that unfolds naturally. b. Use dialogue and descriptions of actions, thoughts, and feelings to develop experiences and events or show the response of characters to situations. c. Use temporal words and phrases to signal event order. d. Provide a sense of closure.	3. Write narratives to develop real or imagined experiences or events using effective technique, descriptive details, and clear event sequences. a. Orient the reader by establishing a situationand introducing a narrator and/or characters; organize an event sequence that unfolds naturally. b. Use dialogue and description to develop experiences and events or show the responses of characters to situations. c. Use a variety of transitional words and phrases to manage the sequence of events. d. Use concrete words and phrases and sensory details to convey experiences and events precisely. e. Provide a conclusion that follows from the narrated experiences or events.	3. Write narratives to develop real or imagined experiences or events using effective technique, descriptive details, and clear event sequences. a. Orient the reader by establishing a situation and introducing a narrator and/or characters; organize an event sequence that unfolds naturally. b. Use narrative techniques, such as dialogue, description, and pacing, to develop experiences and events or show the responses of characters to situations. c. Use a variety of transitional words, phrases, and clauses to manage the sequence of events. d. Use concrete words and phrases and sensory details to convey experiences and events precisely. e. Provide a conclusion that follows from the narrated experiences or events.

Writing Standards K–5

W

Grade 3 students:	Grade 4 students:	Grade 5 students:
Production and Distribution of Writing		
4. With guidance and support from adults, produce writing in which the development and organization are appropriate to task and purpose. (Grade-specific expectations for writing types are defined in standards 1–3 above.)	4. Produce clear and coherent writing in which the development and organization are appropriate to task, purpose, and audience. (Grade-specific expectations for writing types are defined in standards 1–3 above.)	4. Produce clear and coherent writing in which the development and organization are appropriate to task, purpose, and audience. (Grade-specific expectations for writing types are defined in standards 1–3 above.)
5. With guidance and support from peers and adults, develop and strengthen writing as needed by planning, revising, and editing. (Editing for conventions should demonstrate command of Language standards 1–3 up to and including grade 3 on pages 28 and 29.)	5. With guidance and support from peers and adults, develop and strengthen writing as needed by planning, revising, and editing. (Editing for conventions should demonstrate command of Language standards 1–3 up to and including grade 4 on pages 28 and 29.)	5. With guidance and support from peers and adults, develop and strengthen writing as needed by planning, revising, editing, rewriting, or trying a new approach. (Editing for conventions should demonstrate command of Language standards 1–3 up to and including grade 5 on pages 28 and 29.)
6. With guidance and support from adults, use technology to produce and publish writing (using keyboarding skills) as well as to interact and collaborate with others.	6. With some guidance and support from adults, use technology, including the Internet, to produce and publish writing as well as to interact and collaborate with others; demonstrate sufficient command of keyboarding skills to type a minimum of one page in a single sitting.	6. With some guidance and support from adults, use technology, including the Internet, to produce and publish writing as well as to interact and collaborate with others; demonstrate sufficient command of keyboarding skills to type a minimum of two pages in a single sitting.
Research to Build and Present Knowledge		
7. Conduct short research projects that build knowledge about a topic.	7. Conduct short research projects that build knowledge through investigation of different aspects of a topic.	7. Conduct short research projects that use several sources to build knowledge through investigation of different aspects of a topic.
8. Recall information from experiences or gather information from print and digital sources; take brief notes on sources and sort evidence into provided categories.	8. Recall relevant information from experiences or gather relevant information from print and digital sources; take notes and categorize information, and provide a list of sources.	8. Recall relevant information from experiences or gather relevant information from print and digital sources; summarize or paraphrase information in notes and finished work, and provide a list of sources.
9. (Begins in grade 4)	9. Draw evidence from literary or informational texts to support analysis, reflection, and research. a. Apply *grade 4 Reading standards* to literature (e.g., "Describe in depth a character, setting, or event in a story or drama, drawing on specific details in the text [e.g., a character's thoughts, words, or actions]."). b. Apply *grade 4 Reading standards* to informational texts (e.g., "Explain how an author uses reasons and evidence to support particular points in a text").	9. Draw evidence from literary or informational texts to support analysis, reflection, and research. a. Apply *grade 5 Reading standards* to literature (e.g., "Compare and contrast two or more characters, settings, or events in a story or a drama, drawing on specific details in the text [e.g., how characters interact]"). b. Apply *grade 5 Reading standards* to informational texts (e.g., "Explain how an author uses reasons and evidence to support particular points in a text, identifying which reasons and evidence support which point[s]").
Range of Writing		
10. Write routinely over extended time frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two) for a range of discipline-specific tasks, purposes, and audiences.	10. Write routinely over extended time frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two) for a range of discipline-specific tasks, purposes, and audiences.	10. Write routinely over extended time frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two) for a range of discipline-specific tasks, purposes, and audiences.

College and Career Readiness Anchor Standards for Speaking and Listening

The K–5 standards on the following pages define what students should understand and be able to do by the end of each grade. They correspond to the College and Career Readiness (CCR) anchor standards below by number. The CCR and grade-specific standards are necessary complements—the former providing broad standards, the latter providing additional specificity—that together define the skills and understandings that all students must demonstrate.

Comprehension and Collaboration

1. Prepare for and participate effectively in a range of conversations and collaborations with diverse partners, building on others' ideas and expressing their own clearly and persuasively.
2. Integrate and evaluate information presented in diverse media and formats, including visually, quantitatively, and orally.
3. Evaluate a speaker's point of view, reasoning, and use of evidence and rhetoric.

Presentation of Knowledge and Ideas

4. Present information, findings, and supporting evidence such that listeners can follow the line of reasoning and the organization, development, and style are appropriate to task, purpose, and audience.
5. Make strategic use of digital media and visual displays of data to express information and enhance understanding of presentations.
6. Adapt speech to a variety of contexts and communicative tasks, demonstrating command of formal English when indicated or appropriate.

Note on range and content of student speaking and listening

To build a foundation for college and career readiness, students must have ample opportunities to take part in a variety of rich, structured conversations—as part of a whole class, in small groups, and with a partner. Being productive members of these conversations requires that students contribute accurate, relevant information; respond to and develop what others have said; make comparisons and contrasts; and analyze and synthesize a multitude of ideas in various domains.

New technologies have broadened and expanded the role that speaking and listening play in acquiring and sharing knowledge and have tightened their link to other forms of communication. Digital texts confront students with the potential for continually updated content and dynamically changing combinations of words, graphics, images, hyperlinks, and embedded video and audio.

Speaking and Listening Standards K–5

The following standards for K–5 offer a focus for instruction each year to help ensure that students gain adequate mastery of a range of skills and applications. *Students advancing through the grades are expected to meet each year's grade-specific standards and retain or further develop skills and understandings mastered in preceding grades.*

Kindergartners:	Grade 1 students:	Grade 2 students:
Comprehension and Collaboration		
1. Participate in collaborative conversations with diverse partners about *kindergarten topics and texts* with peers and adults in small and larger groups. a. Follow agreed-upon rules for discussions (e.g., listening to others and taking turns speaking about the topics and texts under discussion). b. Continue a conversation through multiple exchanges.	1. Participate in collaborative conversations with diverse partners about *grade 1 topics and texts* with peers and adults in small and larger groups. a. Follow agreed-upon rules for discussions (e.g., listening to others with care, speaking one at a time about the topics and texts under discussion). b. Build on others' talk in conversations by responding to the comments of others through multiple exchanges. c. Ask questions to clear up any confusion about the topics and texts under discussion.	1. Participate in collaborative conversations with diverse partners about *grade 2 topics and texts* with peers and adults in small and larger groups. a. Follow agreed-upon rules for discussions (e.g., gaining the floor in respectful ways, listening to others with care, speaking one at a time about the topics and texts under discussion). b. Build on others' talk in conversations by linking their comments to the remarks of others. c. Ask for clarification and further explanation as needed about the topics and texts under discussion.
2. Confirm understanding of a text read aloud or information presented orally or through other media by asking and answering questions about key details and requesting clarification if something is not understood.	2. Ask and answer questions about key details in a text read aloud or information presented orally or through other media.	2. Recount or describe key ideas or details from a text read aloud or information presented orally or through other media.
3. Ask and answer questions in order to seek help, get information, or clarify something that is not understood.	3. Ask and answer questions about what a speaker says in order to gather additional information or clarify something that is not understood.	3. Ask and answer questions about what a speaker says in order to clarify comprehension, gather additional information, or deepen understanding of a topic or issue.
Presentation of Knowledge and Ideas		
4. Describe familiar people, places, things, and events and, with prompting and support, provide additional detail.	4. Describe people, places, things, and events with relevant details, expressing ideas and feelings clearly.	4. Tell a story or recount an experience with appropriate facts and relevant, descriptive details, speaking audibly in coherent sentences.
5. Add drawings or other visual displays to descriptions as desired to provide additional detail.	5. Add drawings or other visual displays to descriptions when appropriate to clarify ideas, thoughts, and feelings.	5. Create audio recordings of stories or poems; add drawings or other visual displays to stories or recounts of experiences when appropriate to clarify ideas, thoughts, and feelings.
6. Speak audibly and express thoughts, feelings, and ideas clearly.	6. Produce complete sentences when appropriate to task and situation.	6. Produce complete sentences when appropriate to task and situation in order to provide requested detail or clarification.

Speaking and Listening Standards K–5

SL

Grade 3 students:	Grade 4 students:	Grade 5 students:
Comprehension and Collaboration		
1. Engage effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on *grade 3 topics and texts*, building on others' ideas and expressing their own clearly. a. Come to discussions prepared, having read or studied required material; explicitly draw on that preparation and other information known about the topic to explore ideas under discussion. b. Follow agreed-upon rules for discussions (e.g., gaining the floor in respectful ways, listening to others with care, speaking one at a time about the topics and texts under discussion). c. Ask questions to check understanding of information presented, stay on topic, and link their comments to the remarks of others. d. Explain their own ideas and understanding in light of the discussion.	1. Engage effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on *grade 4 topics and texts*, building on others' ideas and expressing their own clearly. a. Come to discussions prepared, having read or studied required material; explicitly draw on that preparation and other information known about the topic to explore ideas under discussion. b. Follow agreed-upon rules for discussions and carry out assigned roles. c. Pose and respond to specific questions to clarify or follow up on information, and make comments that contribute to the discussion and link to the remarks of others. d. Review the key ideas expressed and explain their own ideas and understanding in light of the discussion.	1. Engage effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on *grade 5 topics and texts*, building on others' ideas and expressing their own clearly. a. Come to discussions prepared, having read or studied required material; explicitly draw on that preparation and other information known about the topic to explore ideas under discussion. b. Follow agreed-upon rules for discussions and carry out assigned roles. c. Pose and respond to specific questions by making comments that contribute to the discussion and elaborate on the remarks of others. d. Review the key ideas expressed and draw conclusions in light of information and knowledge gained from the discussions.
2. Determine the main ideas and supporting details of a text read aloud or information presented in diverse media and formats, including visually, quantitatively, and orally.	2. Paraphrase portions of a text read aloud or information presented in diverse media and formats, including visually, quantitatively, and orally.	2. Summarize a written text read aloud or information presented in diverse media and formats, including visually, quantitatively, and orally.
3. Ask and answer questions about information from a speaker, offering appropriate elaboration and detail.	3. Identify the reasons and evidence a speaker provides to support particular points.	3. Summarize the points a speaker makes and explain how each claim is supported by reasons and evidence.
Presentation of Knowledge and Ideas		
4. Report on a topic or text, tell a story, or recount an experience with appropriate facts and relevant, descriptive details, speaking clearly at an understandable pace.	4. Report on a topic or text, tell a story, or recount an experience in an organized manner, using appropriate facts and relevant, descriptive details to support main ideas or themes; speak clearly at an understandable pace.	4. Report on a topic or text or present an opinion, sequencing ideas logically and using appropriate facts and relevant, descriptive details to support main ideas or themes; speak clearly at an understandable pace.
5. Create engaging audio recordings of stories or poems that demonstrate fluid reading at an understandable pace; add visual displays when appropriate to emphasize or enhance certain facts or details.	5. Add audio recordings and visual displays to presentations when appropriate to enhance the development of main ideas or themes.	5. Include multimedia components (e.g., graphics, sound) and visual displays in presentations when appropriate to enhance the development of main ideas or themes.
6. Speak in complete sentences when appropriate to task and situation in order to provide requested detail or clarification.	6. Differentiate between contexts that call for formal English (e.g., presenting ideas) and situations where informal discourse is appropriate (e.g., small-group discussion); use formal English when appropriate to task and situation.	6. Adapt speech to a variety of contexts and tasks, using formal English when appropriate to task and situation.

College and Career Readiness Anchor Standards for Language

The K–5 standards on the following pages define what students should understand and be able to do by the end of each grade. They correspond to the College and Career Readiness (CCR) anchor standards below by number. The CCR and grade-specific standards are necessary complements—the former providing broad standards, the latter providing additional specificity—that together define the skills and understandings that all students must demonstrate.

Conventions of Standard English

1. Demonstrate command of the conventions of standard English grammar and usage when writing or speaking.
2. Demonstrate command of the conventions of standard English capitalization, punctuation, and spelling when writing.

Knowledge of Language

3. Apply knowledge of language to understand how language functions in different contexts, to make effective choices for meaning or style, and to comprehend more fully when reading or listening.

Vocabulary Acquisition and Use

4. Determine or clarify the meaning of unknown and multiple-meaning words and phrases by using context clues, analyzing meaningful word parts, and consulting general and specialized reference materials, as appropriate.
5. Demonstrate understanding of figurative language, word relationships, and nuances in word meanings.
6. Acquire and use accurately a range of general academic and domain-specific words and phrases sufficient for reading, writing, speaking, and listening at the college and career readiness level; demonstrate independence in gathering vocabulary knowledge when encountering an unknown term important to comprehension or expression.

Note on range and content of student language use

To build a foundation for college and career readiness in language, students must gain control over many conventions of standard English grammar, usage, and mechanics as well as learn other ways to use language to convey meaning effectively. They must also be able to determine or clarify the meaning of grade-appropriate words encountered through listening, reading, and media use; come to appreciate that words have nonliteral meanings, shadings of meaning, and relationships to other words; and expand their vocabulary in the course of studying content. The inclusion of Language standards in their own strand should not be taken as an indication that skills related to conventions, effective language use, and vocabulary are unimportant to reading, writing, speaking, and listening; indeed, they are inseparable from such contexts.

Language Standards K–5

The following standards for grades K–5 offer a focus for instruction each year to help ensure that students gain adequate mastery of a range of skills and applications. *Students advancing through the grades are expected to meet each year's grade-specific standards and retain or further develop skills and understandings mastered in preceding grades.* Beginning in grade 3, skills and understandings that are particularly likely to require continued attention in higher grades as they are applied to increasingly sophisticated writing and speaking are marked with an asterisk (*).

Kindergartners:	Grade 1 students:	Grade 2 students:
Conventions of Standard English		
1. Demonstrate command of the conventions of standard English grammar and usage when writing or speaking. a. Print many upper- and lowercase letters. b. Use frequently occurring nouns and verbs. c. Form regular plural nouns orally by adding /s/ or /es/ (e.g., *dog, dogs*; *wish, wishes*). d. Understand and use question words (interrogatives) (e.g., *who, what, where, when, why, how*). e. Use the most frequently occurring prepositions (e.g., *to, from, in, out, on, off, for, of, by, with*). f. Produce and expand complete sentences in shared language activities.	1. Demonstrate command of the conventions of standard English grammar and usage when writing or speaking. a. Print all upper- and lowercase letters. b. Use common, proper, and possessive nouns. c. Use singular and plural nouns with matching verbs in basic sentences (e.g., *He hops*; *We hop*). d. Use personal, possessive, and indefinite pronouns (e.g., *I, me, my; they, them, their; anyone, everything*). e. Use verbs to convey a sense of past, present, and future (e.g., *Yesterday I walked home; Today I walk home; Tomorrow I will walk home*). f. Use frequently occurring adjectives. g. Use frequently occurring conjunctions (e.g., *and, but, or, so, because*). h. Use determiners (e.g., articles, demonstratives). i. Use frequently occurring prepositions (e.g., *during, beyond, toward*). j. Produce and expand complete simple and compound declarative, interrogative, imperative, and exclamatory sentences in response to prompts.	1. Demonstrate command of the conventions of standard English grammar and usage when writing or speaking. a. Use collective nouns (e.g., *group*). b. Form and use frequently occurring irregular plural nouns (e.g., *feet, children, teeth, mice, fish*). c. Use reflexive pronouns (e.g., *myself, ourselves*). d. Form and use the past tense of frequently occurring irregular verbs (e.g., *sat, hid, told*). e. Use adjectives and adverbs, and choose between them depending on what is to be modified. f. Produce, expand, and rearrange complete simple and compound sentences (e.g., *The boy watched the movie; The little boy watched the movie; The action movie was watched by the little boy*).
2. Demonstrate command of the conventions of standard English capitalization, punctuation, and spelling when writing. a. Capitalize the first word in a sentence and the pronoun *I*. b. Recognize and name end punctuation. c. Write a letter or letters for most consonant and short-vowel sounds (phonemes). d. Spell simple words phonetically, drawing on knowledge of sound-letter relationships.	2. Demonstrate command of the conventions of standard English capitalization, punctuation, and spelling when writing. a. Capitalize dates and names of people. b. Use end punctuation for sentences. c. Use commas in dates and to separate single words in a series. d. Use conventional spelling for words with common spelling patterns and for frequently occurring irregular words. e. Spell untaught words phonetically, drawing on phonemic awareness and spelling conventions.	2. Demonstrate command of the conventions of standard English capitalization, punctuation, and spelling when writing. a. Capitalize holidays, product names, and geographic names. b. Use commas in greetings and closings of letters. c. Use an apostrophe to form contractions and frequently occurring possessives. d. Generalize learned spelling patterns when writing words (e.g., cage → badge; boy → boil). e. Consult reference materials, including beginning dictionaries, as needed to check and correct spellings.

Language Standards K–5

Kindergartners:	Grade 1 students:	Grade 2 students:
Knowledge of Language		
3. (Begins in grade 2)	3. (Begins in grade 2)	3. Use knowledge of language and its conventions when writing, speaking, reading, or listening. a. Compare formal and informal uses of English.
Vocabulary Acquisition and Use		
4. Determine or clarify the meaning of unknown and multiple-meaning words and phrases based on *kindergarten reading and content.* a. Identify new meanings for familiar words and apply them accurately (e.g., knowing *duck* is a bird and learning the verb *to duck*). b. Use the most frequently occurring inflections and affixes (e.g., *-ed*, *-s*, *re-*, *un-*, *pre-*, *-ful*, *-less*) as a clue to the meaning of an unknown word.	4. Determine or clarify the meaning of unknown and multiple-meaning words and phrases based on *grade 1 reading and content*, choosing flexibly from an array of strategies. a. Use sentence-level context as a clue to the meaning of a word or phrase. b. Use frequently occurring affixes as a clue to the meaning of a word. c. Identify frequently occurring root words (e.g., *look*) and their inflectional forms (e.g., *looks*, *looked*, *looking*).	4. Determine or clarify the meaning of unknown and multiple-meaning words and phrases based on *grade 2 reading and content*, choosing flexibly from an array of strategies. a. Use sentence-level context as a clue to the meaning of a word or phrase. b. Determine the meaning of the new word formed when a known prefix is added to a known word (e.g., *happy/unhappy*, *tell/retell*). c. Use a known root word as a clue to the meaning of an unknown word with the same root (e.g., *addition*, *additional*). d. Use knowledge of the meaning of individual words to predict the meaning of compound words (e.g., *birdhouse, lighthouse, housefly; bookshelf, notebook, bookmark*). e. Use glossaries and beginning dictionaries, both print and digital, to determine or clarify the meaning of words and phrases.
5. With guidance and support from adults, explore word relationships and nuances in word meanings. a. Sort common objects into categories (e.g., shapes, foods) to gain a sense of the concepts the categories represent. b. Demonstrate understanding of frequently occurring verbs and adjectives by relating them to their opposites (antonyms). c. Identify real-life connections between words and their use (e.g., note places at school that are *colorful*). d. Distinguish shades of meaning among verbs describing the same general action (e.g., *walk, march, strut, prance*) by acting out the meanings.	5. With guidance and support from adults, demonstrate understanding of word relationships and nuances in word meanings. a. Sort words into categories (e.g., colors, clothing) to gain a sense of the concepts the categories represent. b. Define words by category and by one or more key attributes (e.g., a *duck* is a bird that swims; a *tiger* is a large cat with stripes). c. Identify real-life connections between words and their use (e.g., note places at home that are *cozy*). d. Distinguish shades of meaning among verbs differing in manner (e.g., *look, peek, glance, stare, glare, scowl*) and adjectives differing in intensity (e.g., *large, gigantic*) by defining or choosing them or by acting out the meanings.	5. Demonstrate understanding of word relationships and nuances in word meanings. a. Identify real-life connections between words and their use (e.g., describe foods that are *spicy* or *juicy*). b. Distinguish shades of meaning among closely related verbs (e.g., *toss, throw, hurl*) and closely related adjectives (e.g., *thin, slender, skinny, scrawny*).
6. Use words and phrases acquired through conversations, reading and being read to, and responding to texts.	6. Use words and phrases acquired through conversations, reading and being read to, and responding to texts, including using frequently occurring conjunctions to signal simple relationships (e.g., *because*).	6. Use words and phrases acquired through conversations, reading and being read to, and responding to texts, including using adjectives and adverbs to describe (e.g., *When other kids are happy that makes me happy*).

Language Standards K–5

L

Grade 3 students:	Grade 4 students:	Grade 5 students:
Knowledge of Language		
3. Use knowledge of language and its conventions when writing, speaking, reading, or listening. a. Choose words and phrases for effect.* b. Recognize and observe differences between the conventions of spoken and written standard English.	3. Use knowledge of language and its conventions when writing, speaking, reading, or listening. a. Choose words and phrases to convey ideas precisely.* b. Choose punctuation for effect.* c. Differentiate between contexts that call for formal English (e.g., presenting ideas) and situations where informal discourse is appropriate (e.g., small-group discussion).	3. Use knowledge of language and its conventions when writing, speaking, reading, or listening. a. Expand, combine, and reduce sentences for meaning, reader/listener interest, and style. b. Compare and contrast the varieties of English (e.g., dialects, registers) used in stories, dramas, or poems.
Vocabulary Acquisition and Use		
4. Determine or clarify the meaning of unknown and multiple-meaning word and phrases based on *grade 3 reading and content,* choosing flexibly from a range of strategies. a. Use sentence-level context as a clue to the meaning of a word or phrase. b. Determine the meaning of the new word formed when a known affix is added to a known word (e.g., *agreeable/disagreeable, comfortable/uncomfortable, care/careless, heat/preheat*). c. Use a known root word as a clue to the meaning of an unknown word with the same root (e.g., *company, companion*). d. Use glossaries or beginning dictionaries, both print and digital, to determine or clarify the precise meaning of key words and phrases.	4. Determine or clarify the meaning of unknown and multiple-meaning words and phrases based on *grade 4 reading and content,* choosing flexibly from a range of strategies. a. Use context (e.g., definitions, examples, or restatements in text) as a clue to the meaning of a word or phrase. b. Use common, grade-appropriate Greek and Latin affixes and roots as clues to the meaning of a word (e.g., *telegraph, photograph, autograph*). c. Consult reference materials (e.g., dictionaries, glossaries, thesauruses), both print and digital, to find the pronunciation and determine or clarify the precise meaning of key words and phrases.	4. Determine or clarify the meaning of unknown and multiple-meaning words and phrases based on *grade 5 reading and content,* choosing flexibly from a range of strategies. a. Use context (e.g., cause/effect relationships and comparisons in text) as a clue to the meaning of a word or phrase. b. Use common, grade-appropriate Greek and Latin affixes and roots as clues to the meaning of a word (e.g., *photograph, photosynthesis*). c. Consult reference materials (e.g., dictionaries, glossaries, thesauruses), both print and digital, to find the pronunciation and determine or clarify the precise meaning of key words and phrases.
5. Demonstrate understanding of word relationships and nuances in word meanings. a. Distinguish the literal and nonliteral meanings of words and phrases in context (e.g., *take steps*). b. Identify real-life connections between words and their use (e.g., describe people who are *friendly* or *helpful*). c. Distinguish shades of meaning among related words that describe states of mind or degrees of certainty (e.g., *knew, believed, suspected, heard, wondered*).	5. Demonstrate understanding of figurative language, word relationships, and nuances in word meanings. a. Explain the meaning of simple similes and metaphors (e.g., *as pretty as a picture*) in context. b. Recognize and explain the meaning of common idioms, adages, and proverbs. c. Demonstrate understanding of words by relating them to their opposites (antonyms) and to words with similar but not identical meanings (synonyms).	5. Demonstrate understanding of figurative language, word relationships, and nuances in word meanings. a. Interpret figurative language, including similes and metaphors, in context. b. Recognize and explain the meaning of common idioms, adages, and proverbs. c. Use the relationship between particular words (e.g., synonyms, antonyms, homographs) to better understand each of the words.
6. Acquire and use accurately grade-appropriate conversational, general academic, and domain-specific words and phrases, including those that signal spatial and temporal relationships (e.g., *After dinner that night we went looking for them*).	6. Acquire and use accurately grade-appropriate general academic and domain-specific words and phrases, including those that signal precise actions, emotions, or states of being (e.g., *quizzed, whined, stammered*) and that are basic to a particular topic (e.g., *wildlife, conservation,* and	6. Acquire and use accurately grade-appropriate general academic and domain-specific words and phrases, including those that signal contrast, addition, and other logical relationships (e.g., *however, although, nevertheless, similarly, moreover, in addition*).

Language Progressive Skills, by Grade

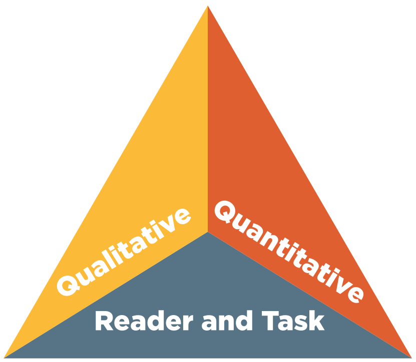
The following skills, marked with an asterisk (*) in Language standards 1–3, are particularly likely to require continued attention in higher grades as they are applied to increasingly sophisticated writing and speaking.

Standard	Grade(s)							
	3	4	5	6	7	8	9–10	11–12
L.3.1f. Ensure subject-verb and pronoun-antecedent agreement.								
L.3.3a. Choose words and phrases for effect.								
L.4.1f. Produce complete sentences, recognizing and correcting inappropriate fragments and run-ons.								
L.4.1g. Correctly use frequently confused words (e.g., *to/too/two; there/their*).								
L.4.3a. Choose words and phrases to convey ideas precisely.[*]								
L.4.3b. Choose punctuation for effect.								
L.5.1d. Recognize and correct inappropriate shifts in verb tense.								
L.5.2a. Use punctuation to separate items in a series.[†]								
L.6.1c. Recognize and correct inappropriate shifts in pronoun number and person.								
L.6.1d. Recognize and correct vague pronouns (i.e., ones with unclear or ambiguous antecedents).								
L.6.1e. Recognize variations from standard English in their own and others' writing and speaking, and identify and use strategies to improve expression in conventional language.								
L.6.2a. Use punctuation (commas, parentheses, dashes) to set off nonrestrictive/parenthetical elements.								
L.6.3a. Vary sentence patterns for meaning, reader/listener interest, and style.[‡]								
L.6.3b. Maintain consistency in style and tone.								
L.7.1c. Place phrases and clauses within a sentence, recognizing and correcting misplaced and dangling modifiers.								
L.7.3a. Choose language that expresses ideas precisely and concisely, recognizing and eliminating wordiness and redundancy.								
L.8.1d. Recognize and correct inappropriate shifts in verb voice and mood.								
L.9–10.1a. Use parallel structure.								

[*]Subsumed by L.7.3a
[†]Subsumed by L.9–10.1a
[‡]Subsumed by L.11–12.3a

Standard 10: Range, Quality, and Complexity of Student Reading K–5

Measuring Text Complexity: Three Factors



Qualitative evaluation of the text: Levels of meaning, structure, language conventionality and clarity, and knowledge demands

Quantitative evaluation of the text: Readability measures and other scores of text complexity

Matching reader to text and task: Reader variables (such as motivation, knowledge, and experiences) and task variables (such as purpose and the complexity generated by the task assigned and the questions posed)

Range of Text Types for K–5

Students in K–5 apply the Reading standards to the following range of text types, with texts selected from a broad range of cultures and periods.

Literature			Informational Text
Stories	**Dramas**	**Poetry**	**Literary Nonfiction and Historical, Scientific, and Technical Texts**
Includes children's adventure stories, folktales, legends, fables, fantasy, realistic fiction, and myth	Includes staged dialogue and brief familiar scenes	Includes nursery rhymes and the subgenres of the narrative poem, limerick, and free verse poem	Includes biographies and autobiographies; books about history, social studies, science, and the arts; technical texts, including directions, forms, and information displayed in graphs, charts, or maps; and digital sources on a range of topics

Texts Illustrating the Complexity, Quality, and Range of Student Reading K–5

	Informational Texts: Literary Nonfiction and Historical, Scientific, and Technical Texts
K*	▪ *My Five Senses* by Aliki (1962)** ▪ *Truck* by Donald Crews (1980) ▪ *I Read Signs* by Tana Hoban (1987) ▪ *What Do You Do With a Tail Like This?* by Steve Jenkins and Robin Page (2003)* ▪ *Amazing Whales!* by Sarah L. Thomson (2005)*
1*	▪ *A Tree Is a Plant* by Clyde Robert Bulla, illustrated by Stacey Schuett (1960)** ▪ *Starfish* by Edith Thacher Hurd (1962) ▪ *Follow the Water from Brook to Ocean* by Arthur Dorros (1991)** ▪ *From Seed to Pumpkin* by Wendy Pfeffer, illustrated by James Graham Hale (2004)* ▪ *How People Learned to Fly* by Fran Hodgkins and True Kelley (2007)*
2–3	▪ *A Medieval Feast* by Aliki (1983) ▪ *From Seed to Plant* by Gail Gibbons (1991) ▪ *The Story of Ruby Bridges* by Robert Coles (1995)* ▪ *A Drop of Water: A Book of Science and Wonder* by Walter Wick (1997) ▪ *Moonshot: The Flight of Apollo 11* by Brian Floca (2009)
4–5	▪ *Discovering Mars: The Amazing Story of the Red Planet* by Melvin Berger (1992) ▪ *Hurricanes: Earth's Mightiest Storms* by Patricia Lauber (1996) ▪ *A History of US* by Joy Hakim (2005) ▪ *Horses* by Seymour Simon (2006) ▪ *Quest for the Tree Kangaroo: An Expedition to the Cloud Forest of New Guinea* by Sy Montgomery (2006)

Note: Given space limitations, the illustrative texts listed above are meant only to show individual titles that are representative of a wide range of topics and genres. (See Appendix B for excerpts of these and other texts illustrative of K–5 text complexity, quality, and range.) At a curricular or instructional level, within and across grade levels, texts need to be selected around topics or themes that generate knowledge and allow students to study those topics or themes in depth. On the next page is an example of progressions of texts building knowledge across grade levels.

*Children at the kindergarten and grade 1 levels should be expected to read texts independently that have been specifically written to correlate to their reading level and their word knowledge. Many of the titles listed above are meant to supplement carefully structured independent reading with books to read along with a teacher or that are read aloud to students to build knowledge and cultivate a joy in reading.

Staying on Topic Within a Grade and Across Grades: How to Build Knowledge Systematically

Building knowledge systematically in English language arts is like giving children various pieces of a puzzle in each grade that, over time, will form one big picture. At a curricular or instructional level, texts—within and across grade levels—need to be selected around topics or themes that systematically develop the knowledge base of students. Within a grade level, there should be an adequate number of titles on a single topic that would allow children to study that topic for a sustained period. The knowledge children have learned about particular topics in early grade levels should then be expanded and developed in subsequent grade levels to ensure an increasingly deeper understanding of these topics. Children in the upper elementary grades will generally be expected to read these texts independently and reflect on them in writing. However, children in the early grades (particularly K–2) should participate in rich, structured conversations with an adult in response to the written texts that are read aloud, orally comparing and contrasting as well as analyzing and synthesizing, in the manner called for by the *Standards*.

Preparation for reading complex informational texts should begin at the very earliest elementary school grades. What follows is one example that uses domain-specific nonfiction titles across grade levels to illustrate how curriculum designers and classroom teachers can infuse the English language arts block with rich, age-appropriate content knowledge and vocabulary in history/social studies, science, and the arts. Having students listen to informational read-alouds in the early grades helps lay the necessary foundation for students' reading and understanding of increasingly complex texts on their own in subsequent grades.

Exemplar Texts on a Topic Across Grades	K	1	2–3	4–5
The Human Body **Students can begin learning about the human body starting in kindergarten and then review and extend their learning during each subsequent grade.**	**The five senses and associated body parts** • *My Five Senses* by Aliki (1989) • *Hearing* by Maria Rius (1985) • *Sight* by Maria Rius (1985) • *Smell* by Maria Rius (1985) • *Taste* by Maria Rius (1985) • *Touch* by Maria Rius (1985) **Taking care of your body: Overview (hygiene, diet, exercise, rest)** • *My Amazing Body: A First Look at Health & Fitness* by Pat Thomas (2001) • *Get Up and Go!* by Nancy Carlson (2008) • *Go Wash Up* by Doering Tourville (2008) • *Sleep* by Paul Showers (1997) • *Fuel the Body* by Doering Tourville (2008)	**Introduction to the systems of the human body and associated body parts** • *Under Your Skin: Your Amazing Body* by Mick Manning (2007) • *Me and My Amazing Body* by Joan Sweeney (1999) • *The Human Body* by Gallimard Jeunesse (2007) • *The Busy Body Book* by Lizzy Rockwell (2008) • *First Encyclopedia of the Human Body* by Fiona Chandler (2004) **Taking care of your body: Germs, diseases, and preventing illness** • *Germs Make Me Sick* by Marilyn Berger (1995) • *Tiny Life on Your Body* by Christine Taylor-Butler (2005) • *Germ Stories* by Arthur Kornberg (2007) • *All About Scabs* by GenichiroYagu (1998)	**Digestive and excretory systems** • *What Happens to a Hamburger* by Paul Showers (1985) • *The Digestive System* by Christine Taylor-Butler (2008) • *The Digestive System* by Rebecca L. Johnson (2006) • *The Digestive System* by Kristin Petrie (2007) **Taking care of your body: Healthy eating and nutrition** • *Good Enough to Eat* by Lizzy Rockwell (1999) • *Showdown at the Food Pyramid* by Rex Barron (2004) **Muscular, skeletal, and nervous systems** • *The Mighty Muscular and Skeletal Systems* Crabtree Publishing (2009) • *Muscles* by Seymour Simon (1998) • *Bones* by Seymour Simon (1998) • *The Astounding Nervous System* Crabtree Publishing (2009) • *The Nervous System* by Joelle Riley (2004)	**Circulatory system** • *The Heart* by Seymour Simon (2006) • *The Heart and Circulation* by Carol Ballard (2005) • *The Circulatory System* by Kristin Petrie (2007) • *The Amazing Circulatory System* by John Burstein (2009) **Respiratory system** • *The Lungs* by Seymour Simon (2007) • *The Respiratory System* by Susan Glass (2004) • *The Respiratory System* by Kristin Petrie (2007) • *The Remarkable Respiratory System* by John Burstein (2009) **Endocrine system** • *The Endocrine System* by Rebecca Olien (2006) • *The Exciting Endocrine System* by John Burstein (2009)

STANDARDS FOR

Literacy in All Subjects

6–12

College and Career Readiness Anchor Standards for Reading

The grades 6–12 standards on the following pages define what students should understand and be able to do by the end of each grade span. They correspond to the College and Career Readiness (CCR) anchor standards below by number. The CCR and grade-specific standards are necessary complements—the former providing broad standards, the latter providing additional specificity—that together define the skills and understandings that all students must demonstrate.

Key Ideas and Details

1. Read closely to determine what the text says explicitly and to make logical inferences from it; cite specific textual evidence when writing or speaking to support conclusions drawn from the text.
2. Determine central ideas or themes of a text and analyze their development; summarize the key supporting details and ideas.
3. Analyze how and why individuals, events, or ideas develop and interact over the course of a text.

Craft and Structure

4. Interpret words and phrases as they are used in a text, including determining technical, connotative, and figurative meanings, and analyze how specific word choices shape meaning or tone.
5. Analyze the structure of texts, including how specific sentences, paragraphs, and larger portions of the text (e.g., a section, chapter, scene, or stanza) relate to each other and the whole.
6. Assess how point of view or purpose shapes the content and style of a text.

Integration of Knowledge and Ideas

7. Integrate and evaluate content presented in diverse formats and media, including visually and quantitatively, as well as in words.*
8. Delineate and evaluate the argument and specific claims in a text, including the validity of the reasoning as well as the relevance and sufficiency of the evidence.
9. Analyze how two or more texts address similar themes or topics in order to build knowledge or to compare the approaches the authors take.

Range of Reading and Level of Text Complexity

10. Read and comprehend complex literary and informational texts independently and proficiently.

*Please see “Research to Build and Present Knowledge” in Writing for additional standards relevant to gathering, assessing, and applying information from print and digital sources.

Note on range and content of student reading

Reading is critical to building knowledge in history/social studies as well as in science and technical subjects. College and career ready reading in these fields requires an appreciation of the norms and conventions of each discipline, such as the kinds of evidence used in history and science; an understanding of domain-specific words and phrases; an attention to precise details; and the capacity to evaluate intricate arguments, synthesize complex information, and follow detailed descriptions of events and concepts. In history/social studies, for example, students need to be able to analyze, evaluate, and differentiate primary and secondary sources. When reading scientific and technical texts, students need to be able to gain knowledge from challenging texts that often make extensive use of elaborate diagrams and data to convey information and illustrate concepts. Students must be able to read complex informational texts in these fields with independence and confidence because the vast majority of reading in college and workforce training programs will be sophisticated nonfiction. It is important to note that these Reading standards are meant to complement the specific content demands of the disciplines, not replace them.

Reading Standards for Literacy in All Subjects

RH

The standards below begin at grade 6; standards for K–5 reading in history/social studies, science, and technical subjects are integrated into the K–5 Reading standards. The CCR anchor standards and high school standards in literacy work in tandem to define college and career readiness expectations—the former providing broad standards, the latter providing additional specificity.

Grades 6–8 students:	Grades 9–10 students:	Grades 11–12 students:
Key Ideas and Details		
1. Cite specific textual evidence to support analysis of primary and secondary sources.	1. Cite specific textual evidence to support analysis of primary and secondary sources, attending to such features as the date and origin of the information.	1. Cite specific textual evidence to support analysis of primary and secondary sources, connecting insights gained from specific details to an understanding of the text as a whole.
2. Determine the central ideas or information of a primary or secondary source; provide an accurate summary of the source distinct from prior knowledge or opinions.	2. Determine the central ideas or information of a primary or secondary source; provide an accurate summary of how key events or ideas develop over the course of the text.	2. Determine the central ideas or information of a primary or secondary source; provide an accurate summary that makes clear the relationships among the key details and ideas.
3. Identify key steps in a text's description of a process related to history/social studies (e.g., how a bill becomes law, how interest rates are raised or lowered).	3. Analyze in detail a series of events described in a text; determine whether earlier events caused later ones or simply preceded them.	3. Evaluate various explanations for actions or events and determine which explanation best accords with textual evidence, acknowledging where the text leaves matters uncertain.
Craft and Structure		
4. Determine the meaning of words and phrases as they are used in a text, including vocabulary specific to domains related to history/social studies.	4. Determine the meaning of words and phrases as they are used in a text, including vocabulary describing political, social, or economic aspects of history/social studies.	4. Determine the meaning of words and phrases as they are used in a text, including analyzing how an author uses and refines the meaning of a key term over the course of a text (e.g., how Madison defines *faction* in *Federalist* No. 10).
5. Describe how a text presents information (e.g., sequentially, comparatively, causally).	5. Analyze how a text uses structure to emphasize key points or advance an explanation or analysis.	5. Analyze in detail how a complex primary source is structured, including how key sentences, paragraphs, and larger portions of the text contribute to the whole.
6. Identify aspects of a text that reveal an author's point of view or purpose (e.g., loaded language, inclusion or avoidance of particular facts).	6. Compare the point of view of two or more authors for how they treat the same or similar topics, including which details they include and emphasize in their respective accounts.	6. Evaluate authors' differing points of view on the same historical event or issue by assessing the authors' claims, reasoning, and evidence.
Integration of Knowledge and Ideas		
7. Integrate visual information (e.g., in charts, graphs, photographs, videos, or maps) with other information in print and digital texts.	7. Integrate quantitative or technical analysis (e.g., charts, research data) with qualitative analysis in print or digital text.	7. Integrate and evaluate multiple sources of information presented in diverse formats and media (e.g., visually, quantitatively, as well as in words) in order to address a question or solve a problem.
8. Distinguish among fact, opinion, and reasoned judgment in a text.	8. Assess the extent to which the reasoning and evidence in a text support the author's claims.	8. Evaluate an author's premises, claims, and evidence by corroborating or challenging them with other information.
9. Analyze the relationship between a primary and secondary source on the same topic.	9. Compare and contrast treatments of the same topic in several primary and secondary sources.	9. Integrate information from diverse sources, both primary and secondary, into a coherent understanding of an idea or event, noting discrepancies among sources.
Range of Reading and Level of Text Complexity		
10. By the end of grade 8, read and comprehend history/social studies texts in the grades 6–8 text complexity band independently and proficiently.	10. By the end of grade 10, read and comprehend history/social studies texts in the grades 9–10 text complexity band independently and proficiently.	10. By the end of grade 12, read and comprehend history/social studies texts in the grades 11–CCR text complexity band independently and proficiently.

Reading Standards for Literacy in All Subjects

RST

Grades 6–8 students:	Grades 9–10 students:	Grades 11–12 students:
Key Ideas and Details		
1. Cite specific textual evidence to support analysis of science and technical texts.	1. Cite specific textual evidence to support analysis of science and technical texts, attending to the precise details of explanations or descriptions.	1. Cite specific textual evidence to support analysis of science and technical texts, attending to important distinctions the author makes and to any gaps or inconsistencies in the account.
2. Determine the central ideas or conclusions of a text; provide an accurate summary of the text distinct from prior knowledge or opinions.	2. Determine the central ideas or conclusions of a text; trace the text's explanation or depiction of a complex process, phenomenon, or concept; provide an accurate summary of the text.	2. Determine the central ideas or conclusions of a text; summarize complex concepts, processes, or information presented in a text by paraphrasing them in simpler but still accurate terms.
3. Follow precisely a multistep procedure when carrying out experiments, taking measurements, or performing technical tasks.	3. Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks, attending to special cases or exceptions defined in the text.	3. Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks; analyze the specific results based on explanations in the text.
Craft and Structure		
4. Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to *grades 6–8 texts and topics.*	4. Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to *grades 9–10 texts and topics*.	4. Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to *grades 11–12 texts and topics*.
5. Analyze the structure an author uses to organize a text, including how the major sections contribute to the whole and to an understanding of the topic.	5. Analyze the structure of the relationships among concepts in a text, including relationships among key terms (e.g., *force, friction, reaction force, energy*).	5. Analyze how the text structures information or ideas into categories or hierarchies, demonstrating understanding of the information or ideas.
6. Analyze the author's purpose in providing an explanation, describing a procedure, or discussing an experiment in a text.	6. Analyze the author's purpose in providing an explanation, describing a procedure, or discussing an experiment in a text, defining the question the author seeks to address.	6. Analyze the author's purpose in providing an explanation, describing a procedure, or discussing an experiment in a text, identifying important issues that remain unresolved.
Integration of Knowledge and Ideas		
7. Integrate quantitative or technical information expressed in words in a text with a version of that information expressed visually (e.g., in a flowchart, diagram, model, graph, or table).	7. Translate quantitative or technical information expressed in words in a text into visual form (e.g., a table or chart) and translate information expressed visually or mathematically (e.g., in an equation) into words.	7. Integrate and evaluate multiple sources of information presented in diverse formats and media (e.g., quantitative data, video, multimedia) in order to address a question or solve a problem.
8. Distinguish among facts, reasoned judgment based on research findings, and speculation in a text.	8. Assess the extent to which the reasoning and evidence in a text support the author's claim or a recommendation for solving a scientific or technical problem.	8. Evaluate the hypotheses, data, analysis, and conclusions in a science or technical text, verifying the data when possible and corroborating or challenging conclusions with other sources of information.
9. Compare and contrast the information gained from experiments, simulations, video, or multimedia sources with that gained from reading a text on the same topic.	9. Compare and contrast findings presented in a text to those from other sources (including their own experiments), noting when the findings support or contradict previous explanations or accounts.	9. Synthesize information from a range of sources (e.g., texts, experiments, simulations) into a coherent understanding of a process, phenomenon, or concept, resolving conflicting information when possible.
Range of Reading and Level of Text Complexity		
10. By the end of grade 8, read and comprehend science/technical texts in the grades 6–8 text complexity band independently and proficiently.	10. By the end of grade 10, read and comprehend science/technical texts in the grades 9–10 text complexity band independently and proficiently.	10. By the end of grade 12, read and comprehend science/technical texts in the grades 11–CCR text complexity band independently and proficiently.

College and Career Readiness Anchor Standards for Writing

The grades 6–12 standards on the following pages define what students should understand and be able to do by the end of each grade span. They correspond to the College and Career Readiness (CCR) anchor standards below by number. The CCR and grade-specific standards are necessary complements—the former providing broad standards, the latter providing additional specificity—that together define the skills and understandings that all students must demonstrate.

Text Types and Purposes*

1. Write arguments to support claims in an analysis of substantive topics or texts using valid reasoning and relevant and sufficient evidence.
2. Write informative/explanatory texts to examine and convey complex ideas and information clearly and accurately through the effective selection, organization, and analysis of content.
3. Write narratives to develop real or imagined experiences or events using effective technique, well-chosen details and well-structured event sequences.

Production and Distribution of Writing

4. Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.
5. Develop and strengthen writing as needed by planning, revising, editing, rewriting, or trying a new approach.
6. Use technology, including the Internet, to produce and publish writing and to interact and collaborate with others.

Research to Build and Present Knowledge

7. Conduct short as well as more sustained research projects based on focused questions, demonstrating understanding of the subject under investigation.
8. Gather relevant information from multiple print and digital sources, assess the credibility and accuracy of each source, and integrate the information while avoiding plagiarism.
9. Draw evidence from literary or informational texts to support analysis, reflection, and research.

Range of Writing

10. Write routinely over extended time frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two) for a range of tasks, purposes, and audiences.

*These broad types of writing include many subgenres.

Note on range and content of student writing

For students, writing is a key means of asserting and defending claims, showing what they know about a subject, and conveying what they have experienced, imagined, thought, and felt. To be college and career ready writers, students must take task, purpose, and audience into careful consideration, choosing words, information, structures, and formats deliberately. They need to be able to use technology strategically when creating, refining, and collaborating on writing. They have to become adept at gathering information, evaluating sources, and citing material accurately, reporting findings from their research and analysis of sources in a clear and cogent manner. They must have the flexibility, concentration, and fluency to produce high-quality first-draft text under a tight deadline and the capacity to revisit and make improvements to a piece of writing over multiple drafts when circumstances encourage or require it. To meet these goals, students must devote significant time and effort to writing, producing numerous pieces over short and long time frames throughout the year.

Writing Standards for Literacy in All Subjects

WHST

The standards below begin at grade 6; standards for K–5 writing in history/social studies, science, and technical subjects are integrated into the K–5 Writing standards. The CCR anchor standards and high school standards in literacy work in tandem to define college and career readiness expectations—the former providing broad standards, the latter providing additional specificity.

Grades 6–8 students:	Grades 9–10 students:	Grades 11–12 students:
Text Types and Purposes		
1. Write arguments focused on *discipline-specific content*. a. Introduce claim(s) about a topic or issue, acknowledge and distinguish the claim(s) from alternate or opposing claims, and organize the reasons and evidence logically. b. Support claim(s) with logical reasoning and relevant, accurate data and evidence that demonstrate an understanding of the topic or text, using credible sources. c. Use words, phrases, and clauses to create cohesion and clarify the relationships among claim(s), counterclaims, reasons, and evidence. d. Establish and maintain a formal style. e. Provide a concluding statement or section that follows from and supports the argument presented.	1. Write arguments focused on *discipline-specific content.* a. Introduce precise claim(s), distinguish the claim(s) from alternate or opposing claims, and create an organization that establishes clear relationships among the claim(s), counterclaims, reasons, and evidence. b. Develop claim(s) and counterclaims fairly, supplying data and evidence for each while pointing out the strengths and limitations of both claim(s) and counterclaims in a discipline-appropriate form and in a manner that anticipates the audience's knowledge level and concerns. c. Use words, phrases, and clauses to link the major sections of the text, create cohesion, and clarify the relationships between claim(s) and reasons, between reasons and evidence, and between claim(s) and counterclaims. d. Establish and maintain a formal style and objective tone while attending to the norms and conventions of the discipline in which they are writing. e. Provide a concluding statement or section that follows from or supports the argument presented.	1. Write arguments focused on *discipline-specific content.* a. Introduce precise, knowledgeable claim(s), establish the significance of the claim(s), distinguish the claim(s) from alternate or opposing claims, and create an organization that logically sequences the claim(s), counterclaims, reasons, and evidence. b. Develop claim(s) and counterclaims fairly and thoroughly, supplying the most relevant data and evidence for each while pointing out the strengths and limitations of both claim(s) and counterclaims in a discipline-appropriate form that anticipates the audience's knowledge level, concerns, values, and possible biases. c. Use words, phrases, and clauses as well as varied syntax to link the major sections of the text, create cohesion, and clarify the relationships between claim(s) and reasons, between reasons and evidence, and between claim(s) and counterclaims. d. Establish and maintain a formal style and objective tone while attending to the norms and conventions of the discipline in which they are writing. e. Provide a concluding statement or section that follows from or supports the argument presented.

Writing Standards for Literacy in All Subjects

WHST

Grades 6–8 students:	Grades 9–10 students:	Grades 11–12 students:
Text Types and Purposes (continued)		
2. Write informative/explanatory texts, including the narration of historical events, scientific procedures/ experiments, or technical processes. a. Introduce a topic clearly, previewing what is to follow; organize ideas, concepts, and information into broader categories as appropriate to achieving purpose; include formatting (e.g., headings), graphics (e.g., charts, tables), and multimedia when useful to aiding comprehension. b. Develop the topic with relevant, well-chosen facts, definitions, concrete details, quotations, or other information and examples. c. Use appropriate and varied transitions to create cohesion and clarify the relationships among ideas and concepts. d. Use precise language and domain-specific vocabulary to inform about or explain the topic. e. Establish and maintain a formal style and objective tone. f. Provide a concluding statement or section that follows from and supports the information or explanation presented.	2. Write informative/explanatory texts, including the narration of historical events, scientific procedures/ experiments, or technical processes. a. Introduce a topic and organize ideas, concepts, and information to make important connections and distinctions; include formatting (e.g., headings), graphics (e.g., figures, tables), and multimedia when useful to aiding comprehension. b. Develop the topic with well-chosen, relevant, and sufficient facts, extended definitions, concrete details, quotations, or other information and examples appropriate to the audience's knowledge of the topic. c. Use varied transitions and sentence structures to link the major sections of the text, create cohesion, and clarify the relationships among ideas and concepts. d. Use precise language and domain-specific vocabulary to manage the complexity of the topic and convey a style appropriate to the discipline and context as well as to the expertise of likely readers. e. Establish and maintain a formal style and objective tone while attending to the norms and conventions of the discipline in which they are writing. f. Provide a concluding statement or section that follows from and supports the information or explanation presented (e.g., articulating implications or the significance of the topic).	2. Write informative/explanatory texts, including the narration of historical events, scientific procedures/ experiments, or technical processes. a. Introduce a topic and organize complex ideas, concepts, and information so that each new element builds on that which precedes it to create a unified whole; include formatting (e.g., headings), graphics (e.g., figures, tables), and multimedia when useful to aiding comprehension. b. Develop the topic thoroughly by selecting the most significant and relevant facts, extended definitions, concrete details, quotations, or other information and examples appropriate to the audience's knowledge of the topic. c. Use varied transitions and sentence structures to link the major sections of the text, create cohesion, and clarify the relationships among complex ideas and concepts. d. Use precise language, domain-specific vocabulary and techniques such as metaphor, simile, and analogy to manage the complexity of the topic; convey a knowledgeable stance in a style that responds to the discipline and context as well as to the expertise of likely readers. e. Provide a concluding statement or section that follows from and supports the information or explanation provided (e.g., articulating implications or the significance of the topic).
3. (See note; not applicable as a separate requirement)	3. (See note; not applicable as a separate requirement)	3. (See note; not applicable as a separate requirement)

Note: Students' narrative skills continue to grow in these grades. The Standards require that students be able to incorporate narrative elements effectively into arguments and informative/explanatory texts. In history/social studies, students must be able to incorporate narrative accounts into their analyses of individuals or events of historical import. In science and technical subjects, students must be able to write precise enough descriptions of the step-by-step procedures they use in their investigations or technical work that others can replicate them and (possibly) reach the same results.

Writing Standards for Literacy in All Subjects

WHST

Grades 6–8 students:	Grades 9–10 students:	Grades 11–12 students:
Production and Distribution of Writing		
4. Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.	4. Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.	4. Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.
5. With some guidance and support from peers and adults, develop and strengthen writing as needed by planning, revising, editing, rewriting, or trying a new approach, focusing on how well purpose and audience have been addressed.	5. Develop and strengthen writing as needed by planning, revising, editing, rewriting, or trying a new approach, focusing on addressing what is most significant for a specific purpose and audience.	5. Develop and strengthen writing as needed by planning, revising, editing, rewriting, or trying a new approach, focusing on addressing what is most significant for a specific purpose and audience.
6. Use technology, including the Internet, to produce and publish writing and present the relationships between information and ideas clearly and efficiently.	6. Use technology, including the Internet, to produce, publish, and update individual or shared writing products, taking advantage of technology's capacity to link to other information and to display information flexibly and dynamically.	6. Use technology, including the Internet, to produce, publish, and update individual or shared writing products in response to ongoing feedback, including new arguments or information.
Research to Build and Present Knowledge		
7. Conduct short research projects to answer a question (including a self-generated question), drawing on several sources and generating additional related, focused questions that allow for multiple avenues of exploration.	7. Conduct short as well as more sustained research projects to answer a question (including a self-generated question) or solve a problem; narrow or broaden the inquiry when appropriate; synthesize multiple sources on the subject, demonstrating understanding of the subject under investigation.	7. Conduct short as well as more sustained research projects to answer a question (including a self-generated question) or solve a problem; narrow or broaden the inquiry when appropriate; synthesize multiple sources on the subject, demonstrating understanding of the subject under investigation.
8. Gather relevant information from multiple print and digital sources, using search terms effectively; assess the credibility and accuracy of each source; and quote or paraphrase the data and conclusions of others while avoiding plagiarism and following a standard format for citation.	8. Gather relevant information from multiple authoritative print and digital sources, using advanced searches effectively; assess the usefulness of each source in answering the research question; integrate information into the text selectively to maintain the flow of ideas, avoiding plagiarism and following a standard format for citation.	8. Gather relevant information from multiple authoritative print and digital sources, using advanced searches effectively; assess the strengths and limitations of each source in terms of the specific task, purpose, and audience; integrate information into the text selectively to maintain the flow of ideas, avoiding plagiarism and overreliance on any one source and following a standard format for citation.
9. Draw evidence from informational texts to support analysis reflection, and research.	9. Draw evidence from informational texts to support analysis, reflection, and research.	9. Draw evidence from informational texts to support analysis, reflection, and research.
Range of Writing		
10. Write routinely over extended time frames (time for reflection and revision) and shorter time frames (a single sitting or a day or two) for a range of discipline-specific tasks, purposes, and audiences.	10. Write routinely over extended time frames (time for reflection and revision) and shorter time frames (a single sitting or a day or two) for a range of discipline-specific tasks, purposes, and audiences.	10. Write routinely over extended time frames (time for reflection and revision) and shorter time frames (a single sitting or a day or two) for a range of discipline-specific tasks, purposes, and audiences.

College and Career Readiness Anchor Standards for Speaking and Listening

The grades 6–12 standards on the following pages define what students should understand and be able to do by the end of each grade. They correspond to the College and Career Readiness (CCR) anchor standards below by number. The CCR and grade-specific standards are necessary complements—the former providing broad standards, the latter providing additional specificity—that together define the skills and understandings that all students must demonstrate.

Comprehension and Collaboration

1. Prepare for and participate effectively in a range of conversations and collaborations with diverse partners, building on others' ideas and expressing their own clearly and persuasively.
2. Integrate and evaluate information presented in diverse media and formats, including visually, quantitatively, and orally.
3. Evaluate a speaker's point of view, reasoning, and use of evidence and rhetoric.

Presentation of Knowledge and Ideas

4. Present information, findings, and supporting evidence such that listeners can follow the line of reasoning and the organization, development, and style are appropriate to task, purpose, and audience.
5. Make strategic use of digital media and visual displays of data to express information and enhance understanding of presentations.
6. Adapt speech to a variety of contexts and communicative tasks, demonstrating command of formal English when indicated or appropriate.

Note on range and content of student speaking and listening

To become college and career ready, students must have ample opportunities to take part in a variety of rich, structured conversations—as part of a whole class, in small groups, and with a partner—built around important content in various domains. They must be able to contribute appropriately to these conversations, to make comparisons and contrasts, and to analyze and synthesize a multitude of ideas in accordance with the standards of evidence appropriate to a particular discipline. Whatever their intended major or profession, high school graduates will depend heavily on their ability to listen attentively to others so that they are able to build on others' meritorious ideas while expressing their own clearly and persuasively.

New technologies have broadened and expanded the role that speaking and listening play in acquiring and sharing knowledge and have tightened their link to other forms of communication. The Internet has accelerated the speed at which connections between speaking, listening, reading, and writing can be made, requiring that students be ready to use these modalities nearly simultaneously. Technology itself is changing quickly, creating a new urgency for students to be adaptable in response to change.

Speaking and Listening Standards for Literacy in All Subjects

SL

The following standards for grades 6–12 offer a focus for instruction in each year to help ensure that students gain adequate mastery of a range of skills and applications. *Students advancing through the grades are expected to meet each year's grade-specific standards and retain or further develop skills and understandings mastered in preceding grades.*

Grade 6 students:	Grade 7 students:	Grade 8 students:
Comprehension and Collaboration		
1. Engage effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on *grade 6 topics, texts, and issues*, building on others' ideas and expressing their own clearly. a. Come to discussions prepared, having read or studied required material; explicitly draw on that preparation by referring to evidence on the topic, text, or issue to probe and reflect on ideas under discussion. b. Follow rules for collegial discussions, set specific goals and deadlines, and define individual roles as needed. c. Pose and respond to specific questions with elaboration and detail by making comments that contribute to the topic, text, or issue under discussion. d. Review the key ideas expressed and demonstrate understanding of multiple perspectives through reflection and paraphrasing.	1. Engage effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on *grade 7 topics, texts, and issues,* building on others' ideas and expressing their own clearly. a. Come to discussions prepared, having read or researched material under study; explicitly draw on that preparation by referring to evidence on the topic, text, or issue to probe and reflect on ideas under discussion. b. Follow rules for collegial discussions, track progress toward specific goals and deadlines, and define individual roles as needed. c. Pose questions that elicit elaboration and respond to others' questions and comments with relevant observations and ideas that bring the discussion back on topic as needed. d. Acknowledge new information expressed by others and, when warranted, modify their own views.	1. Engage effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on *grade 8 topics, texts, and issues,* building on others' ideas and expressing their own clearly. a. Come to discussions prepared, having read or researched material under study; explicitly draw on that preparation by referring to evidence on the topic, text, or issue to probe and reflect on ideas under discussion. b. Follow rules for collegial discussions and decision-making, track progress toward specific goals and deadlines, and define individual roles as needed. c. Pose questions that connect the ideas of several speakers and respond to others' questions and comments with relevant evidence, observations, and ideas. d. Acknowledge new information expressed by others, and, when warranted, qualify or justify their own views in light of the evidence presented.
2. Interpret information presented in diverse media and formats (e.g., visually, quantitatively, orally) and explain how it contributes to a topic, text, or issue under study.	2. Analyze the main ideas and supporting details presented in diverse media and formats (e.g., visually, quantitatively, orally) and explain how the ideas clarify a topic, text, or issue under study.	2. Analyze the purpose of information presented in diverse media and formats (e.g., visually, quantitatively, orally) and evaluate the motives (e.g., social, commercial, political) behind its presentation.
3. Delineate a speaker's argument and specific claims, distinguishing claims that are supported by reasons and evidence from claims that are not.	3. Delineate a speaker's argument and specific claims, evaluating the soundness of the reasoning and the relevance and sufficiency of the evidence.	3. Delineate a speaker's argument and specific claims, evaluating the soundness of the reasoning and relevance and sufficiency of the evidence and identifying when irrelevant evidence is introduced.
Presentation of Knowledge and Ideas		
4. Present claims and findings, sequencing ideas logically and using pertinent descriptions, facts, and details to accentuate main ideas or themes; use appropriate eye contact, adequate volume, and clear pronunciation.	4. Present claims and findings, emphasizing salient points in a focused, coherent manner with pertinent descriptions, facts, details, and examples; use appropriate eye contact, adequate volume, and clear pronunciation.	4. Present claims and findings, emphasizing salient points in a focused, coherent manner with relevant evidence, sound valid reasoning, and well-chosen details; use appropriate eye contact, adequate volume, and clear pronunciation.
5. Include multimedia components (e.g., graphics, images, music, sound) and visual displays in presentations to clarify information.	5. Include multimedia components and visual displays in presentations to clarify claims and findings and emphasize salient points.	5. Integrate multimedia and visual displays into presentations to clarify information, strengthen claims and evidence, and add interest.
6. Adapt speech to a variety of contexts and tasks, demonstrating command of formal English when indicated or appropriate.	6. Adapt speech to a variety of contexts and tasks, demonstrating command of formal English when indicated or appropriate.	6. Adapt speech to a variety of contexts and tasks, demonstrating command of formal English when indicated or appropriate.

Speaking and Listening Standards for Literacy in All Subjects

SL

The CCR anchor standards and high school grade-specific standards work in tandem to define college and career readiness expectations—the former providing broad standards, the latter providing additional specificity.

Grades 9–10 students:	Grades 11–12 students:
Comprehension and Collaboration	
1. Initiate and participate effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on *grades 9–10 topics, texts, and issues,* building on others' ideas and expressing their own clearly and persuasively. a. Come to discussions prepared, having read and researched material under study; explicitly draw on that preparation by referring to evidence from texts and other research on the topic or issue to stimulate a thoughtful, well-reasoned exchange of ideas. b. Work with peers to set rules for collegial discussions and decision-making (e.g., informal consensus, taking votes on key issues, presentation of alternate views), clear goals and deadlines, and individual roles as needed. c. Propel conversations by posing and responding to questions that relate the current discussion to broader themes or larger ideas; actively incorporate others into the discussion; and clarify, verify, or challenge ideas and conclusions. d. Respond thoughtfully to diverse perspectives, summarize points of agreement and disagreement, and, when warranted, qualify or justify their own views and understanding and make new connections in light of the evidence and reasoning presented.	1. Initiate and participate effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on *grades 11–12 topics, texts, and issues,* building on others' ideas and expressing their own clearly and persuasively. a. Come to discussions prepared, having read and researched material under study; explicitly draw on that preparation by referring to evidence from texts and other research on the topic or issue to stimulate a thoughtful, well-reasoned exchange of ideas. b. Work with peers to promote civil, democratic discussions and decision-making, set clear goals and deadlines, and establish individual roles as needed. c. Propel conversations by posing and responding to questions that probe reasoning and evidence; ensure a hearing for a full range of positions on a topic or issue; clarify, verify, or challenge ideas and conclusions; and promote divergent and creative perspectives. d. Respond thoughtfully to diverse perspectives; synthesize comments, claims, and evidence made on all sides of an issue; resolve contradictions when possible; and determine what additional information or research is required to deepen the investigation or complete the task.
2. Integrate multiple sources of information presented in diverse media or formats (e.g., visually, quantitatively, orally) evaluating the credibility and accuracy of each source.	2. Integrate multiple sources of information presented in diverse formats and media (e.g., visually, quantitatively, orally) in order to make informed decisions and solve problems, evaluating the credibility and accuracy of each source and noting any discrepancies among the data.
3. Evaluate a speaker's point of view, reasoning, and use of evidence and rhetoric, identifying any fallacious reasoning or exaggerated or distorted evidence.	3. Evaluate a speaker's point of view, reasoning, and use of evidence and rhetoric, assessing the stance, premises, links among ideas, word choice, points of emphasis, and tone used.
Presentation of Knowledge and Ideas	
4. Present information, findings, and supporting evidence clearly, concisely, and logically such that listeners can follow the line of reasoning and the organization, development, substance, and style are appropriate to purpose, audience, and task.	4. Present information, findings, and supporting evidence, conveying a clear and distinct perspective, such that listeners can follow the line of reasoning, alternative or opposing perspectives are addressed, and the organization, development, substance, and style are appropriate to purpose, audience, and a range of formal and informal tasks.
5. Make strategic use of digital media (e.g., textual, graphical, audio, visual, and interactive elements) in presentations to enhance understanding of findings, reasoning, and evidence and to add interest.	5. Make strategic use of digital media (e.g., textual, graphical, audio, visual, and interactive elements) in presentations to enhance understanding of findings, reasoning, and evidence and to add interest.
6. Adapt speech to a variety of contexts and tasks, demonstrating command of formal English when indicated or appropriate.	6. Adapt speech to a variety of contexts and tasks, demonstrating a command of formal English when indicated or appropriate.

College and Career Readiness Anchor Standards for Language

The grades 6–12 standards on the following pages define what students should understand and be able to do by the end of each grade. They correspond to the College and Career Readiness (CCR) anchor standards below by number. The CCR and grade-specific standards are necessary complements—the former providing broad standards, the latter providing additional specificity—that together define the skills and understandings that all students must demonstrate.

Conventions of Standard English

1. Demonstrate command of the conventions of standard English grammar and usage when writing or speaking.
2. Demonstrate command of the conventions of standard English capitalization, punctuation, and spelling when writing.

Knowledge of Language

3. Apply knowledge of language to understand how language functions in different contexts, to make effective choices for meaning or style, and to comprehend more fully when reading or listening.

Vocabulary Acquisition and Use

4. Determine or clarify the meaning of unknown and multiple-meaning words and phrases by using context clues, analyzing meaningful word parts, and consulting general and specialized reference materials, as appropriate.
5. Demonstrate understanding of figurative language, word relationships, and nuances in word meanings.
6. Acquire and use accurately a range of general academic and domain-specific words and phrases sufficient for reading, writing, speaking, and listening at the college and career readiness level; demonstrate independence in gathering vocabulary knowledge when considering a word or phrase important to comprehension or expression.

Note on range and content of student language use

To be college and career ready in language, students must have firm control over the conventions of standard English. At the same time, they must come to appreciate that language is as at least as much a matter of craft as of rules and be able to choose words, syntax, and punctuation to express themselves and achieve particular functions and rhetorical effects. They must also have extensive vocabularies, built through reading and study, enabling them to comprehend complex texts and engage in purposeful writing about and conversations around content. They need to become skilled in determining or clarifying the meaning of words and phrases they encounter, choosing flexibly from an array of strategies to aid them. They must learn to see an individual word as part of a network of other words—words, for example, that have similar denotations but different connotations. The inclusion of Language standards in their own strand should not be taken as an indication that skills related to conventions, effective language use, and vocabulary are unimportant to reading, writing, speaking, and listening; indeed, they are inseparable from such contexts.

Language Standards for Literacy in All Subjects

L

The following standards for grades 6–12 offer a focus for instruction each year to help ensure that students gain adequate mastery of a range of skills and applications. *Students advancing through the grades are expected to meet each year's grade-specific standards and retain or further develop skills and understandings mastered in preceding grades.* Beginning in grade 3, skills and understandings that are particularly likely to require continued attention in higher grades as they are applied to increasingly sophisticated writing and speaking are marked with an asterisk (*).

Grade 6 students:	Grade 7 students:	Grade 8 students:
Conventions of Standard English		
1. Demonstrate command of the conventions of standard English grammar and usage when writing or speaking. a. Ensure that pronouns are in the proper case (subjective, objective, possessive). b. Use intensive pronouns (e.g., *myself, ourselves*). c. Recognize and correct inappropriate shifts in pronoun number and person.* d. Recognize and correct vague pronouns (i.e., ones with unclear or ambiguous antecedents).* e. Recognize variations from standard English in their own and others' writing and speaking, and identify and use strategies to improve expression in conventional language.*	1. Demonstrate command of the conventions of standard English grammar and usage when writing or speaking. a. Explain the function of phrases and clauses in general and their function in specific sentences. b. Choose among simple, compound, complex, and compound-complex sentences to signal differing relationships among ideas. c. Place phrases and clauses within a sentence, recognizing and correcting misplaced and dangling modifiers.*	1. Demonstrate command of the conventions of standard English grammar and usage when writing or speaking. a. Explain the function of verbals (gerunds, participles, infinitives) in general and their function in particular sentences. b. Form and use verbs in the active and passive voice. c. Form and use verbs in the indicative, imperative, interrogative, conditional, and subjunctive mood. d. Recognize and correct inappropriate shifts in verb voice and mood.*
2. Demonstrate command of the conventions of standard English capitalization, punctuation, and spelling when writing. a. Use punctuation (commas, parentheses, dashes) to set off nonrestrictive/parenthetical elements.* b. Spell correctly.	2. Demonstrate command of the conventions of standard English capitalization, punctuation, and spelling when writing. a. Use a comma to separate coordinate adjectives (e.g., *It was a fascinating, enjoyable movie* but not *He wore an old[,] green shirt*). b. Spell correctly.	2. Demonstrate command of the conventions of standard English capitalization, punctuation, and spelling when writing. a. Use punctuation (comma, ellipsis, dash) to indicate a pause or break. b. Use an ellipsis to indicate an omission. c. Spell correctly.
Knowledge of Language		
3. Use knowledge of language and its conventions when writing, speaking, reading, or listening. a. Vary sentence patterns for meaning, reader/listener interest, and style.* b. Maintain consistency in style and tone.*	3. Use knowledge of language and its conventions when writing, speaking, reading, or listening. a. Choose language that expresses ideas precisely and concisely, recognizing and eliminating wordiness and redundancy.*	3. Use knowledge of language and its conventions when writing, speaking, reading, or listening. a. Use verbs in the active and passive voice and in the conditional and subjunctive mood to achieve particular effects (e.g., emphasizing the actor or the action; expressing uncertainty or describing a state contrary to fact).

Language Standards for Literacy in All Subjects

Grade 6 students:	Grade 7 students:	Grade 8 students:
Vocabulary Acquisition and Use		
4. Determine or clarify the meaning of unknown and multiple-meaning words and phrases based on *grade 6 reading and content*, choosing flexibly from a range of strategies. a. Use context (e.g., the overall meaning of a sentence or paragraph; a word's position or function in a sentence) as a clue to the meaning of a word or phrase. b. Use common, grade-appropriate Greek or Latin affixes and roots as clues to the meaning of a word (e.g., *audience, auditory, audible*). c. Consult reference materials (e.g., dictionaries, glossaries, thesauruses), both print and digital, to find the pronunciation of a word or determine or clarify its precise meaning or its part of speech. d. Verify the preliminary determination of the meaning of a word or phrase (e.g., by checking the inferred meaning in context or in a dictionary).	4. Determine or clarify the meaning of unknown and multiple-meaning words and phrases based on *grade 7 reading and content*, choosing flexibly from a range of strategies. a. Use context (e.g., the overall meaning of a sentence or paragraph; a word's position or function in a sentence) as a clue to the meaning of a word or phrase. b. Use common, grade-appropriate Greek or Latin affixes and roots as clues to the meaning of a word (e.g., *belligerent, bellicose, rebel*). c. Consult general and specialized reference materials (e.g., dictionaries, glossaries, thesauruses), both print and digital, to find the pronunciation of a word or determine or clarify its precise meaning or its part of speech. d. Verify the preliminary determination of the meaning of a word or phrase (e.g., by checking the inferred meaning in context or in a dictionary).	4. Determine or clarify the meaning of unknown and multiple-meaning words or phrases based on *grade 8 reading and content*, choosing flexibly from a range of strategies. a. Use context (e.g., the overall meaning of a sentence or paragraph; a word's position or function in a sentence) as a clue to the meaning of a word or phrase. b. Use common, grade-appropriate Greek or Latin affixes and roots as clues to the meaning of a word (e.g., *precede, recede, secede*). c. Consult general and specialized reference materials (e.g., dictionaries, glossaries, thesauruses), both print and digital, to find the pronunciation of a word or determine or clarify its precise meaning or its part of speech. d. Verify the preliminary determination of the meaning of a word or phrase (e.g., by checking the inferred meaning in context or in a dictionary).
5. Demonstrate understanding of figurative language, word relationships, and nuances in word meanings. a. Interpret figures of speech (e.g., personification) in context. b. Use the relationship between particular words (e.g., cause/effect, part/whole, item/category) to better understand each of the words. c. Distinguish among the connotations (associations) of words with similar denotations (definitions) (e.g., *stingy, scrimping, economical, unwasteful, thrifty*).	5. Demonstrate understanding of figurative language, word relationships, and nuances in word meanings. a. Interpret figures of speech (e.g., literary, biblical, and mythological allusions) in context. b. Use the relationship between particular words (e.g., synonym/antonym, analogy) to better understand each of the words. c. Distinguish among the connotations (associations) of words with similar denotations (definitions) (e.g., *refined, respectful, polite, diplomatic, condescending*).	5. Demonstrate understanding of figurative language, word relationships, and nuances in word meanings. a. Interpret figures of speech (e.g. verbal irony, puns) in context. b. Use the relationship between particular words to better understand each of the words. c. Distinguish among the connotations (associations) of words with similar denotations (definitions) (e.g., *bullheaded, willful, firm, persistent, resolute*).
6. Acquire and use accurately grade-appropriate general academic and domain-specific words and phrases; gather vocabulary knowledge when considering a word or phrase important to comprehension or expression.	6. Acquire and use accurately grade-appropriate general academic and domain-specific words and phrases; gather vocabulary knowledge when considering a word or phrase important to comprehension or expression.	6. Acquire and use accurately grade-appropriate general academic and domain-specific words and phrases; gather vocabulary knowledge when considering a word or phrase important to comprehension or expression.

Language Standards for Literacy in All Subjects

L

Grades 9–10 students:	Grades 11–12 students:
Vocabulary Acquisition and Use	
4. Determine or clarify the meaning of unknown and multiple-meaning words and phrases based on *grades 9–10 reading and content*, choosing flexibly from a range of strategies. a. Use context (e.g., the overall meaning of a sentence, paragraph, or text; a word's position or function in a sentence) as a clue to the meaning of a word or phrase. b. Identify and correctly use patterns of word changes that indicate different meanings or parts of speech (e.g., *analyze, analysis, analytical; advocate, advocacy*). c. Consult general and specialized reference materials (e.g., dictionaries, glossaries, thesauruses), both print and digital, to find the pronunciation of a word or determine or clarify its precise meaning, its part of speech, or its etymology. d. Verify the preliminary determination of the meaning of a word or phrase (e.g., by checking the inferred meaning in context or in a dictionary).	4. Determine or clarify the meaning of unknown and multiple-meaning words and phrases based on *grades 11–12 reading and content*, choosing flexibly from a range of strategies. a. Use context (e.g., the overall meaning of a sentence, paragraph, or text; a word's position or function in a sentence) as a clue to the meaning of a word or phrase. b. Identify and correctly use patterns of word changes that indicate different meanings or parts of speech (e.g., *conceive, conception, conceivable*). c. Consult general and specialized reference materials (e.g., dictionaries, glossaries, thesauruses), both print and digital, to find the pronunciation of a word or determine or clarify its precise meaning, its part of speech, its etymology, or its standard usage. d. Verify the preliminary determination of the meaning of a word or phrase (e.g., by checking the inferred meaning in context or in a dictionary).
5. Demonstrate understanding of figurative language, word relationships, and nuances in word meanings. a. Interpret figures of speech (e.g., euphemism, oxymoron) in context and analyze their role in the text. b. Analyze nuances in the meaning of words with similar denotations.	5. Demonstrate understanding of figurative language, word relationships, and nuances in word meanings. a. Interpret figures of speech (e.g., hyperbole, paradox) in context and analyze their role in the text. b. Analyze nuances in the meaning of words with similar denotations.
6. Acquire and use accurately general academic and domain-specific words and phrases, sufficient for reading, writing, speaking, and listening at the college and career readiness level; demonstrate independence in gathering vocabulary knowledge when considering a word or phrase important to comprehension or expression.	6. Acquire and use accurately general academic and domain-specific words and phrases, sufficient for reading, writing, speaking, and listening at the college and career readiness level; demonstrate independence in gathering vocabulary knowledge when considering a word or phrase important to comprehension or expression.

Language Progressive Skills, by Grade

The following skills, marked with an asterisk (*) in Language standards 1–3, are particularly likely to require continued attention in higher grades as they are applied to increasingly sophisticated writing and speaking.

Standard	Grade(s)							
	3	4	5	6	7	8	9–10	11–12
L.3.1f. Ensure subject-verb and pronoun-antecedent agreement.								
L.3.3a. Choose words and phrases for effect.								
L.4.1f. Produce complete sentences, recognizing and correcting inappropriate fragments and run-ons.								
L.4.1g. Correctly use frequently confused words (e.g., *to/too/two; there/their*).								
L.4.3a. Choose words and phrases to convey ideas precisely.*								
L.4.3b. Choose punctuation for effect.								
L.5.1d. Recognize and correct inappropriate shifts in verb tense.								
L.5.2a. Use punctuation to separate items in a series.†								
L.6.1c. Recognize and correct inappropriate shifts in pronoun number and person.								
L.6.1d. Recognize and correct vague pronouns (i.e., ones with unclear or ambiguous antecedents).								
L.6.1e. Recognize variations from standard English in their own and others' writing and speaking, and identify and use strategies to improve expression in conventional language.								
L.6.2a. Use punctuation (commas, parentheses, dashes) to set off nonrestrictive/parenthetical elements.								
L.6.3a. Vary sentence patterns for meaning, reader/listener interest, and style.‡								
L.6.3b. Maintain consistency in style and tone.								
L.7.1c. Place phrases and clauses within a sentence, recognizing and correcting misplaced and dangling modifiers.								
L.7.3a. Choose language that expresses ideas precisely and concisely, recognizing and eliminating wordiness and redundancy.								
L.8.1d. Recognize and correct inappropriate shifts in verb voice and mood.								
L.9–10.1a. Use parallel structure.								

* Subsumed by L.7.3a
† Subsumed by L.9–10.1a
‡ Subsumed by L.11–12.3a

STANDARDS FOR

Literacy in All Subjects

Appendix A

Research Supporting Key Elements of the Standards
Glossary of Key Terms

COMMON CORE STATE STANDARDS FOR

English Language Arts & Literacy in All Subjects

Appendix A:

Research Supporting
Key Elements of the Standards

Glossary of Key Terms

Reading

One of the key requirements of the Common Core State Standards for Reading is that all students must be able to comprehend texts of steadily increasing complexity as they progress through school. By the time they complete the core, students must be able to read and comprehend independently and proficiently the kinds of complex texts commonly found in college and careers. The first part of this section makes a research-based case for why the complexity of what students read matters. In brief, while reading demands in college, workforce training programs, and life in general have held steady or increased over the last half century, K–12 texts have actually declined in sophistication, and relatively little attention has been paid to students' ability to read complex texts independently. These conditions have left a serious gap between many high school seniors' reading ability and the reading requirements they will face after graduation. The second part of this section addresses how text complexity can be measured and made a regular part of instruction. It introduces a three-part model that blends qualitative and quantitative measures of text complexity with reader and task considerations. The section concludes with three annotated examples showing how the model can be used to assess the complexity of various kinds of texts appropriate for different grade levels.

Why Text Complexity Matters

In 2006, ACT, Inc., released a report called *Reading Between the Lines* that showed which skills differentiated those students who equaled or exceeded the benchmark score (21 out of 36) in the reading section of the ACT college admissions test from those who did not. Prior ACT research had shown that students achieving the benchmark score or better in reading—which only about half (51 percent) of the roughly half million test takers in the 2004–2005 academic year had done—had a high probability (75 percent chance) of earning a C or better in an introductory, credit-bearing course in U.S. history or psychology (two common reading-intensive courses taken by first-year college students) and a 50 percent chance of earning a B or better in such a course.[1]

Surprisingly, what chiefly distinguished the performance of those students who had earned the benchmark score or better from those who had not was not their relative ability in making inferences while reading or answering questions related to particular cognitive processes, such as determining main ideas or determining the meaning of words and phrases in context. Instead, the clearest differentiator was students' ability to answer questions associated with complex texts. Students scoring below benchmark performed no better than chance (25 percent correct) on four-option multiple-choice questions pertaining to passages rated as "complex" on a three-point qualitative rubric described in the report. These findings held for male and female students, students from all racial/ethnic groups, and students from families with widely varying incomes. The most important implication of this study was that a pedagogy focused only on "higher-order" or "critical" thinking was insufficient to ensure that students were ready for college and careers: what students could read, in terms of its complexity, was at least as important as what they could do with what they read.

The ACT report is one part of an extensive body of research attesting to the importance of text complexity in reading achievement. The clear, alarming picture that emerges from the evidence, briefly summarized below[2], is that while the reading demands of college, workforce training programs, and citizenship have held steady or risen over the past fifty years or so, K–12 texts have, if anything, become less demanding. This finding is the impetus behind the Standards' strong emphasis on increasing text complexity as a key requirement in reading.

College, Careers, and Citizenship: Steady or Increasing Complexity of Texts and Tasks

Research indicates that the demands that college, careers, and citizenship place on readers have either held steady or increased over roughly the last fifty years. The difficulty of college textbooks, as measured by Lexile scores, has not decreased in any block of time since 1962; it has, in fact, increased over that period (Stenner, Koons, & Swartz, in press). The word difficulty of every scientific journal and magazine from 1930 to 1990 examined by Hayes and Ward (1992) had actually increased, which is important in part because, as a 2005 College Board study (Milewski, Johnson, Glazer, & Kubota, 2005) found, college professors assign more readings from periodicals than do high school teachers. Workplace reading, measured in Lexiles, exceeds grade 12 complexity significantly, although there is considerable variation (Stenner, Koons, & Swartz, in press). The vocabulary difficulty of newspapers remained stable over the 1963–1991 period Hayes and his colleagues (Hayes, Wolfer, & Wolfe, 1996) studied.

Furthermore, students in college are expected to read complex texts with substantially greater independence (i.e., much less scaffolding) than are students in typical K–12 programs. College students are held more accountable for what they read on their own than are most students in high school (Erickson & Strommer, 1991; Pritchard, Wilson, & Yamnitz, 2007). College instructors assign readings, not necessarily explicated in class, for which students might be held accountable through exams, papers, presentations, or class discussions. Students in high school, by contrast, are

[1]In the 2008–2009 academic year, only 53 percent of students achieved the reading benchmark score or higher; the increase from 2004–2005 was not statistically significant. See ACT, Inc. (2009).
[2]Much of the summary found in the next two sections is heavily influenced by Marilyn Jager Adams's painstaking review of the relevant literature. See Adams (2009).

rarely held accountable for what they are able to read independently (Heller & Greenleaf, 2007). This discrepancy in task demand, coupled with what we see below is a vast gap in text complexity, may help explain why only about half of the students taking the ACT Test in the 2004–2005 academic year could meet the benchmark score in reading (which also was the case in 2008–2009, the most recent year for which data are available) and why so few students in general are prepared for postsecondary reading (ACT, Inc., 2006, 2009).

K–12 Schooling: Declining Complexity of Texts and a Lack of Reading of Complex Texts Independently

Despite steady or growing reading demands from various sources, K–12 reading texts have actually trended downward in difficulty in the last half century. Jeanne Chall and her colleagues (Chall, Conard, & Harris, 1977) found a thirteen-year decrease from 1963 to 1975 in the difficulty of grade 1, grade 6, and (especially) grade 11 texts. Extending the period to 1991, Hayes, Wolfer, and Wolfe (1996) found precipitous declines (relative to the period from 1946 to 1962) in average sentence length and vocabulary level in reading textbooks for a variety of grades. Hayes also found that while science books were more difficult to read than literature books, only books for Advanced Placement (AP) classes had vocabulary levels equivalent to those of even newspapers of the time (Hayes & Ward, 1992). Carrying the research closer to the present day, Gary L. Williamson (2006) found a 350L (Lexile) gap between the difficulty of end-of-high school and college texts—a gap equivalent to 1.5 standard deviations and more than the Lexile difference between grade 4 and grade 8 texts on the National Assessment of Educational Progress (NAEP). Although legitimate questions can be raised about the tools used to measure text complexity (e.g., Mesmer, 2008), what is relevant in these numbers is the general, steady decline—over time, across grades, and substantiated by several sources—in the difficulty and likely also the sophistication of content of the texts students have been asked to read in school since 1962.

There is also evidence that current standards, curriculum, and instructional practice have not done enough to foster the independent reading of complex texts so crucial for college and career readiness, particularly in the case of informational texts. K–12 students are, in general, given considerable scaffolding—assistance from teachers, class discussions, and the texts themselves (in such forms as summaries, glossaries, and other text features)—with reading that is already less complex overall than that typically required of students prior to 1962.[3] What is more, students today are asked to read very little expository text—as little as 7 and 15 percent of elementary and middle school instructional reading, for example, is expository (Hoffman, Sabo, Bliss, & Hoy, 1994; Moss & Newton, 2002; Yopp & Yopp, 2006)—yet much research supports the conclusion that such text is harder for most students to read than is narrative text (Bowen & Roth, 1999; Bowen, Roth, & McGinn, 1999, 2002; Heller & Greenleaf, 2007; Shanahan & Shanahan, 2008), that students need sustained exposure to expository text to develop important reading strategies (Afflerbach, Pearson, & Paris, 2008; Kintsch, 1998, 2009; McNamara, Graesser, & Louwerse, in press; Perfetti, Landi, & Oakhill, 2005; van den Broek, Lorch, Linderholm, & Gustafson, 2001; van den Broek, Risden, & Husebye-Hartmann, 1995), and that expository text makes up the vast majority of the required reading in college and the workplace (Achieve, Inc., 2007). Worse still, what little expository reading students are asked to do is too often of the superficial variety that involves skimming and scanning for particular, discrete pieces of information; such reading is unlikely to prepare students for the cognitive demand of true understanding of complex text.

The Consequences: Too Many Students Reading at Too Low a Level

The impact that low reading achievement has on students' readiness for college, careers, and life in general is significant. To put the matter bluntly, a high school graduate who is a poor reader is a postsecondary student who must struggle mightily to succeed. The National Center for Education Statistics (NCES) (Wirt, Choy, Rooney, Provasnik, Sen, & Tobin, 2004) reports that although needing to take one or more remedial/developmental courses of any sort lowers a student's chance of eventually earning a degree or certificate, "the need for remedial reading appears to be the most serious barrier to degree completion" (p. 63). Only 30 percent of 1992 high school seniors who went on to enroll in postsecondary education between 1992 and 2000 and then took any remedial reading course went on to receive a degree or certificate, compared to 69 percent of the 1992 seniors who took no postsecondary remedial courses and 57 percent of those who took one remedial course in a subject other than reading or mathematics. Considering that 11 percent of those high school seniors required at least one remedial reading course, the societal impact of low reading achievement is as profound as its impact on the aspirations of individual students.

Reading levels among the adult population are also disturbingly low. The 2003 National Assessment of Adult Literacy (Kutner, Greenberg, Jin, Boyle, Hsu, & Dunleavy, 2007) reported that 14 percent of adults read prose texts at "below basic" level, meaning they could exhibit "no more than the most simple and concrete literacy skills"; a similarly small number (13 percent) could read prose texts at the "proficient level," meaning they could perform "more complex and challenging literacy activities" (p. 4). The percent of "proficient" readers had actually declined in a statistically significant way from 1992 (15 percent). This low and declining achievement rate may be connected to a general lack of reading. As reported by the National Endowment for the Arts (2004), the percent of U.S. adults reading literature dropped from 54.0 in 1992 to 46.7 in 2002, while the percent of adults reading *any* book also declined by 7 percent

[3]As also noted in "Key Considerations in Implementing Text Complexity," below, it is important to recognize that scaffolding often is entirely appropriate. The expectation that scaffolding will occur with particularly challenging texts is built into the Standards' grade-by-grade text complexity expectations, for example. The general movement, however, should be toward *decreasing scaffolding* and *increasing independence* both within and across the text complexity bands defined in the Standards.

during the same time period. Although the decline occurred in all demographic groups, the steepest decline by far was among 18-to-24- and 25-to-34-year-olds (28 percent and 23 percent, respectively). In other words, the problem of lack of reading is not only getting worse but doing so at an accelerating rate. Although numerous factors likely contribute to the decline in reading, it is reasonable to conclude from the evidence presented above that the deterioration in overall reading ability, abetted by a decline in K–12 text complexity and a lack of focus on independent reading of complex texts, is a contributing factor.

Being able to read complex text independently and proficiently is essential for high achievement in college and the workplace and important in numerous life tasks. Moreover, current trends suggest that if students cannot read challenging texts with understanding—if they have not developed the skill, concentration, and stamina to read such texts—they will read less in general. In particular, if students cannot read complex expository text to gain information, they will likely turn to text-free or text-light sources, such as video, podcasts, and tweets. These sources, while not without value, cannot capture the nuance, subtlety, depth, or breadth of ideas developed through complex text. As Adams (2009) puts it, “There may one day be modes and methods of information delivery that are as efficient and powerful as text, but for now there is no contest. To grow, our students must read lots, and more specifically they must read lots of ‘complex’ texts—texts that offer them new language, new knowledge, and new modes of thought” (p. 182). A turning away from complex texts is likely to lead to a general impoverishment of knowledge, which, because knowledge is intimately linked with reading comprehension ability, will accelerate the decline in the ability to comprehend complex texts and the decline in the richness of text itself. This bodes ill for the ability of Americans to meet the demands placed upon them by citizenship in a democratic republic and the challenges of a highly competitive global marketplace of goods, services, and ideas.

It should be noted also that the problems with reading achievement are not “equal opportunity” in their effects: students arriving at school from less-educated families are disproportionately represented in many of these statistics (Bettinger & Long, 2009). The consequences of insufficiently high text demands and a lack of accountability for independent reading of complex texts in K–12 schooling are severe for everyone, but they are disproportionately so for those who are already most isolated from text before arriving at the schoolhouse door.

The Standards’ Approach to Text Complexity

To help redress the situation described above, the Standards define a three-part model for determining how easy or difficult a particular text is to read as well as grade-by-grade specifications for increasing text complexity in successive years of schooling (Reading standard 10). These are to be used together with grade-specific standards that require increasing sophistication in students’ reading comprehension ability (Reading standards 1–9). The Standards thus approach the intertwined issues of what and how student read.

A Three-Part Model for Measuring Text Complexity

As signaled by the graphic at right, the Standards’ model of text complexity consists of three equally important parts.

(1) Qualitative dimensions of text complexity. In the Standards, *qualitative dimensions* and *qualitative factors* refer to those aspects of text complexity best measured or only measurable by an attentive human reader, such as levels of meaning or purpose; structure; language conventionality and clarity; and knowledge demands.

(2) Quantitative dimensions of text complexity. The terms *quantitative dimensions* and *quantitative factors* refer to those aspects of text complexity, such as word length or frequency, sentence length, and text cohesion, that are difficult if not impossible for a human reader to evaluate efficiently, especially in long texts, and are thus today typically measured by computer software.

(3) Reader and task considerations. While the prior two elements of the model focus on the inherent complexity of text, variables specific to particular readers (such as motivation, knowledge, and experiences) and to particular tasks (such as purpose and the complexity of the task assigned and the questions posed) must also be considered when determining whether a text is appropriate for a given student. Such assessments are best made by teachers employing their professional judgment, experience, and knowledge of their students and the subject.

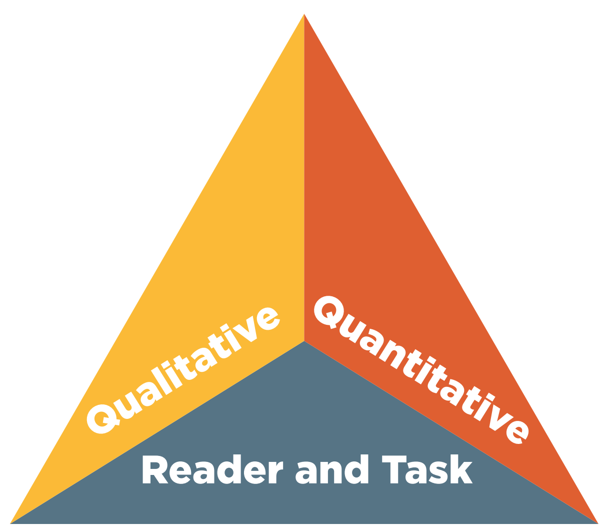


Figure 1: The Standards’ Model of Text Complexity

The Standards presume that all three elements will come into play when text complexity and appropriateness are determined. The following pages begin with a brief overview of just some of the currently available tools, both qualitative and quantitative, for measuring text complexity, continue with some important considerations for using text complexity with students, and conclude with a series of examples showing how text complexity measures, balanced with reader and task considerations, might be used with a number of different texts.

Qualitative and Quantitative Measures of Text Complexity

The qualitative and quantitative measures of text complexity described below are representative of the best tools presently available. However, each should be considered only provisional; more precise, more accurate, and easier-to-use tools are urgently needed to help make text complexity a vital, everyday part of classroom instruction and curriculum planning.

Qualitative Measures of Text Complexity

Using qualitative measures of text complexity involves making an informed decision about the difficulty of a text in terms of one or more factors discernible to a human reader applying trained judgment to the task. In the Standards, qualitative measures, along with professional judgment in matching a text to reader and task, serve as a necessary complement and sometimes as a corrective to quantitative measures, which, as discussed below, cannot (at least at present) capture all of the elements that make a text easy or challenging to read and are not equally successful in rating the complexity of all categories of text.

Built on prior research, the four qualitative factors described below are offered here as a first step in the development of robust tools for the qualitative analysis of text complexity. These factors are presented as continua of difficulty rather than as a succession of discrete “stages” in text complexity. Additional development and validation would be needed to translate these or other dimensions into, for example, grade-level- or grade-band-specific rubrics. The qualitative factors run from easy (left-hand side) to difficult (right-hand side). Few, if any, authentic texts will be low or high on all of these measures, and some elements of the dimensions are better suited to literary or to informational texts.

(1) ***Levels of Meaning (literary texts) or Purpose (informational texts).*** Literary texts with a single level of meaning tend to be easier to read than literary texts with multiple levels of meaning (such as satires, in which the author’s literal message is intentionally at odds with his or her underlying message). Similarily, informational texts with an explicitly stated purpose are generally easier to comprehend than informational texts with an implicit, hidden, or obscure purpose.

(2) ***Structure.*** Texts of low complexity tend to have simple, well-marked, and conventional structures, whereas texts of high complexity tend to have complex, implicit, and (particularly in literary texts) unconventional structures. Simple literary texts tend to relate events in chronological order, while complex literary texts make more frequent use of flashbacks, flash-forwards, and other manipulations of time and sequence. Simple informational texts are likely not to deviate from the conventions of common genres and subgenres, while complex informational texts are more likely to conform to the norms and conventions of a specific discipline. Graphics tend to be simple and either unnecessary or merely supplementary to the meaning of texts of low complexity, whereas texts of high complexity tend to have similarly complex graphics, graphics whose interpretation is essential to understanding the text, and graphics that provide an independent source of information within a text. (Note that many books for the youngest students rely heavily on graphics to convey meaning and are an exception to the above generalization.)

(3) ***Language Conventionality and Clarity.*** Texts that rely on literal, clear, contemporary, and conversational language tend to be easier to read than texts that rely on figurative, ironic, ambiguous, purposefully misleading, archaic or otherwise unfamiliar language or on general academic and domain-specific vocabulary.

(4) ***Knowledge Demands.*** Texts that make few assumptions about the extent of readers’ life experiences and the depth of their cultural/literary and content/discipline knowledge are generally less complex than are texts that make many assumptions in one or more of those areas.

Figure 2: Qualitative Dimensions of Text Complexity

Levels of Meaning (literary texts) or Purpose (informational texts)

- Single level of meaning → Multiple levels of meaning
- Explicitly stated purpose → Implicit purpose, may be hidden or obscure

Structure

- Simple → Complex
- Explicit → Implicit
- Conventional → Unconventional (chiefly literary texts)
- Events related in chronological order → Events related out of chronological order (chiefly literary texts)
- Traits of a common genre or subgenre → Traits specific to a particular discipline (chiefly informational texts)
- Simple graphics → Sophisticated graphics
- Graphics unnecessary or merely supplementary to understanding the text → Graphics essential to understanding the text and may provide information not otherwise conveyed in the text

Language Conventionality and Clarity

- Literal → Figurative or ironic
- Clear → Ambiguous or purposefully misleading
- Contemporary, familiar → Archaic or otherwise unfamiliar
- Conversational → General academic and domain-specific

Knowledge Demands: Life Experiences (literary texts)

- Simple theme → Complex or sophisticated themes
- Single themes → Multiple themes
- Common, everyday experiences or clearly fantastical situations → Experiences distinctly different from one's own
- Single perspective → Multiple perspectives
- Perspective(s) like one's own → Perspective(s) unlike or in opposition to one's own

Knowledge Demands: Cultural/Literary Knowledge (chiefly literary texts)

- Everyday knowledge and familiarity with genre conventions required → Cultural and literary knowledge useful
- Low intertextuality (few if any references/allusions to other texts) → High intertextuality (many references/allusions to other texts)

Knowledge Demands: Content/Discipline Knowledge (chiefly informational texts)

- Everyday knowledge and familiarity with genre conventions required → Extensive, perhaps specialized discipline-specific content knowledge required
- Low intertextuality (few if any references to/citations of other texts) → High intertextuality (many references to/citations of other texts)

Adapted from ACT, Inc. (2006). *Reading between the lines: What the ACT reveals about college readiness in reading.* Iowa City, IA: Author; Carnegi Council on Advancing Adolescent Literacy. (2010). *Time to act: An agenda for advancing adolescent literacy for college and career success.* New York: Carnegie Corporation of New York; Chall, J. S., Bissex, G. L., Conrad, S. S., & Harris-Sharples, S. (1996). *Qualitative assessment of text difficulty: A practical guide for teachers and writers.* Cambridge, UK: Brookline Books; Hess, K., & Biggam, S. (2004). A discussion of "increasing text complexity." Published by the New Hampshire, Rhode Island, and Vermont departments of education as part of the New England Common Assessment Program (NECAP). Retrieved from www.nciea.org/publications/TextComplexity_KH05.pdf

Quantitative Measures of Text Complexity

A number of quantitative tools exist to help educators assess aspects of text complexity that are better measured by algorithm than by a human reader. The discussion is not exhaustive, nor is it intended as an endorsement of one method or program over another. Indeed, because of the limits of each of the tools, new or improved ones are needed quickly if text complexity is to be used effectively in the classroom and curriculum.

Numerous formulas exist for measuring the readability of various types of texts. Such formulas, including the widely used Flesch-Kincaid Grade Level test, typically use word length and sentence length as proxies for semantic and syntactic complexity, respectively (roughly, the complexity of the meaning and sentence structure). The assumption behind these formulas is that longer words and longer sentences are more difficult to read than shorter ones; a text with many long words and/or sentences is thus rated by these formulas as harder to read than a text with many short words and/or sentences would be. Some formulas, such as the Dale-Chall Readability Formula, substitute word frequency for word length as a factor, the assumption here being that less familiar words are harder to comprehend than familiar words. The higher the proportion of less familiar words in a text, the theory goes, the harder that text is to read. While these readability formulas are easy to use and readily available—some are even built into various word processing applications—their chief weakness is that longer words, less familiar words, and longer sentences are not inherently hard to read. In fact, series of short, choppy sentences can pose problems for readers precisely because these sentences lack the cohesive devices, such as transition words and phrases, that help establish logical links among ideas and thereby reduce the inference load on readers.

Like Dale-Chall, the Lexile Framework for Reading, developed by MetaMetrics, Inc., uses word frequency and sentence length to produce a single measure, called a Lexile, of a text's complexity. The most important difference between the Lexile system and traditional readability formulas is that traditional formulas only assign a score to texts, whereas the Lexile Framework can place both readers and texts on the same scale. Certain reading assessments yield Lexile scores based on student performance on the instrument; some reading programs then use these scores to assign texts to students. Because it too relies on word familiarity and sentence length as proxies for semantic and syntactic complexity, the Lexile Framework, like traditional formulas, may underestimate the difficulty of texts that use simple, familiar language to convey sophisticated ideas, as is true of much high-quality fiction written for adults and appropriate for older students. For this reason and others, it is possible that factors other than word familiarity and sentence length contribute to text difficulty. In response to such concerns, MetaMetrics has indicated that it will release the qualitative ratings it assigns to some of the texts it rates and will actively seek to determine whether one or more additional factors can and should be added to its quantitative measure. Other readability formulas also exist, such as the ATOS formula associated with the Accelerated Reader program developed by Renaissance Learning. ATOS uses word difficulty (estimated grade level), word length, sentence length, and text length (measured in words) as its factors. Like the Lexile Framework, ATOS puts students and texts on the same scale.

A nonprofit service operated at the University of Memphis, Coh-Metrix attempts to account for factors in addition to those measured by readability formulas. The Coh-Metrix system focuses on the cohesiveness of a text—basically, how tightly the text holds together. A high-cohesion text does a good deal of the work for the reader by signaling relationships among words, sentences, and ideas using repetition, concrete language, and the like; a low-cohesion text, by contrast, requires the reader him- or herself to make many of the connections needed to comprehend the text. High-cohesion texts are not necessarily "better" than low-cohesion texts, but they are easier to read.

The standard Coh-Metrix report includes information on more than sixty indices related to text cohesion, so it can be daunting to the layperson or even to a professional educator unfamiliar with the indices. Coh-Metrix staff have worked to isolate the most revealing, informative factors from among the many they consider, but these "key factors" are not yet widely available to the public, nor have the results they yield been calibrated to the Standards' text complexity grade bands. The greatest value of these factors may well be the promise they offer of more advanced and usable tools yet to come.

Reader and Task Considerations

The use of qualitative and quantitative measures to assess text complexity is balanced in the Standards' model by the expectation that educators will employ professional judgment to match texts to particular students and tasks. Numerous considerations go into such matching. For example, harder texts may be appropriate for highly knowledgeable or skilled readers, and easier texts may be suitable as an expedient for building struggling readers' knowledge or reading skill up to the level required by the Standards. Highly motivated readers are often willing to put in the extra effort required to read harder texts that tell a story or contain information in which they are deeply interested. Complex tasks may require the kind of information contained only in similarly complex texts.

Numerous factors associated with the individual reader are relevant when determining whether a given text is appropriate for him or her. The RAND Reading Study Group identified many such factors in the 2002 report *Reading for Understanding*:

> The reader brings to the act of reading his or her cognitive capabilities (attention, memory, critical analytic ability, inferencing, visualization); motivation (a purpose for reading, interest in the content, self-efficacy as a reader); knowledge (vocabulary and topic knowledge, linguistic and discourse knowledge, knowledge of

comprehension strategies); and experiences.

As part of describing the activity of reading, the RAND group also named important task-related variables, including the reader's purpose (which might shift over the course of reading), "the type of reading being done, such as skimming (getting the gist of the text) or studying (reading the text with the intent of retaining the information for a period of time)," and the intended outcome, which could include "an increase in knowledge, a solution to some real-world problem, and/or engagement with the text."[4]

Key Considerations in Implementing Text Complexity

Texts and Measurement Tools

The tools for measuring text complexity are at once useful and imperfect. Each of the qualitative and quantitative tools described above has its limitations, and none is completely accurate. The development of new and improved text complexity tools should follow the release of the Standards as quickly as possible. In the meantime, the Standards recommend that multiple quantitative measures be used whenever possible and that their results be confirmed or overruled by a qualitative analysis of the text in question.

Certain measures are less valid or inappropriate for certain kinds of texts. Current quantitative measures are suitable for prose and dramatic texts. Until such time as quantitative tools for capturing poetry's difficulty are developed, determining whether a poem is appropriately complex for a given grade or grade band will necessarily be a matter of a qualitative assessment meshed with reader-task considerations. Furthermore, texts for kindergarten and grade 1 may not be appropriate for quantitative analysis, as they often contain difficult-to-assess features designed to aid early readers in acquiring written language. The Standards' poetry and K–1 text exemplars were placed into grade bands by expert teachers drawing on classroom experience.

Many current quantitative measures underestimate the challenge posed by complex narrative fiction. Quantitative measures of text complexity, particularly those that rely exclusively or in large part on word- and sentence-level factors, tend to assign sophisticated works of literature excessively low scores. For example, as illustrated in example 2 below, some widely used quantitative measures, including the Flesch-Kincaid Grade Level test and the Lexile Framework for Reading, rate the Pulitzer Prize–winning novel *Grapes of Wrath* as appropriate for grades 2–3. This counterintuitive result emerges because works such as *Grapes* often express complex ideas in relatively commonplace language (familiar words and simple syntax), especially in the form of dialogue that mimics everyday speech. Until widely available quantitative tools can better account for factors recognized as making such texts challenging, including multiple levels of meaning and mature themes, preference should likely be given to qualitative measures of text complexity when evaluating narrative fiction intended for students in grade 6 and above.

Measures of text complexity must be aligned with college and career readiness expectations for all students. Qualitative scales of text complexity should be anchored at one end by descriptions of texts representative of those required in typical first-year credit-bearing college courses and in workforce training programs. Similarly, quantitative measures should identify the college- and career-ready reading level as one endpoint of the scale. MetaMetrics, for example, has realigned its Lexile ranges to match the Standards' text complexity grade bands and has adjusted upward its trajectory of reading comprehension development through the grades to indicate that all students should be reading at the college and career readiness level by no later than the end of high school.

Figure 3: Text Complexity Grade Bands and Associated Lexile Ranges (in Lexiles)

Text Complexity Grade Band in the Standards	Old Lexile Ranges	Lexile Ranges Aligned to CCR expectations
K–1	N/A	N/A
2–3	450–725	450–790
4–5	645–845	770–980
6–8	860–1010	955–1155
9–10	960–1115	1080–1305
11–CCR	1070–1220	1215–1355

[4]RAND Reading Study Group. (2002). *Reading for understanding: Toward an R&D program in reading comprehension.* Santa Monica, CA: RAND. The quoted text appears in pages xiii–xvi.

Readers and Tasks

Students' ability to read complex text does not always develop in a linear fashion. Although the progression of Reading standard 10 (see below) defines required grade-by-grade growth in students' ability to read complex text, the development of this ability in individual students is unlikely to occur at an unbroken pace. Students need opportunities to stretch their reading abilities but also to experience the satisfaction and pleasure of easy, fluent reading within them, both of which the Standards allow for. As noted above, such factors as students' motivation, knowledge, and experiences must also come into play in text selection. Students deeply interested in a given topic, for example, may engage with texts on that subject across a range of complexity. Particular tasks may also require students to read harder texts than they would normally be required to. Conversely, teachers who have had success using particular texts that are easier than those required for a given grade band should feel free to continue to use them so long as the general movement during a given school year is toward texts of higher levels of complexity.

Students reading well above and well below grade-band level need additional support. Students for whom texts within their text complexity grade band (or even from the next higher band) present insufficient challenge must be given the attention and resources necessary to develop their reading ability at an appropriately advanced pace. On the other hand, students who struggle greatly to read texts within (or even below) their text complexity grade band must be given the support needed to enable them to read at a grade-appropriate level of complexity.

Even many students on course for college and career readiness are likely to need scaffolding as they master higher levels of text complexity. As they enter each new grade band, many students are likely to need at least some extra help as they work to comprehend texts at the high end of the range of difficulty appropriate to the band. For example, many students just entering grade 2 will need some support as they read texts that are advanced for the grades 2–3 text complexity band. Although such support is educationally necessary and desirable, instruction must move generally toward *decreasing scaffolding* and *increasing independence*, with the goal of students reading independently and proficiently within a given grade band by the end of the band's final year (continuing the previous example, the end of grade 3).

The Standards' Grade-Specific Text Complexity Demands

As illustrated in figure 4, text complexity in the Standards is defined in grade bands: grades 2–3, 4–5, 6–8, 9–10, and 11–CCR.[5] Students in the first year(s) of a given band are expected by the end of the year to read and comprehend proficiently within the band, with scaffolding as needed at the high end of the range. Students in the last year of a band are expected by the end of the year to read and comprehend independently and proficiently within the band.

Figure 4: The Progression of Reading Standard 10

Grade(s)	Reading Standard 10 (individual text types omitted)
K	Actively engage in group reading activities with purpose and understanding.
1	With prompting and support, read prose and poetry [informational texts] of appropriate complexity for grade 1.
2	By the end of the year, read and comprehend literature [informational texts] in the grades 2–3 text complexity band proficiently, with scaffolding as needed at the high end of the range.
3	By the end of the year, read and comprehend literature [informational texts] at the high end of the grades 2–3 text complexity band independently and proficiently.
4	By the end of the year, read and comprehend literature [informational texts] in the grades 4–5 text complexity band proficiently, with scaffolding as needed at the high end of the range.
5	By the end of the year, read and comprehend literature [informational texts] at the high end of the grades 4–5 text complexity band independently and proficiently.
6	By the end of the year, read and comprehend literature [informational texts, history/social studies texts, science/technical texts] in the grades 6–8 text complexity band proficiently, with scaffolding as needed at the high end of the range.
7	By the end of the year, read and comprehend literature [informational texts, history/social studies texts, science/technical texts] in the grades 6–8 text complexity band proficiently, with scaffolding as needed at the high end of the range.
8	By the end of the year, read and comprehend literature [informational texts, history/social studies texts, science/technical texts] at the high end of the grades 6–8 text complexity band independently and proficiently.
9–10	By the end of grade 9, read and comprehend literature [informational texts, history/social studies texts, science/technical texts] in the grades 9–10 text complexity band proficiently, with scaffolding as needed at the high end of the range. By the end of grade 10, read and comprehend literature [informational texts, history/social studies texts, science/technical texts] at the high end of the grades 9–10 text complexity band independently and proficiently.
11–12	By the end of grade 11, read and comprehend literature [informational texts, history/social studies texts, science/technical texts] in the grades 11–CCR text complexity band proficiently, with scaffolding as needed at the high end of the range. By the end of grade 12, read and comprehend literature [informational texts, history/social studies texts, science/technical texts] at the high end of the grades 11–CCR text complexity band independently and proficiently.

[5]As noted above in "Key Considerations in Implementing Text Complexity," K–1 texts are not amenable to quantitative measure. Furthermore, students in those grades are acquiring the code at varied rates. Hence, the Standards' text complexity requirements begin formally with grade 2.

Writing

Definitions of the Standards' Three Text Types

Argument

Arguments are used for many purposes—to change the reader's point of view, to bring about some action on the reader's part, or to ask the reader to accept the writer's explanation or evaluation of a concept, issue, or problem. An argument is a reasoned, logical way of demonstrating that the writer's position, belief, or conclusion is valid. In English language arts, students make claims about the worth or meaning of a literary work or works. They defend their interpretations or judgments with evidence from the text(s) they are writing about. In history/social studies, students analyze evidence from multiple primary and secondary sources to advance a claim that is best supported by the evidence, and they argue for a historically or empirically situated interpretation. In science, students make claims in the form of statements or conclusions that answer questions or address problems. Using data in a scientifically acceptable form, students marshal evidence and draw on their understanding of scientific concepts to argue in support of their claims. Although young children are not able to produce fully developed logical arguments, they develop a variety of methods to extend and elaborate their work by providing examples, offering reasons for their assertions, and explaining cause and effect. These kinds of expository structures are steps on the road to argument. In grades K–5, the term "opinion" is used to refer to this developing form of argument.

Informational/Explanatory Writing

Informational/explanatory writing conveys information accurately. This kind of writing serves one or more closely related purposes: to increase readers' knowledge of a subject, to help readers better understand a procedure or process, or to provide readers with an enhanced comprehension of a concept. Informational/explanatory writing addresses matters such as types (*What are the different types of poetry?*) and components (*What are the parts of a motor?*); size, function, or behavior (*How big is the United States? What is an X-ray used for? How do penguins find food?*); how things work (*How does the legislative branch of government function?*); and why things happen (*Why do some authors blend genres?*). To produce this kind of writing, students draw from what they already know and from primary and secondary sources. With practice, students become better able to develop a controlling idea and a coherent focus on a topic and more skilled at selecting and incorporating relevant examples, facts, and details into their writing. They are also able to use a variety of techniques to convey information, such as naming, defining, describing, or differentiating different types or parts; comparing or contrasting ideas or concepts; and citing an anecdote or a scenario to illustrate a point. Informational/explanatory writing includes a wide array of genres, including academic genres such as literary analyses, scientific and historical reports, summaries, and précis writing as well as forms of workplace and functional writing such as instructions, manuals, memos, reports, applications, and résumés. As students advance through the grades, they expand their repertoire of informational/explanatory genres and use them effectively in a variety of disciplines and domains.

Although information is provided in both arguments and explanations, the two types of writing have different aims. Arguments seek to make people believe that something is true or to persuade people to change their beliefs or behavior. Explanations, on the other hand, start with the assumption of truthfulness and answer questions about why or how. Their aim is to make the reader understand rather than to persuade him or her to accept a certain point of view. In short, arguments are used for persuasion and explanations for clarification.

Like arguments, explanations provide information about causes, contexts, and consequences of processes, phenomena, states of affairs, objects, terminology, and so on. However, in an argument, the writer not only gives information but also presents a case with the "pros" (supporting ideas) and "cons" (opposing ideas) on a debatable issue. Because an argument deals with whether the main claim is true, it demands empirical descriptive evidence, statistics, or definitions for support. When writing an argument, the writer supports his or her claim(s) with sound reasoning and relevant and sufficient evidence.

Narrative Writing

Narrative writing conveys experience, either real or imaginary, and uses time as its deep structure. It can be used for many purposes, such as to inform, instruct, persuade, or entertain. In English language arts, students produce narratives that take the form of creative fictional stories, memoirs, anecdotes, and autobiographies. Over time, they learn to provide visual details of scenes, objects, or people; to depict specific actions (for example, movements, gestures,

Creative Writing beyond Narrative

The narrative category does not include all of the possible forms of creative writing, such as many types of poetry. The Standards leave the inclusion and evaluation of other such forms to teacher discretion.

postures, and expressions); to use dialogue and interior monologue that provide insight into the narrator's and characters' personalities and motives; and to manipulate pace to highlight the significance of events and create tension and suspense. In history/social studies, students write narrative accounts about individuals. They also construct event models of what happened, selecting from their sources only the most relevant information. In science, students write narrative descriptions of the step-by-step procedures they follow in their investigations so that others can replicate their procedures and (perhaps) reach the same results. With practice, students expand their repertoire and control of different narrative strategies.

Texts that Blend Types

Skilled writers many times use a blend of these three text types to accomplish their purposes. For example, *The Longitude Prize*, included above and in Appendix B, embeds narrative elements within a largely expository structure. Effective student writing can also cross the boundaries of type, as does the grade 12 student sample "Fact vs. Fiction and All the Grey Space In Between" found in Appendix C.

The Special Place of Argument in the Standards

While all three text types are important, the Standards put particular emphasis on students' ability to write sound arguments on substantive topics and issues, as this ability is critical to college and career readiness. English and education professor Gerald Graff (2003) writes that "argument literacy" is fundamental to being educated. The university is largely an "argument culture," Graff contends; therefore, K–12 schools should "teach the conflicts" so that students are adept at understanding and engaging in argument (both oral and written) when they enter college. He claims that because argument is not standard in most school curricula, only 20 percent of those who enter college are prepared in this respect. Theorist and critic Neil Postman (1997) calls argument the soul of an education because argument forces a writer to evaluate the strengths and weaknesses of multiple perspectives. When teachers ask students to consider two or more perspectives on a topic or issue, something far beyond surface knowledge is required: students must think critically and deeply, assess the validity of their own thinking, and anticipate counterclaims in opposition to their own assertions.

"Argument" and "Persuasion"

When writing to persuade, writers employ a variety of persuasive strategies. One common strategy is an appeal to the credibility, character, or authority of the writer (or speaker). When writers establish that they are knowledgeable and trustworthy, audiences are more likely to believe what they say. Another is an appeal to the audience's self-interest, sense of identity, or emotions, any of which can sway an audience. A logical argument, on the other hand, convinces the audience because of the perceived merit and reasonableness of the claims and proofs offered rather than either the emotions the writing evokes in the audience or the character or credentials of the writer. The Standards place special emphasis on writing logical arguments as a particularly important form of college- and career-ready writing.

The unique importance of argument in college and careers is asserted eloquently by Joseph M. Williams and Lawrence McEnerney (n.d.) of the University of Chicago Writing Program. As part of their attempt to explain to new college students the major differences between good high school and college writing, Williams and McEnerney define *argument* not as "wrangling" but as "a serious and focused conversation among people who are intensely interested in getting to the bottom of things *cooperatively*":

> Those values are also an integral part of your education in college. For four years, you are asked to read, do research, gather data, analyze it, think about it, and then communicate it to readers in a form . . . which enables them to assess it and use it. You are asked to do this not because we expect you all to become professional scholars, but because in just about any profession you pursue, you will do research, think about what you find, make decisions about complex matters, and then explain those decisions—usually in writing—to others who have a stake in your decisions being sound ones. In an Age of Information, what most professionals do is research, think, and make arguments. (And part of the value of doing your own thinking and writing is that it makes you much better at evaluating the thinking and writing of others.) (ch. 1)

In the process of describing the special value of argument in college- and career-ready writing, Williams and McEnerney also establish argument's close links to research in particular and to knowledge building in general, both of which are also heavily emphasized in the Standards.

Much evidence supports the value of argument generally and its particular importance to college and career readiness. A 2009 ACT national curriculum survey of postsecondary instructors of composition, freshman English, and survey of American literature courses (ACT, Inc., 2009) found that "write to argue or persuade readers" was virtually tied with "write to convey information" as the most important type of writing needed by incoming college students. Other curriculum surveys, including those conducted by the College Board (Milewski, Johnson, Glazer, & Kubota, 2005) and the states of Virginia and Florida[6], also found strong support for writing arguments as a key part of instruction. The 2007 writing framework for the National Assessment of Educational Progress (NAEP) (National Assessment Governing Board, 2006) assigns persuasive writing the single largest targeted allotment of assessment time at grade 12 (40 percent, versus 25 percent for narrative writing and 35 percent for informative writing). (The 2011 prepublication framework [National Assessment Governing Board, 2007] maintains the 40 percent figure for persuasive writing at grade 12, allotting 40 percent to writing to explain and 20 percent to writing to convey experience.) Writing arguments or writing to persuade is also an important element in standards frameworks for numerous high-performing nations.[7]

Specific skills central to writing arguments are also highly valued by postsecondary educators. A 2002 survey of instructors of freshman composition and other introductory courses across the curriculum at California's community colleges, California State University campuses, and University of California campuses (Intersegmental Committee of the Academic Senates of the California Community Colleges, the California State University, and the University of California, 2002) found that among the most important skills expected of incoming students were articulating a clear thesis; identifying, evaluating, and using evidence to support or challenge the thesis; and considering and incorporating counterarguments into their writing. On the 2009 ACT national curriculum survey (ACT, Inc., 2009), postsecondary faculty gave high ratings to such argument-related skills as "develop ideas by using some specific reasons, details, and examples," "take and maintain a position on an issue," and "support claims with multiple and appropriate sources of evidence."

The value of effective argument extends well beyond the classroom or workplace, however. As Richard Fulkerson (1996) puts it in *Teaching the Argument in Writing*, the proper context for thinking about argument is one "in which the goal is not victory but a good decision, one in which all arguers are at risk of needing to alter their views, one in which a participant takes seriously and fairly the views different from his or her own" (pp. 16–17). Such capacities are broadly important for the literate, educated person living in the diverse, information-rich environment of the twenty-first century.

[6]Unpublished data collected by Achieve, Inc.

[7]See, for example, frameworks from Finland, Hong Kong, and Singapore as well as Victoria and New South Wales in Australia.

Speaking and Listening

The Special Role of Speaking and Listening in K–5 Literacy

If literacy levels are to improve, the aims of the English language arts classroom, especially in the earliest grades, must include oral language in a purposeful, systematic way, in part because it helps students master the printed word. Besides having intrinsic value as modes of communication, listening and speaking are necessary prerequisites of reading and writing (Fromkin, Rodman, & Hyams, 2006; Hulit, Howard, & Fahey, 2010; Pence & Justice, 2007; Stuart, Wright, Grigor, & Howey, 2002). The interrelationship between oral and written language is illustrated in the table below, using the distinction linguists make between *receptive language* (language that is heard, processed, and understood by an individual) and *expressive language* (language that is generated and produced by an individual).

Figure 14: Receptive and Expressive Oral and Written Language

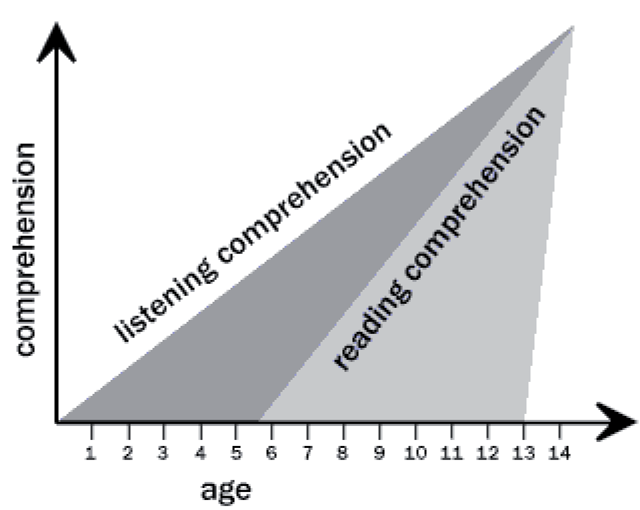
	Receptive Language	Expressive Language
Oral Language	Listening	Speaking
Written Language	Reading (decoding + comprehension)	Writing (handwriting, spelling, written composition)

Oral language development precedes and is the foundation for written language development; in other words, oral language is primary and written language builds on it. Children's oral language competence is strongly predictive of their facility in learning to read and write: listening and speaking vocabulary and even mastery of syntax set boundaries as to what children can read and understand no matter how well they can decode (Catts, Adolf, & Weismer, 2006; Hart & Risley, 1995; Hoover & Gough, 1990: Snow, Burns, & Griffin, 1998).

For children in preschool and the early grades, receptive and expressive abilities do not develop simultaneously or at the same pace: receptive language generally precedes expressive language. Children need to be able to understand words before they can produce and use them.

Oral language is particularly important for the youngest students. Hart and Risley (1995), who studied young children in the context of their early family life and then at school, found that the total number of words children had heard as preschoolers predicted how many words they understood and how fast they could learn new words in kindergarten. Preschoolers who had heard more words had larger vocabularies once in kindergarten. Furthermore, when the students were in grade 3, their early language competence from the preschool years still accurately predicted their language and reading comprehension. The preschoolers who had heard more words, and subsequently had learned more words orally, were better readers. In short, early language advantage persists and manifests itself in higher levels of literacy. A meta-analysis by Sticht and James (1984) indicates that the importance of oral language extends well beyond the earliest grades. As illustrated in the graphic below, Sticht and James found evidence strongly suggesting that children's listening comprehension outpaces reading comprehension until the middle school years (grades 6–8).

Figure 15: Listening and Reading Comprehension, by Age



The research strongly suggests that the English language arts classroom should explicitly address the link between oral and written language, exploiting the influence of oral language on a child's later ability to read by allocating instructional time to building children's listening skills, as called for in the Standards. The early grades should not focus on decoding alone, nor should the later grades pay attention only to building reading comprehension. Time should be devoted to reading fiction and content-rich selections aloud to young children, just as it is to providing those same children with the skills they will need to decode and encode.

This focus on oral language is of greatest importance for the children most at risk—children for whom English is a second language and children who have not been exposed at home to the kind of language found in written texts (Dickinson & Smith, 1994). Ensuring that all children in the United States have access to an excellent education requires that issues of oral language come to the fore in elementary classrooms.

Read-Alouds and the Reading-Speaking-Listening Link

Generally, teachers will encourage children in the upper elementary grades to read texts independently and reflect on them in writing. However, children in the early grades—particularly kindergarten through grade 3—benefit from participating in rich, structured conversations with an adult in response to written texts that are read aloud, orally comparing and contrasting as well as analyzing and synthesizing (Bus, Van Ijzendoorn, & Pellegrini, 1995; Feitelstein, Goldstein, Iraqui, & Share, 1993; Feitelstein, Kita, & Goldstein, 1986; Whitehurst et al., 1988). The Standards acknowledge the importance of this aural dimension of early learning by including a robust set of K–3 Speaking and Listening standards and by offering in Appendix B an extensive number of read-aloud text exemplars appropriate for K–1 and for grades 2–3.

Because, as indicated above, children's listening comprehension likely outpaces reading comprehension until the middle school years, it is particularly important that students in the earliest grades build knowledge through being read to as well as through reading, with the balance gradually shifting to reading independently. By reading a story or nonfiction selection aloud, teachers allow children to experience written language without the burden of decoding, granting them access to content that they may not be able to read and understand by themselves. Children are then free to focus their mental energy on the words and ideas presented in the text, and they will eventually be better prepared to tackle rich written content on their own. Whereas most titles selected for kindergarten and grade 1 will need to be read aloud exclusively, some titles selected for grades 2–5 may be appropriate for read-alouds as well as for reading independently. Reading aloud to students in the upper grades should not, however, be used as a substitute for independent reading by students; read-alouds at this level should supplement and enrich what students are able to read by themselves.

Language

Overview

The Standards take a hybrid approach to matters of conventions, knowledge of language, and vocabulary. As noted in the table below, certain elements important to reading, writing, and speaking and listening are included in those strands to help provide a coherent set of expectations for those modes of communication.

Figure 16: Elements of the Language Standards
in the Reading, Writing, and Speaking and Listening Strands

Strand	Standard
Reading	**R.CCR.4.** Interpret words and phrases as they are used in a text, including determining technical, connotative, and figurative meanings, and analyze how specific word choices shape meaning or tone.
Writing	**W.CCR.5.** Develop and strengthen writing as needed by planning, revising, editing, rewriting, or trying a new approach.
Speaking and Listening	**SL.CCR.6.** Adapt speech to a variety of contexts and communicative tasks, demonstrating command of formal English when indicated or appropriate.

In many respects, however, conventions, knowledge of language, and vocabulary extend across reading, writing, speaking, and listening. Many of the conventions-related standards are as appropriate to formal spoken English as they are to formal written English. Language choice is a matter of craft for both writers and speakers. New words and phrases are acquired not only through reading and being read to but also through direct vocabulary instruction and (particularly in the earliest grades) through purposeful classroom discussions around rich content.

The inclusion of Language standards in their own strand should not be taken as an indication that skills related to conventions, knowledge of language, and vocabulary are unimportant to reading, writing, speaking, and listening; indeed, they are inseparable from such contexts.

Conventions and Knowledge of Language

Teaching and Learning the Conventions of Standard English

Development of Grammatical Knowledge

Grammar and usage development in children and in adults rarely follows a linear path. Native speakers and language learners often begin making new errors and seem to lose their mastery of particular grammatical structures or print conventions as they learn new, more complex grammatical structures or new usages of English, such as in college-level persuasive essays (Bardovi-Harlig, 2000; Bartholomae, 1980; DeVilliers & DeVilliers, 1973; Shaughnessy, 1979). These errors are often signs of language development as learners synthesize new grammatical and usage knowledge with their current knowledge. Thus, students will often need to return to the same grammar topic in greater complexity as they move through K–12 schooling and as they increase the range and complexity of the texts and communicative contexts in which they read and write. The Standards account for the recursive, ongoing nature of grammatical knowledge in two ways. First, the Standards return to certain important language topics in higher grades at greater levels of sophistication. For instance, instruction on verbs in early elementary school (K–3) should address simple present, past, and future tenses; later instruction should extend students' knowledge of verbs to other tenses (progressive and perfect tenses[8] in grades 4 and 5), mood (modal auxiliaries in grade 4 and grammatical mood in grade 8) and voice (active and passive voice in grade 8). Second, the Standards identify with an asterisk (*) certain skills and understandings that students are to be introduced to in basic ways at lower grades but that are likely in need of being

[8]Though progressive and perfect are more correctly *aspects* of verbs rather than *tenses*, the Standards use the more familiar notion here and throughout for the sake of accessibility.

retaught and relearned in subsequent grades as students' writing and speaking matures and grows more complex. (See "Progressive Language Skills in the Standards," below.)

Making Appropriate Grammar and Usage Choices in Writing and Speaking

Students must have a strong command of the grammar and usage of spoken and written standard English to succeed academically and professionally. Yet there is great variety in the language and grammar features of spoken and written standard English (Biber, 1991; Krauthamer, 1999), of academic and everyday standard English, and of the language of different disciplines (Schleppegrell, 2001). Furthermore, in the twenty-first century, students must be able to communicate effectively in a wide range of print and digital texts, each of which may require different grammatical and usage choices to be effective. Thus, grammar and usage instruction should acknowledge the many varieties of English that exist and address differences in grammatical structure and usage between these varieties in order to help students make purposeful language choices in their writing and speaking (Fogel & Ehri, 2000; Wheeler & Swords, 2004). Students must also be taught the *purposes* for using particular grammatical features in particular disciplines or texts; if they are taught simply to vary their grammar and language to keep their writing "interesting," they may actually become more confused about how to make effective language choices (Lefstein, 2009). The Standards encourage this sort of instruction in a number of ways, most directly through a series of grade-specific standards associated with Language CCR standard 3 that, beginning in grade 1, focuses on making students aware of language variety.

Using Knowledge of Grammar and Usage for Reading and Listening Comprehension

Grammatical knowledge can also aid reading comprehension and interpretation (Gargani, 2006; Williams, 2000, 2005). Researchers recommend that students be taught to use knowledge of grammar and usage, as well as knowledge of vocabulary, to comprehend complex academic texts (García & Beltrán, 2003; Short & Fitzsimmons, 2007; RAND Reading Study Group, 2002). At the elementary level, for example, students can use knowledge of verbs to help them understand the plot and characters in a text (Williams, 2005). At the secondary level, learning the grammatical structures of nonstandard dialects can help students understand how accomplished writers such as Harper Lee, Langston Hughes, and Mark Twain use various dialects of English to great advantage and effect, and can help students analyze setting, character, and author's craft in great works of literature. Teaching about the grammatical patterns found in specific disciplines has also been shown to help English language learners' reading comprehension in general and reading comprehension in history classrooms in particular (Achugar, Schleppegrell, & Oteíza, 2007; Gargani, 2006).

As students learn more about the patterns of English grammar in different communicative contexts throughout their K–12 academic careers, they can develop more complex understandings of English grammar and usage. Students can use this understanding to make more purposeful and effective choices in their writing and speaking and more accurate and rich interpretations in their reading and listening.

Progressive Language Skills in the Standards

While all of the Standards are cumulative, certain Language skills and understandings are more likely than others to need to be retaught and relearned as students advance through the grades. Beginning in grade 3, the Standards note such "progressive" skills and understandings with an asterisk (*) in the main document; they are also summarized in the table on pages 29 and 55 of that document as well as on page 34 of this appendix. These skills and understandings should be mastered at a basic level no later than the end of the grade in which they are introduced in the Standards. In subsequent grades, as their writing and speaking become more sophisticated, students will need to learn to apply these skills and understandings in more advanced ways.

The following example shows how one such task—ensuring subject-verb agreement, formally introduced in the Standards in grade 3—can become more challenging as students' writing matures. The sentences in the table below are taken verbatim from the annotated writing samples found in Appendix C. The example is illustrative only of a general development of sophistication and not meant to be exhaustive, to set firm grade-specific expectations, or to establish a precise hierarchy of increasing difficulty in subject-verb agreement.

Figure 17: Example of Subject-Verb Agreement Progression across Grades

Example	Condition
Horses are so beautiful and fun to ride. [Horses, grade 3]	Subject and verb next to each other
When I started out the door, I noticed that Tigger and Max were follow-ing me to school. [Glowing Shoes, grade 4]	Compound subject joined by *and*
A mother or female horse is called a mare. [Horses, grade 3]	Compound subject joined by *or*; each subject takes a singular verb[1]
The first thing to do is research, research, research! [Zoo Field Trip, grade 4]	Intervening phrase between subject and verb
If the watershed for the pools is changed, the condition of the pools changes. [A Geographical Report, grade 7]	Intervening phrase between each subject and verb suggesting a different number for the verb than the subject calls for
Another was the way to the other evil places. [Getting Shot and Living Through It, grade 5] *All his stories are the same type.* [Author Response: Roald Dahl, grade 5] *All the characters that Roald Dahl ever made were probably fake charac-ters.* [Author Response: Roald Dahl, grade 5] *One of the reasons why my cat Gus is the best pet is because he is a cuddle bug.* [A Pet Story About My Cat . . . Gus, grade 6]	Indefinite pronoun as subject, with increasing distance between subject and verb

[1]In this particular example, *or female horse* should have been punctuated by the student as a nonrestrictive appositive, but the sentence as is illustrates the notion of a compound subject joined by or.

Figure 18: Language Progressive Skills, by Grade

The following standards, marked with an asterisk (*) in the main Standards document, are particularly likely to require continued attention in higher grades as they are applied to increasingly sophisticated writing and speaking.

Standard	Grade(s)							
	3	4	5	6	7	8	9–10	11–12
L.3.1f. Ensure subject-verb and pronoun-antecedent agreement.								
L.3.3a. Choose words and phrases for effect.								
L.4.1f. Produce complete sentences, recognizing and correcting inappropriate fragments and run-ons.								
L.4.1g. Correctly use frequently confused words (e.g., *to/too/two; there/their*).								
L.4.3a. Choose words and phrases to convey ideas precisely.*								
L.4.3b. Choose punctuation for effect.								
L.5.1d. Recognize and correct inappropriate shifts in verb tense.								
L.5.2a. Use punctuation to separate items in a series.†								
L.6.1c. Recognize and correct inappropriate shifts in pronoun number and person.								
L.6.1d. Recognize and correct vague pronouns (i.e., ones with unclear or ambiguous antecedents).								
L.6.1e. Recognize variations from standard English in their own and others' writing and speaking, and identify and use strategies to improve expression in conventional language.								
L.6.2a. Use punctuation (commas, parentheses, dashes) to set off nonrestrictive/parenthetical elements.								
L.6.3a. Vary sentence patterns for meaning, reader/listener interest, and style.‡								
L.6.3b. Maintain consistency in style and tone.								
L.7.1c. Place phrases and clauses within a sentence, recognizing and correcting misplaced and dangling modifiers.								
L.7.3a. Choose language that expresses ideas precisely and concisely, recognizing and eliminating wordiness and redundancy.								
L.8.1d. Recognize and correct inappropriate shifts in verb voice and mood.								
L.9–10.1a. Use parallel structure.								

* Subsumed by L.7.3a
† Subsumed by L.9–10.1a
‡ Subsumed by L.11–12.3a

Vocabulary

Acquiring Vocabulary

> Words are not just words. They are the nexus—the interface—between communication and thought. When we read, it is through words that we build, refine, and modify our knowledge. What makes vocabulary valuable and important is not the words themselves so much as the understandings they afford.
>
> Marilyn Jager Adams (2009, p. 180)

The importance of students acquiring a rich and varied vocabulary cannot be overstated. Vocabulary has been empirically connected to reading comprehension since at least 1925 (Whipple, 1925) and had its importance to comprehension confirmed in recent years (National Institute of Child Health and Human Development, 2000). It is widely accepted among researchers that the difference in students' vocabulary levels is a key factor in disparities in academic achievement (Baumann & Kameenui, 1991; Becker, 1977; Stanovich, 1986) but that vocabulary instruction has been neither frequent nor systematic in most schools (Biemiller, 2001; Durkin, 1978; Lesaux, Kieffer, Faller, & Kelley, 2010; Scott & Nagy, 1997).

Research suggests that if students are going to grasp and retain words and comprehend text, they need incremental, repeated exposure in a variety of contexts to the words they are trying to learn. When students make multiple connections between a new word and their own experiences, they develop a nuanced and flexible understanding of the word they are learning. In this way, students learn not only what a word means but also how to use that word in a variety of contexts, and they can apply appropriate senses of the word's meaning in order to understand the word in different contexts (Landauer & Dumais, 1997; Landauer, McNamara, Dennis, & Kintsch, 2007; Nagy, Herman, & Anderson, 1985).

Initially, children readily learn words from oral conversation because such conversations are context rich in ways that aid in vocabulary acquisition: in discussions, a small set of words (accompanied by gesture and intonation) is used with great frequency to talk about a narrow range of situations children are exposed to on a day-to-day basis. Yet as children reach school age, new words are introduced less frequently in conversation, and consequently vocabulary acquisition eventually stagnates by grade 4 or 5 unless students acquire additional words from written context (Hayes & Ahrens, 1988).

Written language contains literally thousands of words more than are typically used in conversational language. Yet writing lacks the interactivity and nonverbal context that make acquiring vocabulary through oral conversation relatively easy, which means that purposeful and ongoing concentration on vocabulary is needed (Hayes & Ahrens, 1988). In fact, at most between 5 and 15 percent of new words encountered upon first reading are retained, and the weaker a student's vocabulary is the smaller the gain (Daneman & Green, 1986; Hayes & Ahrens, 1988; Herman, Anderson, Pearson, & Nagy, 1987; Sternberg & Powell, 1983). Yet research shows that if students are truly to understand what they read, they must grasp upward of 95 percent of the words (Betts, 1946; Carver, 1994; Hu & Nation, 2000; Laufer, 1988).

The challenge in reaching what we might call "lexical dexterity" is that, in any given instance, it is not the entire spectrum of a word's history, meanings, usages, and features that matters but only those aspects that are relevant at that moment. Therefore, for a reader to grasp the meaning of a word, two things must happen: first, the reader's internal representation of the word must be sufficiently complete and well articulated to allow the intended meaning to be known to him or her; second, the reader must understand the context well enough to select the intended meaning from the realm of the word's possible meanings (which in turn depends on understanding the surrounding words of the text).

Key to students' vocabulary development is building rich and flexible word knowledge. Students need plentiful opportunities to use and respond to the words they learn through playful informal talk, discussion, reading or being read to, and responding to what is read. Students benefit from instruction about the connections and patterns in language. Developing in students an analytical attitude toward the logic and sentence structure of their texts, alongside an awareness of word parts, word origins, and word relationships, provides students with a sense of how language works such that syntax, morphology, and etymology can become useful cues in building meaning as students encounter new words and concepts (Beck, McKeown, & Kucan, 2008). Although direct study of language is essential to student progress, most word learning occurs indirectly and unconsciously through normal reading, writing, listening, and speaking (Miller, 1999; Nagy, Anderson, & Herman, 1987).

As students are exposed to and interact with language throughout their school careers, they are able to acquire understandings of word meanings, build awareness of the workings of language, and apply their knowledge to comprehend and produce language.

Three Tiers of Words

Isabel L. Beck, Margaret G. McKeown, and Linda Kucan (2002, 2008) have outlined a useful model for conceptualizing categories of words readers encounter in texts and for understanding the instructional and learning challenges that words in each category present. They describe three levels, or *tiers*, of words in terms of the words' commonality (more to less frequently occurring) and applicability (broader to narrower).

While the term *tier* may connote a hierarchy, a ranking of words from least to most important, the reality is that all three tiers of words are vital to comprehension and vocabulary development, although learning tier two and three words typically requires more deliberate effort (at least for students whose first language is English) than does learning tier one words.

- **Tier One words** are the words of everyday speech usually learned in the early grades, albeit not at the same rate by all children. They are not considered a challenge to the average native speaker, though English language learners of any age will have to attend carefully to them. While Tier One words are important, they are not the focus of this discussion.

- **Tier Two words** (what the Standards refer to as *general academic* words) are far more likely to appear in written texts than in speech. They appear in all sorts of texts: informational texts (words such as *relative*, *vary*, *formulate*, *specificity*, and *accumulate*), technical texts (*calibrate*, *itemize*, *periphery*), and literary texts (*misfortune*, *dignified*, *faltered*, *unabashedly*). Tier Two words often represent subtle or precise ways to say relatively simple things—*saunter* instead of *walk*, for example. Because Tier Two words are found across many types of texts, they are highly generalizable.

- **Tier Three words** (what the Standards refer to as *domain-specific* words) are specific to a domain or field of study (*lava*, *carburetor*, *legislature*, *circumference*, *aorta*) and key to understanding a new concept within a text. Because of their specificity and close ties to content knowledge, Tier Three words are far more common in informational texts than in literature. Recognized as new and "hard" words for most readers (particularly student readers), they are often explicitly defined by the author of a text, repeatedly used, and otherwise heavily scaffolded (e.g., made a part of a glossary).

Tier Two Words and Access to Complex Texts

Because Tier Three words are obviously unfamiliar to most students, contain the ideas necessary to a new topic, and are recognized as both important and specific to the subject area in which they are instructing students, teachers often define Tier Three words prior to students encountering them in a text and then reinforce their acquisition throughout a lesson. Unfortunately, this is not typically the case with Tier Two words, which by definition are not unique to a particular discipline and as a result are not the clear responsibility of a particular content area teacher. What is more, many Tier Two words are far less well defined by contextual clues in the texts in which they appear and are far less likely to be defined explicitly within a text than are Tier Three words. Yet Tier Two words are frequently encountered in complex written texts and are particularly powerful because of their wide applicability to many sorts of reading. Teachers thus need to be alert to the presence of Tier Two words and determine which ones need careful attention.

Tier Three Words and Content Learning

This normal process of word acquisition occurs up to four times faster for Tier Three words when students have become familiar with the domain of the discourse and encounter the word in different contexts (Landauer & Dumais, 1997). Hence, vocabulary development for these words occurs most effectively through a coherent course of study in which subject matters are integrated and coordinated across the curriculum and domains become familiar to the student over several days or weeks.

Examples of Tier Two and Tier Three Words in Context

The following annotated samples call attention to Tier Two and Tier Three words in particular texts and, by singling them out, foreground the importance of these words to the meaning of the texts in which they appear. Both samples appear without annotations in Appendix B.

Example 1: *Volcanoes* (Grades 4–5 Text Complexity Band

Excerpt

> In **early times**, no one knew how **volcanoes formed** or why they **spouted red-hot molten** rock. In **modern times**, scientists began to study **volcanoes**. They still don't know all the answers, but they know much about how a **volcano** works.

Our planet made up of many layers of rock. The top layers of solid rock are called the crust. Deep beneath the crust is the mantle, where it is so hot that some rock melts. The melted, or molten, rock is called magma.

Volcanoes are formed when magma pushes its way up through the crack in Earth's crust. This is called a volcanic eruption. When magma pours forth on the surface, it is called lava.

Simon, Seymour. *Volcanoes.* New York: HarperCollins, 2006. (2006)

Of the Tier Two words, among the most important to the overall meaning of the excerpt is layers. An understanding of the word layers is necessary both to visualize the structure of the crust ("the top layers of solid rock are called the crust") and to grasp the notion of the planet being composed of layers, of which the crust and the mantle are uppermost. Perhaps equally important are the word spouted and the phrase pours forth; an understanding of each of these is needed to visualize the action of a volcano. The same could be said of the word surface. Both layers and surface are likely to reappear in middle and high school academic texts in both literal and figurative contexts ("this would seem plausible on the surface"; "this story has layers of meaning"), which would justify more intensive instruction in them in grades 4–5.

Tier Three words often repeat; in this excerpt, all of the Tier Three words except mantle and lava appear at least twice. Volcano(es) appears four times—five if volcanic is counted. As is also typical with Tier Three words, the text provides the reader with generous support in determining meaning, including explicit definitions (e.g., "the melted, or molten, rock is called magma") and repetition and overlapping sentences (e.g., . . . called the crust. Deep beneath the crust . . .).

Example 2: *Freedom Walkers* (Grades 6–8 Text Complexity Band)

Excerpt

From the Introduction: "Why They Walked"

Not so long ago in Montgomery, Alabama, the color of your skin determined where you could sit on a public bus. If you happened to be an African American, you had to sit in the back of the bus, even if there were empty seats up front.

Back then, racial segregation was the rule throughout the American South. Strict laws—called "Jim Crow" laws—enforced a system of white supremacy that discriminated against blacks and kept them in their place as second-class citizens.

People were separated by race from the moment they were born in segregated hospitals until the day they were buried in segregated cemeteries. Blacks and whites did not attend the same schools, worship in the same churches, eat in the same restaurants, sleep in the same hotels, drink from the same water fountains, or sit together in the same movie theaters.

In Montgomery, it was against the law for a white person and a Negro to play checkers on public property or ride together in a taxi.

Most southern blacks were denied their right to vote. The biggest obstacle was the poll tax, a special tax that was required of all voters but was too costly for many blacks and for poor whites as well. Voters also had to pass a literacy test to prove that they could read, write, and understand the U.S. Constitution. These tests were often rigged to disqualify even highly educated blacks. Those who overcame the obstacles and insisted on registering as voters faced threats, harassment and even physical violence. As a result, African Americans in the South could not express their grievances in the voting booth, which for the most part, was closed to them. But there were other ways to protest, and one day a half century ago, the black citizens in Montgomery rose up in protest and united to demand their rights—by walking peacefully.

It all started on a bus.

Freedman, Russell. *Freedom Walkers: The Story of the Montgomery Bus Boycott.* New York: Holiday House, 2006. (2006)

The first Tier Two word encountered in the excerpt, determined, is essential to understanding the overall meaning of the text. The power of determined here lies in the notion that skin color in Montgomery, Alabama, at that time was the causal agent for all that follows. The centrality of determined to the topic merits the word intensive attention. Its study is further merited by the fact that it has multiple meanings, is likely to appear in future literary and informational texts, and is part of a family of related words (*determine*, *determination*, *determined*, *terminate*, *terminal*).

Understanding the excerpt's Tier Three words is also necessary to comprehend the text fully. As was the case in example 1, these words are often repeated and defined in context. **Segregation**, for example, is introduced in the second paragraph, and while determining its meaning from the sentence in which it appears might be difficult, several closely related concepts (**white supremacy**, **discriminated**, **second-class**) appears in the next sentence to provide more context.

Glossary of Key Terms

Every effort has been made to ensure that the phrasing of the Standards is as clear and free of jargon as possible. When used, specialized and discipline-specific terms (e.g., *simile*, *stanza*, *declarative sentence*) typically conform to their standard definition, and readers are advised to consult high-quality dictionaries or standard resources in the field for clarification. The terms defined below are limited to those words and phrases particularly important to the Standards and that have a meaning unique to this document. *CCSS* refers to the main Common Core State Standards document; the names of various sections (e.g., "Reading") refer to parts of this appendix.

Definitions of many important terms associated with reading foundational skills appear in Reading Foundational Skills, pages 17–22. Descriptions of the Standards' three writing types (argument, informative/explanatory writing, and narrative) can be found in Writing, pages 23–24.

Domain-specific words and phrases – Vocabulary specific to a particular field of study (domain), such as the human body (CCSS, p. 33); in the Standards, *domain-specific words and phrases* are analogous to Tier Three words (Language, p. 33).

Editing – A part of writing and preparing presentations concerned chiefly with improving the clarity, organization, concision, and correctness of expression relative to task, purpose, and audience; compared to *revising*, a smaller-scale activity often associated with surface aspects of a text; see also *revising*, *rewriting*

Emergent reader texts – Texts consisting of short sentences comprised of learned sight words and CVC words; may also include rebuses to represent words that cannot yet be decoded or recognized; see also *rebus*

Evidence – Facts, figures, details, quotations, or other sources of data and information that provide support for claims or an analysis and that can be evaluated by others; should appear in a form and be derived from a source widely accepted as appropriate to a particular discipline, as in details or quotations from a text in the study of literature and experimental results in the study of science

Focused question – A query narrowly tailored to task, purpose, and audience, as in a research query that is sufficiently precise to allow a student to achieve adequate specificity and depth within the time and format constraints

Formal English – See *standard English*

General academic words and phrases – Vocabulary common to written texts but not commonly a part of speech; in the Standards, *general academic words and phrases* are analogous to Tier Two words and phrases (Language, p. 33)

Independent(ly) – A student performance done without *scaffolding* from a teacher, other adult, or peer; in the Standards, often paired with *proficient(ly)* to suggest a successful student performance done without *scaffolding*; in the Reading standards, the act of reading a text without scaffolding, as in an assessment; see also *proficient(ly)*, *scaffolding*

More sustained research project – An investigation intended to address a relatively expansive query using several sources over an extended period of time, as in a few weeks of instructional time

Point of view – Chiefly in literary texts, the narrative point of view (as in first- or third-person narration); more broadly, the position or perspective conveyed or represented by an author, narrator, speaker, or character

Print or digital (texts, sources) – Sometimes added for emphasis to stress that a given standard is particularly likely to be applied to electronic as well as traditional texts; the Standards are generally assumed to apply to both

Proficient(ly) – A student performance that meets the criterion established in the Standards as measured by a teacher or assessment; in the Standards, often paired with *independent(ly)* to suggest a successful student performance done without *scaffolding*; in the Reading standards, the act of reading a text with comprehension; see also *independent(ly)*, *scaffolding*

Rebus – A mode of expressing words and phrases by using pictures of objects whose names resemble those words

Revising – A part of writing and preparing presentations concerned chiefly with a reconsideration and reworking of the content of a text relative to task, purpose, and audience; compared to *editing*, a larger-scale activity often associated with the overall content and structure of a text; see also *editing*, *rewriting*

Rewriting – A part of writing and preparing presentations that involves largely or wholly replacing a previous, unsatisfactory effort with a new effort, better aligned to task, purpose, and audience, on the same or a similar topic or theme; compared to *revising*, a larger-scale activity more akin to replacement than refinement; see also *editing*, *revising*

SECTION 6

Wisconsin Research and Resources

Guiding Principles for Teaching and Learning:

Research, Probing Questions, Resources, and References

1. Every student has the right to learn.

It is our collective responsibility as an education community to make certain each child receives a high-quality, challenging education designed to maximize potential; an education that reflects and stretches his or her abilities and interests. This belief in the right of every child to learn forms the basis of equitable teaching and learning. The five principles that follow cannot exist without this commitment guiding our work.

2. Instruction must be rigorous and relevant.

To understand the world in which we live, there are certain things we all must learn. Each school subject is made up of a core of essential knowledge that is deep, rich, and vital. Every student, regardless of age or ability, must be taught this essential knowledge. What students learn is fundamentally connected to how they learn, and successful instruction blends the content of a discipline with processes of an engaging learning environment that changes to meet the dynamic needs of all students.

3. Purposeful assessment drives instruction and affects learning.

Assessment is an integral part of teaching and learning. Purposeful assessment practices help teachers and students understand where they have been, where they are, and where they might go next. No one assessment can provide sufficient information to plan teaching and learning. Using different types of assessments as part of instruction results in useful information about student understanding and progress. Educators should use this information to guide their own practice and in partnership with students and their families to reflect on learning and set future goals.

4. Learning is a collaborative responsibility.

Teaching and learning are both collaborative processes. Collaboration benefits teaching and learning when it occurs on several levels: when students, teachers, family members, and the community collectively prioritize education and engage in activities that support local schools, educators, and students; when educators collaborate with their colleagues to support innovative classroom practices and set high expectations for themselves and their students; and when students are given opportunities to work together toward academic goals in ways that enhance learning.

5. Students bring strengths and experiences to learning.

Every student learns. Although no two students come to school with the same culture, learning strengths, background knowledge, or experiences, and no two students learn in exactly the same way, every student's unique personal history enriches classrooms, schools, and the community. This diversity is our greatest education asset.

6. Responsive environments engage learners.

Meaningful learning happens in environments where creativity, awareness, inquiry, and critical thinking are part of instruction. Responsive learning environments adapt to the individual needs of each student and encourage learning by promoting collaboration rather than isolation of learners. Learning environments, whether classrooms, schools, or other systems, should be structured to promote engaged teaching and learning.

Guiding Principle 1:
Every student has the right to learn.

It is our collective responsibility as an education community to make certain each child receives a high-quality, challenging education designed to maximize potential, an education that reflects and stretches his or her abilities and interests. This belief in the right of every child to learn forms the basis of equitable teaching and learning. The five principles that follow cannot exist without this commitment guiding our work.

Every student's right to learn provides the overarching vision for Wisconsin's Guiding Principles for education. To be successful, education must be committed to serving the learning needs of students from various social, economic, cultural, linguistic, and developmental backgrounds. For all students to have a guaranteed right to learn, schooling must be equitable.

Research Summary

Focusing on Equity

The belief that each student has the right to learn despite differences in educational needs and backgrounds has important implications for ensuring an equitable education for all students. In the education research literature, the term *educational equality* refers to the notion that all students should have access to an education of similar quality—the proxy for which is frequently educational *inputs* such as funding, facilities, resources, and quality teaching and learning. In contrast, the term *educational equity* connotes the requirement that all students receive an education that allows them to achieve at a standard level or attain standard educational *outcomes* (Brighouse & Swift, 2008). Importantly, equality in terms of educational resources or inputs may not guarantee equity in educational outcomes because not all students reach the same level of achievement with the same access to resources (Brighouse & Swift, 2008). To serve students of varying economic, social, developmental, or linguistic backgrounds, achieving equity in education may require more resources to meet the greater educational needs of certain students (Berne & Stiefel, 1994).

The research literature offers several components that provide a framework for understanding what an equitable education for all students looks like at the classroom level. These components include a call for all students to be provided with the following:

- Access to resources and facilities
- Instruction in all areas tailored to their needs
- Curriculum that is rigorous and relevant
- Educators who are culturally sensitive and respectful
- Interactions with staff and other students that are positive and encouraging in an atmosphere of learning
- Assessment that is varied to give each student the opportunity to demonstrate learning (Education Northwest, 2011)

Access

Access to resources and facilities largely refers to various legal mandates that all children have the right to attend school and participate in all school activities. Since the landmark ruling *Brown v. Board of Education of Topeka* (1954), court decisions and federal regulations have mandated equality of access to all educational opportunities for students regardless of race, ethnicity, or gender (Civil Rights Act, 1964), disability (Education for All Handicapped Children Act, 1975), or language (*Lau v. Nichols, 1974*). Equity in the provision of educational resources and funding was improved with the passage of Title I of the Elementary and Secondary Education Act (ESEA; 1965), which provided additional resources for economically disadvantaged students to meet their learning needs. Since Title I, research on equity in education has grown, and with the reauthorization of ESEA in the No Child Left Behind Act in 2001, equity in educational outcomes for all students was emphasized in the law. Access to an equitable education is a legal right for all children, and the quality of that access in classroom instruction is a moral and ethical right.

Instruction

Instruction that is tailored to meet all students' needs goes beyond simply providing equal access to education. High-quality instruction has increasingly been defined in the literature as a key factor in student achievement. High-quality instruction includes differentiated instructional strategies, teaching to students' learning styles, and provision of instructional support for students who are educationally, socially, or linguistically challenged. Differentiated instruction involves utilizing unique instructional strategies for meeting individual student needs as well as modifying curriculum for both high- and low-performing students. Assessing and teaching to student learning styles is one form of differentiation. Research has shown the value of adapting instructional strategies to different student learning styles (Gardner, 1999) and supports the practice of classroom differentiation (Mulroy & Eddinger, 2003; Tomlinson, 2005).

Curriculum

Designing curriculum that is rigorous and relevant provides an important foundation for a high-quality learning environment by helping make standards-based content accessible to all students. A relevant, rigorous curriculum has been found to be important for all students. Although advanced and rigorous curriculum is generally viewed to be an important factor of academic success for high-achieving students, research also indicates that using challenging, interesting, and varied curriculum for students of all achievement levels improves student achievement (Daggett, 2005). Rigorous curriculum can be adapted for low-performing students in a way that challenges them and helps them meet learning standards. For example, the universal design for learning (UDL) offers strategies for making the general curriculum accessible to special education students (Rose, Hasselbring, Stahl, & Zabala, 2009). Similarly, research on lesson scaffolding emphasizes strategies for providing a rigorous content curriculum to student who are culturally or linguistically diverse or who need additional context to understand certain concepts (Gibbons, 2002).

Climate

Interactions with staff and students that are positive and focused on learning are part of an emotionally safe school climate, but the literature also supports the need for a climate of high academic expectations (Haycock, 2001). Schools with large numbers of high-poverty and racially diverse students have shown significant academic growth when teachers and staff members create an environment of high expectations for achievement (Reeves, 2010). In addition, research on school climate has asserted the need for students to feel emotionally safe and respected as well as physically safe in school (Gronna & Chin-Chance, 1999).

A positive, respectful learning environment with high expectations and curricular and instructional supports for all students offers an avenue to genuine educational equity.

Probing Questions

- What are some of the needs and challenges your school faces in moving toward a fully equitable education for all students?
- How could you provide leadership in your school to work to ensure an equitable education for all students?

Resources

A variety of resources are available for teachers and leaders on educational equity for all students. A few websites and links are highlighted below:

The School Improvement Center developed activities to help districts develop an equity framework. These resources can be found at Actualizing Equity: The Equity Framework: http://www.gapsc.com/EducatorPreparation/NoChildLeftBehind/Admin/Files/conference_032010/Actualizing_Equity.pdf.

The Education Equality Project developed a website with useful resources for educators. It can be found at http://www.edequality.org.

The Equity Center has a website with a variety of resources. The resources can be found at http://educationnorthwest.org/project/Equity%20Program/resource/.

The Midwest Equity Assistance Center has a website with many resources. It can be found at http://www.meac.org/Publications.html.

The Office for Civil Rights has a useful website for educators. It can be found at http://www2.ed.gov/about/offices/list/ocr/index.html.

Southern Poverty Law Center, Teaching Tolerance Program. Resources can be found at http://www.splcenter.org/what-we-do/teaching-tolerance.

References

Berne, R., & Stiefel, L. (1994). Measuring educational equity at the school level: The finance perspective. *Educational Evaluation and Policy Analysis*, 16(4), 405–421.

Brighouse, H., & Swift, A. (2008). Putting educational equality in its place. *Education, Finance and Policy*, 3(4), 444–446.

Brown v. Board of Education of Topeka, 347 U.S. 483 (1954).

Civil Rights Act, Title IX, Pub. L. No. 88-352, 78 Stat. 241 (1964).

Daggett, W. R. (2005). *Achieving academic excellence through rigor and relevance* [White paper]. Rexford, NY: International Center for Leadership in Education. Retrieved June 3, 2011, from http://www.leadered.com/pdf/Academic_Excellence.pdf

Education Northwest. (2011). *Key components of educational equity* [Website]. Retrieved June 3, 2011, from http://educationnorthwest.org/equity-program/educational

Education for All Handicapped Children Act, Pub. L. No. 94-142 (1975).

Elementary and Secondary Education Act of 1965, Pub. L. No. 89-10) (1965).

Gardner, H. (1999). *Intelligence reframed: Multiple intelligences for the 21st century*. New York: Basic Books.

Gibbons, P. (2002). *Scaffolding language, scaffolding learning: Teaching second language learners in the mainstream classroom*. Westport, CT: Heinemann.

Gronna, S. S., & Chin-Chance, S. A. (1999, April). *Effects of school safety and school characteristics on grade 8 achievement*. Paper presented at the American Educational Research Association, Montreal, Quebec, Canada. (ERIC Document Reproduction Service No. 430292). Retrieved June 3, 2011, from http://www.eric.ed.gov/PDFS/ED430292.pdf

Haycock, K. (2001). Closing the achievement gap. *Educational Leadership*, 58(6), 6–11.

Lau v. Nichols, 414 U.S. 565 (1974).

Mulroy, H., & Eddinger, K. (2003, March). *Differentiation and literacy.* Paper presented at the Institute on Inclusive Education, Rochester, NY.

No Child Left Behind Act of 2001, Pub. L. No. 107-110, 115 Stat. 1425 (2002). Retrieved June 3, 2011, from http://www.ed.gov/policy/elsec/leg/esea02/107-110.pdf

Reeves, D. B. (2010). The 90/90/90 schools: A case study. In D. B. Reeves, *Accountability in action* (2nd ed., 185–196). Denver, CO: Advanced Learning Press.

Rose, D., Hasselbring, T., Stahl, S., & Zabala, J. (2009). Assistive technology, NIMAS, and UDL: From some students to all students. In D. Gordon, J. Gravel, & L. Schifter (Eds.), *A policy reader in universal design for learning* (pp. 133–154). Cambridge, MA: Harvard Education Press.

Tomlinson, C. A. (2005). Grading and differentiation: Paradox or good practice? *Theory Into Practice,* 44(3) 262–269.

Guiding Principle 2:
Instruction must be rigorous and relevant.

To understand the world in which we live, there are certain things we all must learn. Each school subject is made up of a core of essential knowledge that is deep, rich, and vital. Every student, regardless of age or ability, must be taught this essential knowledge. What students learn is fundamentally connected to how they learn, and successful instruction blends the content of a discipline with processes of an engaging learning environment that changes to meet the dynamic needs of all students.

Research Summary

Instruction should connect directly to students' lives and must deeply engage them with the content in order for students to be better prepared for college and careers. To succeed in postsecondary education and in a 21st century economy, students must be afforded opportunities to practice higher-order thinking skills, such as how to analyze an argument, weigh evidence, recognize bias (their own and others' bias), distinguish fact from opinion, balance competing principles, work collaboratively with others, and be able to communicate clearly what they understand (Wagner, 2006). In order to accomplish these goals, instruction must be rigorous and meaningful.

The definition of *rigor* varies greatly in both research and practice. Bower and Powers (2009) conducted a study to determine the essential components of rigor. They defined *rigor* through their research as "how the standard curriculum is delivered within the classroom to ensure students are not only successful on standardized assessments but also able to apply this knowledge to new situations both within the classroom and in the real world." They also identified higher-order thinking and real-world application as two critical aspects of rigor, suggesting that it is not enough for students to know how to memorize information and perform on multiple-choice and short-answer tests. Students must have deep and rich content knowledge, but rigor also includes the ability to apply that knowledge in authentic ways.

Teaching and learning approaches that involve students collaborating on projects that culminate with a product or presentation are a way to bring rigor into the classroom. Students can take on real problems, use what they know and research to come up with real solutions to real problems. They must engage with their subject and with their peers.

In August 2010, the Institutes of Education Sciences reported the results of a randomized control trial showing that a problem-based curriculum boosted high school students' knowledge of economics. This research suggests that students using this learning system and its variants score similarly on standardized tests as students who follow more traditional classroom practices. The research also suggests that students learning through problem-solving and projects are more adept at applying what they know and are more deeply engaged.

The notion of a meaningful curriculum is not a new one. John Dewey (1990), writing in 1902, called for a curriculum that involves a critical but balanced understanding of the culture and the prior knowledge of each child in order to extend learning. According to Spillane (2000), presenting content in more authentic ways—disciplinary and other real-world contexts—has become a central theme of current reform movements. Schools should be places where "the work students are asked to do [is] work worth doing" (Darling-Hammond, 2006, p. 21). Research collected by the International Center for Leadership in Education shows that "students understand and retain knowledge best when they have applied it in a practical, relevant setting" (Daggett, 2005, p. 2). A skilled 21st century educator helps students master learning targets and standards using purposefully crafted lessons and teaches with appropriate instructional strategies incorporated. The students understand why they are learning particular skills and content and are engaged in learning opportunities that allow them to use their inquiry skills, creativity, and critical thinking to solve problems.

According to Brown, Collins, and Duguid (1989), instruction connected to individual contexts has been found to have a significant impact on learning. Research conducted by Sanbonmatsu, Shavitt, and Sherman (1991) and Petty and Cacioppo (1984) also contends that student learning is directly influenced by how well it is connected to a context. Much of this research began with the analysis of how people learn when they find the ideas significant to their own world. It begins to show the importance of connecting content and instruction to the world of the students. Weaver and Cottrell (1988) point out that how content is presented can affect how students retain it. They state instruction that connects the content to the students' lives and experiences helps students to internalize meaning. Sass (1989) and Keller (1987) suggest that if teachers can make the content familiar to the students and link it to what they are familiar with, students' learning will increase. Shulman and Luechauer (1993) contend that these connections must be done by engaging students with rigorous content in interactive learning environments.

Higher-Order Thinking

Higher-order thinking, according to Newmann (1990), "challenges the student to interpret, analyze, or manipulate information" (p. 45). This definition suggests that instruction must be designed to engage students through multiple levels in order for them to gain a better understanding of the content. An analysis of the research by Lewis and Smith (1993) led to their definition of *higher-order thinking*: "when a person takes new information and information stored in memory and interrelates and/or rearranges and extends this information to achieve a purpose or find possible answers in perplexing situations" (p. 44). This definition emphasizes the level of complexity necessary to help students reach a deeper and higher level of understanding of the content. Shulman (1987) points out teachers will need an in-depth knowledge of their content to be able to fit these types of strategies to their instruction.

Real-World Application

VanOers and Wardekker (1999) indicate that connecting instruction to real-world applications gives meaning to learning, makes it practical, and can help to develop connections with the greater community. Incorporating real-world examples becomes more authentic to students because they will be able to connect the learning to the bigger picture rather than just the classroom. Newmann and Wehlage (1993) describe the three criteria developed by Archbald and Newmann (1988) for this type of authentic learning: "Students construct meaning and produce knowledge, students use disciplined inquiry to construct meaning, and students aim their work toward production of discourse, products, and performances that have value or meaning beyond success in school" (p. 8) These criteria, when reflected upon by teachers, can be a useful tool to ensure that instruction is authentic and engaging for all students.

Authentic Learning

Authentic learning builds on the concept of "learning by doing" to increase a student's engagement. To succeed, this method needs to have meaning or value to the student, embody in-depth learning in the subject and allow the student to use what he or she learned to produce something new and innovative (Lemke & Coughlin, 2009). For example, in project-based learning, students collaborate to create their own projects that demonstrate their knowledge (Bell, 2010). Students start by developing a question that will guide their work. The teacher acts as the supervisor. The goal is greater understanding of the topic, deeper learning, higher-level reading, and increased motivation (Bell, 2010). Research has shown that students who engage in project-based learning outscore their traditionally educated peers in standardized testing (Bell, 2010).

Constructivist learning is also a way to bring authenticity to the classroom. Richard Mayer (2004) defines constructivist learning as an "active process in which learners are active sense makers who seek to build coherent and organized knowledge." Students co-construct their learning, with the teacher serving as a guide or facilitator (oftentimes using technology as a facilitating tool). The teacher doesn't function in a purely didactic manner. Neo and Neo (2009) state that constructivism helps students develop problem-solving skills, critical thinking and creative skills and apply them in meaningful ways. Inquiry-based instruction, a type of constructivist learning, has students identify real world problems and then pose and find answers to their own questions. A study by Minner, Levy and Century (2010) has shown this method can improve student performance. They found inquiry-based instruction has a larger impact (approximately 25-30% higher) on a student's initial understanding and retention of content than any other variable.

Another form of authentic learning involves video simulated learning or gaming. Research has shown that video games can provide a rich learning context by fostering creative thinking. The games can show players how to manage complex problems and how their decisions can affect the outcome (Sharritt, 2008). This form of learning also can engage students in collaboration and interaction with peers.

Multimodal Instruction

Multimodal teaching leverages various presentation formats—such as printed material, videos, PowerPoints, and computers—to appeal to different learning styles (Birch, 2009; Moreno & Mayer, 2007). It accommodates a more diverse curriculum and can provide a more engaging and interactive learning environment (Birch, 2009). According to research, an effective way of learning is by utilizing different modalities within the classroom, which can help students understand difficult concepts—therefore improving how they learn (Moreno & Mayer, 2007).

An example of multimodal learning that incorporates technology is digital storytelling. Digital storytelling is the practice of telling stories by using technology tools (e.g., digital cameras, authoring tools, computers) to create multimedia stories (Sadik, 2008). Researchers have found that using this form of learning facilitates student engagement, deep learning, project-based learning, and effective integration of technology into instruction (Sadik, 2008).

Probing Questions

- Research emphasizes the need for higher-order thinking embedded in instructional practice. How might you learn to incorporate higher-order thinking strategies into your practice?
- The research also suggests the need to connect learning experiences to the real world of the students. How can you use real-world examples in your practice to better engage students in their learning?

Resources

The Rigor/Relevance Framework created by Daggett (2005) is a useful tool to create units, lessons, and assessments that ask students to engage with content at a higher, deeper level. The model and examples are available on the following website: http://www.leadered.com/rrr.html.

Newmann's Authentic Intellectual Work Framework (Newmann, Secada & Wehlage, 1995) gives teachers the tools to analyze instructional practices and student work in regard to indicators of rigor. The research and tools are available at the Center for Authentic Intellectual Work website: http://centerforaiw.com/.

References

Archbald, D., & Newmann, F. M. (1988). *Beyond standardized testing: Assessing authentic academic achievement in the secondary school.* Reston, VA: National Association of Secondary School Principals.

Bell, S. (2010). Project-based learning for the 21st century: Skills for the future. *The Clearing House*, 83, 39–43.

Birch, D. (2009). PowerPoint with audio: A breeze to enhance the student learning experience. *E-Journal of Business Education & Scholarship of Teaching*, 3(1), 36–42.

Bower, H. A., & Powers, J. D. (2009, Fall). What is rigor? A qualitative analysis of one school's definition. *Academic Leadership Live: The Online Journal*, 7(4). Retrieved June 3, 2011, from http://www.academicleadership.org/article/What_is_Rigor_A_Qualitative_Analysis_of_One_School_s_Definition

Brown, J. S., Collins, A., & Duguid, P. (1989). Situated cognition and the culture of learning. *Educational Researcher,* 18(1), 32–42.

Daggett, W. R. (2005). *Achieving academic excellence through rigor and relevance*. Rexford, NY: International Center for Leadership in Education.

Darling-Hammond, L. (2006). Securing the right to learn: Policy and practice for powerful teaching and learning. *Educational Researcher*, 35(7), 13–24.

Dewey, J. (1990). *School and society [and] The child and the curriculum*. Chicago: University of Chicago Press.

Finkelstein, Neal, Thomas Hanson, Chun-Wei Huang, Becca Hirschman, and Min Huang. (2010). Effects of problem based economics on high school economics instruction." *Institute For Education Sciences.* West Ed.

Keller, J. M. (1987). Strategies for stimulating the motivation to learn. *Performance & Instruction*, 26(8), 1–7.

Lemke, C., & Coughlin, E. (2009, September). The change agents: Technology is empowering 21st century students in four key ways. *Educational Leadership*, 67(1), 54–59.

Lewis, A., & Smith, D. (1993). Defining higher order thinking. *Theory Into Practice*, 32(3), 131–137.

Mayer, R.E. (2004). *Should There Be a Three-Strikes Rule Against Pure Discovery Learning? The Case for Guided Methods of Instruction.* American Psychologist, 59(1), 14-19.

Minner, Daphne D., Abigail Jurist Levy, and Jeanne Century. "Inquiry-Based Science Instruction—What Is It and Does It Matter? Results from a Research Synthesis Years 1984 to 2002." *JOURNAL OF RESEARCH IN SCIENCE TEACHING* 47.4 (April 2010): 474-96.

Moreno, R., & Mayer, R. (2007). *Interactive multimodal learning environments* [Special issue on interactive learning environment-contemporary issues and trends]. Educational Psychology Review, 19, 309–326.

Neo, M. & Neo, T.K. (2009). *Engaging students in multimedia-mediated Constructivist learning-Students' perceptions*. Educational Technology & Society, 12(2), 254-266.

Newmann, F. M. (1990). Higher order thinking in teaching social studies: A rationale for the assessment of classroom thoughtfulness. *Journal of Curriculum Studies*, 22(2), 41–56.

Newmann, F. M., Secada, W. G., & Wehlage, G. G. (1995). *A guide to authentic instruction and assessment: Vision, standards, and scoring*. Madison, WI: Wisconsin Center for Education Research.

Newmann, F. M., & Wehlage, G. G. (1993, April). Five standards of authentic instruction. *Educational Leadership*, 50(7), 8–12.

Petty, R. E., & Cacioppo, J. T. (1984). The effects of involvement on responses to argument quality: Central and peripheral routes to persuasion. *Journal of Personality and Social Psychology*, 46(1), 69–81.

Sadik, A. (2008). Digital storytelling: A meaningful technology-integrated approach for engaged student learning. *Educational Technology Research & Development*, 56, 487–506.

Sanbonmatsu, D. M., Shavitt, S., & Sherman, S. J. (1991). The role of personal relevance in the formation of distinctiveness-based illusory correlations. *Personality and Social Psychology Bulletin*, 17(2), 124–132.

Sass, E. J. (1989). Motivation in the college classroom: What students tell us. *Teaching of Psychology*, 16(2), 86–88.

Sharritt, M. J. (2008). Forms of learning in collaborative video game play. *Research and Practice in Technology Enhanced Learning*, 3(2), 97–138.

Shulman, L. S. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review*, 57(1), 1–22.

Shulman, G., & Luechauer, D. (1993). The empowering educator: A CQI approach to classroom leadership. In D. L. Hubbard (Ed.), *Continuous quality improvement: Making the transition to education* (pp. 424–453). Maryville, MO: Prescott.

Spillane, J. P. (2000). A fifth-grade teacher's reconstruction of mathematics and literacy teaching: Exploring interactions among identity, learning, and subject matter. *Elementary School Journal*, 100(4), 307–330.

VanOers, B., & Wardekker, K. (1999). On becoming an authentic learner: Semiotic activity in the early grades. *Journal of Curriculum Studies*, 31(2), 229–249.

Wagner, T. (2006, January 11). Rigor on trial [Commentary]. *Education Week*, 25(18), 28–29. Retrieved June 3, 2011, from http://www.edweek.org/ew/articles/2006/01/11/18wagner.h25.html?tkn=NXVFlUJgch3u9KNoYbF2gM%2BinCPa3hvbbWkj&print=1

Weaver, R. L., & Cottrell, H. W. (1988). Motivating students: Stimulating and sustaining student effort. *College Student Journal*, 22, 22–32.

Wentling, R. M., & Waight, C. L. (2001). Initiative that assist and barriers that hinder the successful transition of minority youth into the workplace in the USA. *Journal of Education and Work*, 14(1), 71–89.

Guiding Principle 3:
Purposeful assessment drives instruction and affects learning.

Assessment is an integral part of teaching and learning. Purposeful assessment practices help teachers and students understand where they have been, where they are, and where they might go next. No one assessment can provide sufficient information to plan teaching and learning. Using different types of assessments as part of instruction results in useful information about student understanding and progress. Educators should use this information to guide their own practice and in partnership with students and their families to reflect on learning and set future goals.

Research Summary

Assessment informs teachers, administrators, parents, and other stakeholders about student achievement. It provides valuable information for designing instruction; acts as an evaluation for students, classrooms, and schools; and informs policy decisions. Instruments of assessment can provide formative or summative data, and they can use traditional or authentic designs. Research on assessment emphasizes that the difference between formative and summative assessment has to do with how the data from the assessment is used.

Dunn and Mulvenon (2009) define summative assessment as assessment "data for the purposes of assessing academic progress at the end of a specified time period (i.e., a unit of material or an entire school year) and for the purposes of establishing a student's academic standing relative to some established criterion" (p. 3).

The Council of Chief State School Officers (CCSSO) (2008) define formative assessment as a process "used by teachers and students during instruction that provides feedback to adjust ongoing teaching and learning to improve students' achievement of intended instructional outcomes" (p. 3).

Wisconsin's approach to balanced assessment www.dpi.wi.gov/oea/balanced emphasizes the importance of identifying the purposes for administering an assessment. Identifying the purpose or data needed establishes whether a particular assessment is being used formatively or summatively. There can be multiple purposes for giving a particular assessment, but identifying how the data will be used helps to ensure that the assessment is collecting the data that is needed for educators, students and their families.

Assessments, whether formative or summative, can be designed as traditional or authentic tools. Traditional assessment uses tools such as paper and pencil tests, while authentic assessment focuses on evaluating student learning in a more "real life" situation. The bulk of the research on assessment design focuses on authentic assessment.

Formative Assessment

Using formative assessment as a regular part of instruction has been shown to improve student learning from early childhood to university education. It has been shown to increase learning for both low-performing and high-performing students. Black and Wiliam's (1998) seminal study found that the use of formative assessment produces significant learning gains for low-achieving students. Other researchers have shown similar results for students with special learning needs (McCurdy & Shapiro, 1992; Fuchs & Fuchs, 1986). Research also supports the use of formative assessment in kindergarten classes (Bergan, Sladeczek, Schwarz, & Smith, 1991), and university students (Martinez & Martinez, 1992).

Formative assessment provides students with information on the gaps that exist between their current knowledge and the stated learning goals (Ramaprasad, 1983). By providing feedback on specific errors it helps students understand that their low performance can be improved and is not a result of lack of ability (Vispoel & Austin, 1995). Studies emphasize that formative assessment is most effective when teachers use it to provide specific and timely feedback on errors and suggestions for improvement (Wininger, 2005), when students understand the learning objectives and assessment criteria, and when students have the opportunity to reflect on their work (Ross, 2006; Ruiz-Primo & Furtak, 2006). Recent research supports the use of web-based formative assessment for improving student achievement (Wang, 2007).

A number of studies emphasize the importance of teacher professional development on formative assessment in order to gain maximum student achievement benefits (Atkins, Black & Coffey, 2001; Black & Wiliam, 1998). A 2009 article in *Educational Measurement* asserts that teachers are better at analyzing formative assessment data than at using it to design instruction. Research calls for more professional development on assessment for teachers (Heritage, Kim, Vendlinski, & Herman, 2009).

Authentic Assessment

Generating rich assessment data can be accomplished through the use of an authentic assessment design as well as through traditional tests. Authentic assessments require students to "use prior knowledge, recent learning, and relevant skills to solve realistic, complex problems" (DiMartino & Castaneda, 2007, p. 39). Research on authentic assessment often explores one particular form, such as portfolios (Berryman & Russell, 2001; Tierney et al., 1998); however, several studies examined more than one form of authentic assessment: portfolios, project-based assessment, use of rubrics, teacher observation, and student demonstration (Darling-Hammond, Rustique-Forrester, & Pecheone, 2005; Herman, 1997; Wiggins, 1990). Authentic assessment tools can be used to collect both formative and summative data. These data can provide a more complete picture of student learning.

Balanced Assessment

Wisconsin's Next Generation Assessment Task Force (2009) defines the purpose and characteristics of a balanced assessment system:

Purpose: to provide students, educators, parents, and the public with a range of information about academic achievement and to determine the best practices and policies that will result in improvements to student learning.

Characteristics: includes a continuum of strategies and tools that are designed specifically to meet discrete needs–daily classroom instruction, periodic checkpoints during the year, and annual snapshots of achievement. (p. 6)

A balanced assessment system is an important component of quality teaching and learning. Stiggins (2007) points out that a variety of quality assessments must be available to teachers in order to form a clearer picture of student achievement of the standards. Popham (2008) believes that when an assessment is of high quality, it can accurately detect changes in student achievement and can contribute to continuous improvement of the educational system.

Probing Questions

- How might you use questioning and discussion in your classroom in a way that gives you formative assessment information on all students?
- How can you use assignments and tests as effective formative assessment?
- How could you design and implement a balanced assessment system that includes pre- and post assessments for learning?

Resources

Rick Stiggins, founder and director of the Assessment Training Institute, provides resources on the practice of assessment at http://www.assessmentinst.com/author/rick-stiggins/.

Margaret Heritage's books Formative Assessment for Literacy and Academic Language (2008, coauthored with Alison Bailey) and Formative Assessment: Making It Happen in the Classroom (2010) provide resources and practices. These books are available through bookstores.

ASCD has publications on assessment at http://www.ascd.org/SearchResults.aspx?s=assessment&c=1&n=10&p=0.

The National Middle Schools Association provides assessment information through a search for "assessment" at http://www.nmsa.org/.

Boston (2002) recommends the following resources for assessment:

- A Practical Guide to Alternative Assessment, by J. R. Herman, P. L. Aschbacher, and L. Winters. Available at a variety of booksellers.
- Improving Classroom Assessment: A Toolkit for Professional Developers http://educationnorthwest.org/resource/700
- Classroom Assessment and the National Science Education Standards http:www.nap.edu/catalog/9847.html

References

Atkins, J. M., Black, P., & Coffey, J. (2001). *Classroom assessment and the National Science Education Standards.* Washington, DC: National Academy Press.

Bergen, J. R., Sladeczek, I. E., Schwarz, R. D., & Smith, A. N. (1991). Effects of a measurement and planning system on kindergartners' cognitive development and educational programming. *American Educational Research Journal*, 28(3), 683–714.

Berryman, L., & Russell, D. R. (2001). Portfolios across the curriculum: Whole school assessment in Kentucky. *English Journal*, 90(6), 76–83.

Black, P., & Wiliam, D. (1998). Assessment and classroom learning. *Assessment in Education*, 5(1), 7–74.

Boston, C. (2002). The concept of formative assessment. *Practical Assessment, Research, and Evaluation*, 8(9). Retrieved June 3, 2011, from http://pareonline.net/getvn.asp?v=8&n=9

Council of Chief State School Officers. (2008). *Attributes of effective formative assessment.* Washington, DC: Author. Retrieved June 3, 2011, from http://www.ccsso.org/Documents/2008/Attributes_of_Effective_2008.pdf

Darling-Hammond, L., Rustique-Forrester, E., & Pecheone, R. (2005). *Multiple measure approaches to high school graduation*. Stanford, CA: School Redesign Network at Stanford University.

DiMartino, J., & Castaneda, A. (2007). Assessing applied skills. *Educational Leadership*, 64(7), 38–42.

Dunn, K. E., & Mulvenon, S. W. (2009). A critical review of research on formative assessment: The limited scientific evidence of the impact of formative assessment in education. *Practical Assessment, Research, and Evaluation*, 14(7). Retrieved June 3, 2011, from http://pareonline.net/pdf/v14n7.pdf

Fuchs, L. S., & Fuchs, D. (1986). Effects of systematic formative evaluation: A meta-analysis. *Exceptional Children*, 52(2), 199–208.

Heritage, M., Kim, J., Vendlinski, T., & Herman, J. (2009). From evidence to action: A seamless process in formative assessment? *Educational Measurement: Issues and Practice*, 28(3), 24–31.

Herman, J. (1997). Assessing new assessments: Do they measure up? *Theory Into Practice*, 36(4), 196–204.

Martinez, J. G. R., & Martinez, N. C. (1992). Re-examining repeated testing and teacher effects in a remedial mathematics course. *British Journal of Educational Psychology*, 62(3), 356–363.

McCurdy, B. L., & Shapiro, E. S. (1992). A comparison of teacher monitoring, peer monitoring, and self-monitoring with curriculum-based measurement in reading among student with learning disabilities. *Journal of Special Education*, 26(2), 162–180.

Next Generation Assessment Task Force. (2009). *Crafting a balanced system of assessment in Wisconsin*. Madison: Wisconsin Department of Public Instruction. Retrieved June 3, 2011, from http://www.dpi.state.wi.us/oea/pdf/NGTFbr.pdf

Popham, W. J. (2008). *Transformative assessment.* Alexandria, VA: Association for Supervision and Curriculum Development.

Ramaprasad, A. (1983). On the definition of feedback. *Behavioral Science*, 28(1), 4–13.

Ross, J. A. (2006). The reliability, validity, and utility of self-assessment. *Practical Assessment, Research and Evaluation*, 11(10). Retrieved June 3, 2011, from http://pareonline.net/pdf/v11n10.pdf

Ruiz-Primo, M. A., & Furtak, E. M. (2006). Informal formative assessment and scientific inquiry: Exploring teachers' practices and student learning. *Educational Assessment*, 11(2), 205–235.

Stiggins, R. J. (2007, November–December). Assessment for learning: A key to student motivation and learning. EDge, 2(2), 1–20.

Tierney, R., Clark, C., Fenner, L., Herter, R. J., Simpson, C. S., & Wiser, B. (1998). Portfolios: Assumptions, tensions, and possibilities. *Reading Research Quarterly*, 33(4), 474–486.

Vispoel, W. P., & Austin, J. R. (1995). Success and failure in junior high school: A critical incident approach to understanding students' attributional beliefs. *American Educational Research Journal*, 32(2), 377–412.

Wang, T. H. (2007). What strategies are effective for formative assessment in a e-learning environment? *Journal of Computer Assisted Learning*, 23(1), 171–186.

Wiggins, G. (1990). The case for authentic assessment. *Practical Assessment, Research, and Evaluation*, 2(2). Retrieved June 3, 2011, from http://pareonline.net/getvn.asp?v=2&n=2

Guiding Principle 4:
Learning is a collaborative responsibility.

Teaching and learning are both collaborative processes. Collaboration benefits teaching and learning when it occurs on several levels: when students, teachers, family members, and the community collectively prioritize education and engage in activities that support local schools, educators, and students; when educators collaborate with their colleagues to support innovative classroom practices and set high expectations for themselves and their students; and when students are given opportunities to work together toward academic goals in ways that enhance learning.

Research Summary

Collaborative learning is an approach to teaching and learning that requires learners to work together to deliberate, discuss, and create meaning. Smith and MacGregor (1992) define the term as follows:

"Collaborative learning" is an umbrella term for a variety of educational approaches involving joint intellectual effort by students, or students and teachers together. Usually, students are working in groups of two or more, mutually searching for understanding, solutions, or meanings, or creating a product. Collaborative learning activities vary widely, but most center on students' exploration or application of the course material, not simply the teacher's presentation or explication of it. (p. 1)

Collaborative learning has been practiced and studied since the early 1900s. The principles are based on the theories of John Dewey (2009), Lev Vygotsky (1980), and Benjamin Bloom (1956). Their collective work focusing on how students learn has led educators to develop more student-focused learning environments that put students at the center of instruction. Vygotsky specifically stated that learning is a social act and must not be done in isolation. This principle is the foundation of collaborative learning.

The research of Vygotsky (1980) and Jerome Bruner (1985) indicates that collaborative learning environments are one of the necessities for learning. Slavin's (1989) research also suggests that students and teachers learn more, are more engaged, and feel like they get more out of their classes when working in a collaborative environment. Totten, Sills, Digby, and Russ (1991) found that those involved in collaborative learning understand content at deeper levels and have higher rates of achievement and retention than learners who work alone. They suggest that collaborative learning gives students opportunities to internalize their learning.

A meta-analysis from the Cooperative Learning Center at the University of Minnesota concluded that having students work collaboratively has significantly more impact on learning than having students work alone (Johnson, Maruyama, Johnson, Nelson, & Skon, 1981). An analysis of 122 studies on cooperative learning revealed:

- More students learn more material when they work together—talking through the material with each other and making sure that all group members understand—than when students compete with one another or work alone individualistically.
- More students are motivated to learn the material when they work together than when students compete or work alone individualistically (and the motivation tends to be more intrinsic).
- Students have more positive attitudes when they work together than when they compete or work alone individualistically.
- Students are more positive about the subject being studied, the teacher, and themselves as learners in that class and are more accepting of each other (male or female, handicapped or not, bright or struggling, or from different ethnic backgrounds) when they work together.

Collaboration can be between teachers, between students, and between teacher and student.

Teacher-Teacher Collaboration

It is critical for teachers to have the time to collaborate. Professional learning communities, which provide teachers with established time to collaborate with other teachers, have become a more common practice in recent years. Louis and Kruse (1995) conducted a case study analysis that highlighted some of the positive outcomes associated with professional learning communities, including a reduction in teacher isolation, increases in teacher commitment and sense of shared responsibility, and a better understanding of effective instructional practices. Professional learning communities encourage collaborative problem solving and allow teachers to gain new strategies and skills to improve and energize their teaching and classrooms.

Another example of teacher-to-teacher collaboration is lesson study. This professional development process began in Japan. Lesson study is a collaborative approach to designing and studying classroom lessons and practice. The most critical components of lesson study are observation of the lesson, collection of data about teaching and learning, and a collaborative analysis of the data to further impact instruction (Lewis, 2002; Lewis & Tsuchida, 1998; Wang-Iverson & Yoshida, 2005). Some of these characteristics are similar to other forms of professional development—analyzing student work, cognitive coaching, and action research, to name a few—but the fact that it focuses on teachers observing a live lesson that was collaboratively developed is different than any other form of professional development. Lesson study is a way for teachers to work together, collect data, and analyze data to reflect on teaching and learning (Lewis, 2002).

Student-Student Collaboration

Collaborative learning not only allows students to engage deeply with content but also helps students build the interpersonal skills needed to be successful in college and careers. Johnson, Johnson, and Holubec (1993) state that collaborative learning provides students with the opportunity to develop social skills. They found that many of the outcomes expected as part of a collaborative learning activity corresponded with goals for student content understanding and skill attainment. The strategies associated with collaborative learning—such as role assignments, collaborative problem solving, and task and group processing—all build the social skills that students need to be successful when working with others. Additionally, these skills are important in preparing students for the world of work, where collaborative writing and problem-solving are key elements of many careers.

There is a plethora of instructional and learning strategies that encourage student collaboration, including peer teaching, peer learning, reciprocal learning, team learning, study circles, study groups, and work groups, to name just a few (Johnson & Johnson, 1986). Collaborative inquiry, which combines many of the elements of student collaboration just mentioned, is a research-based strategy in which learners work together through various phases "of planning, reflection, and action as they explore an issue or question of importance to the group" (Goodnough, 2005 88). Collaborative inquiry brings together many perspectives to solve a problem, engaging students in relevant learning around an authentic question. It allows students to work together toward a common purpose to explore, make meaning, and understand the world around them (Lee & Smagorinsky, 2000).

Teacher-Student Collaboration

The purpose for collaboration in an educational setting is to learn and unpack content together to develop a shared understanding. Harding-Smith (1993) points out that collaborative learning approaches are based on the idea that learning must be a social act. It is through interaction that learning occurs. Johnson and Johnson (1986) similarly emphasize that when students and teachers talk and listen to each other, they gain a deeper understanding of the content and can develop the skills necessary to negotiate meaning throughout their lives.

Collaboration requires a shift from teacher-led instruction to instruction and learning that is designed by both teachers and students. Collaboration between student and teacher plays a critical role in helping students reflect and engage in their own learning experiences. The constructivist learning movement is one current example of efforts to increase the amount of collaboration between student and teacher occurring in the classroom. Mayer (2004) defines constructivist learning as an "active process in which learners are active sense makers who seek to build coherent and organized knowledge" (p. 14). Students co-construct their learning, with the teacher serving as a guide or facilitator. The teacher does not function in a purely didactic (i.e., lecturing) role. Neo and Neo (2009) found that constructivism helps students develop problem-solving skills, critical thinking, and creative skills and apply them in meaningful ways.

Probing Questions

- How can you use collaborative learning processes to engage students in their learning?
- How might you create space for teacher-teacher collaboration within your context?

Resources

All Things PLC website provides a number of resources on professional learning communities. Links to these resources can be found at http://www.allthingsplc.info/.

The Wisconsin Center for Education Research hosts a website with many resources for collaborative and small group learning. It can be found at http://www.wcer.wisc.edu/archive/cl1/cl/..

The Texas Collaborative for Teaching Excellence has created a professional development module about collaborative learning, which provides readings, research, and resources. It can be found at http://www.texascollaborative.org/Collaborative_Learning_Module.htm.

A review of research on professional learning communities, presented at the National School Reform Faculty research forum in 2006, contains findings that outline what is known about professional learning communities and how they should be structured. This paper is available at http://www.nsrfharmony.org/research.vescio_ross_adams.pdf.

References

Bloom, B. S. (Ed.) (1956). *Taxonomy of educational objectives. Handbook 1: Cognitive domain*. White Plains, NY: Longman.

Bruner, J. (1985). Vygotsky: An historical and conceptual perspective. In J. V. Wetsch (Ed.), *Culture, communication, and cognition: Vygotskian perspectives* (pp. 21–34). London: Cambridge University Press.

Dewey, J. (2009). *Democracy and education: An introduction to the philosophy of education*. New York: Cosimo Classics.

Goodnough, Karen. (2005). Fostering teacher learning through collaborative inquiry. *The Clearing House* 79(2), 88-92.

Harding-Smith, T. (1993). *Learning together: An introduction to collaborative learning*. New York: HarperCollins.

Johnson, R. T., & Johnson, D. W. (1986). Action research: Cooperative learning in the science classroom. *Science and Children*, 24(2), 31–32.

Johnson, D. W., Johnson, R. T., & Holubec, E. J. (1993). *Circles of learning: Cooperation in the classroom*. Edina, MN: Interaction.

Johnson, D. W., Maruyama, G., Johnson, R. T., Nelson, D., & Skon, L. (1981). Effects of cooperative, competitive, and individualistic goal structures on achievement: A meta-analysis. *Psychological Bulletin*, 89(1), 47–62.

Lee, C. D., & Smagorinsky, P. (Eds.). (2000). *Vygotskian perspectives on literacy research: Constructing meaning through collaborative inquiry*. Cambridge, England: Cambridge University Press.

Lewis, C. (2002). *Lesson study: A handbook of teacher-led instructional change*. Philadelphia: Research for Better Schools.

Lewis, C., & Tsuchida, I. (1998, Winter). A lesson is like a swiftly flowing river: Research lessons and the improvement of Japanese education. *American Educator*, 14–17, 50–52.

Wang-Iverson, P., & Yoshida, M. (2005). Building our understanding of lesson study. Philadelphia: Research for Better Schools.

Louis, K. S., & Kruse, S. D. (1995). *Professionalism and community: Perspectives on reforming urban schools*. Thousand Oaks, CA: Corwin Press.

Mayer, R. E. (2004). Should there be a three strikes rule against pure discovery? The case for guided methods of instruction. *American Psychologist*, 59(1), 14–19.

Neo, M., & Neo, T.-K. (2009). Engaging students in multimedia-mediated constructivist learning: Students' perceptions. *Educational Technology and Society*, 12(2), 254–266.

Slavin, R. E. (1989). Research on cooperative learning: An international perspective. *Scandinavian Journal of Educational Research*, 33(4), 231–243.

Smith, B. L., & MacGregor, J. T. (1992). *What is collaborative learning?* Olympia, WA: Washington Center for Improving the Quality of Undergraduate Education. Retrieved June 3, 2011, from http://learningcommons.evergreen.edu/pdf/collab.pdf

Totten, S., Sills, T., Digby, A., & Russ, P. (1991). *Cooperative learning: A guide to research*. New York: Garland.

Vygotsky, L. (1980). *Mind in society: The development of higher psychological processes*. Cambridge, MA: Harvard University Press.

Guiding Principle 5:
Students bring strengths and experiences to learning.

Every student learns. Although no two students come to school with the same culture, learning strengths, background knowledge, or experiences, and no two students learn in exactly the same way, every student's unique personal history enriches classrooms, schools, and the community. This diversity is our greatest education asset.

Research Summary

The authors of the groundbreaking work *How People Learn: Brain, Mind, Experience, and School* (Bransford, Brown, & Cocking, 2000) found that students' preconceptions may clash with new concepts and information they learn in school. If those preconceptions are not addressed, students may fail to grasp what is being taught or may learn only to pass a test. In other words, a student might enter kindergarten believing the world is flat because he or she has seen a flat map. Despite the presentation of geographic names and principles, the student still maintains the fundamental preconception about the shape of the world. Developing competence—or in this case, a knowledge of the shape of the world—requires that students have a deep foundation of factual knowledge, a context or conceptual framework to place it in, and the opportunity to explore how it connects to the real world. Ultimately, a metacognitive approach—one that pushes students to think about their own thought processes—can help them take control of their own learning.

As educational research on how people learn advances, so does our approach to teaching and learning. Strategies to advance teaching and learning are constantly evolving into new and innovative ways to reach learners. When a teacher uses students' interests, curiosity, and areas of confidence as starting points in planning instruction, learning is more productive. Teachers who are cognizant of these issues—and reflect on how to use them as strengths upon which they can build—ensure that all students have access to the content. Areas to consider are student strengths, gender, background knowledge, and connections to the home environment.

Building on Student Strengths

Teaching to students' strengths can improve student engagement (Sternberg, 2000, Sternberg & Grigorenko, 2000). Many students have strengths that are unrecognized and neglected in traditional schooling. Students in underrepresented minority groups have culturally relevant knowledge that teachers can use to promote learning. Sternberg et al. (2000) found that conventional instruction in school systematically discriminates against students with creative and practical strengths and tends to favor students with strong memory and analytical abilities. This research, combined with Sternberg's earlier (1988) research showing that teaching for diverse styles of learning produces superior results, suggests that capitalizing on the various strengths that all students bring to the classroom can positively affect students' learning. When students are taught in a way that fits *how* they think, they do better in school (Sternberg, 2000; Sternberg & Grigorenko, 2000). Sternberg and O'Hara (2000) found that when students were taught in a way that incorporated analytical thinking, creative thinking (creating, imagining, and inventing) and practical thinking (applying, implementing, and putting into practice)—students achieved at higher levels than when taught using conventional instructional methods.

Gender Considerations

Changing instruction might help alleviate the gender gap in literacy achievement. Research conducted by Sax (2005) reveals that boys fall behind girls in reading and writing early on and never catch up. Sax (2007) found that this dynamic plays a role in higher high school dropout rates for males, particularly black males. The college graduation rate for females approaches twice that of males in Hispanic and black populations. Many classrooms are a better fit for the verbal-emotive, sit-still, take-notes, listen-carefully, multitasking girl (Sax, 2005). The characteristics that boys bring to learning—impulsivity, single-task focus, spatial-kinesthetic learning, and physical aggression—often are viewed as problems.

Researchers such as Blum (1997) have identified more than 100 structural differences between the male and female brains. Altering strategies to accommodate more typically male assets—for example, the use of multimodal teaching (discussed on pages 10-11 of this report); the use of various display formats, such as printed material, videos, presentations, and computers; and an interactive learning environment to appeal to different learning styles—can help bridge the gap between what students are thinking and what they are able to put down on paper. Sadik's (2008) research suggests that using multimodal instructional strategies like digital storytelling—allowing students to incorporate digital cameras, creative and editing tools, computers, and other technology to design multimedia presentations—deepens students' learning.

Background Knowledge

Bransford et al. (2000) note in *How People Learn*, learning depends on how prior knowledge is incorporated into building new knowledge, and thus teachers must take into account students' prior knowledge. Jensen's (2008) research on the brain and learning demonstrates that expertise cannot be developed merely through exposure to information. Students must connect the information to their prior knowledge to internalize and deepen their understanding. Teachers can connect academic learning with real-life experiences. Service learning, project-based learning, school-based enterprises, and student leadership courses are some examples of how schools are trying to make the curriculum relevant. The key to making the curriculum relevant is asking the students to help connect the academics to their lives; this approach gets students actively engaged in their learning, which builds a stronger connection and commitment to school. Bell (2010) suggests that strategies such as project-based approaches to learning can help ensure that content and skills are taught together and connected to prior knowledge, which helps students understand how to develop and apply new skills in various contexts.

Connections to the Home Environment

Cochran-Smith (2004) emphasizes family histories, traditions, and stories as an important part of education. Often, children enter school and find themselves in a place that does not recognize or value the knowledge or experience they bring from their homes or communities. This situation can create a feeling of disconnect for students—a dissonance obliging them to live in and navigate between two different worlds, each preventing them from full participation or success in the other. Districts and schools can alleviate this dissonance by valuing and taking advantage of the unique experiences that each student brings to the classroom. Emphasizing connections to parents and community, recognizing and utilizing student strengths and experiences, and incorporating varied opportunities within the curriculum can help alleviate this dissonance.

Ferguson (2001) points out that it is particularly important to establish connections that not only bring the parents into the school environment but also encourage school understanding and participation within the community. Social distinctions often grow out of differences in attitudes, values, behaviors, and family and community practices (Ferguson, 2001). Students need to feel their unique knowledge and experience is valued by the school, and parents and community members need to feel they are respected and welcome within the school.

Although much attention has been paid to No Child Left Behind (NCLB) requirements for annual achievement tests and high-quality teachers, the law also includes important requirements for schools, districts, and states to organize programs of parental involvement and to communicate with parents and the public about student achievement and the quality of schools. Epstein (2005) offers perspectives on the NCLB requirements for family involvement; provides a few examples from the field; suggests modifications that are needed in the law; and encourages sociologists of education to take new directions in research on school, family, and community partnerships.

Probing Questions

- What are some ways that you currently use students' background knowledge to inform instruction?
- Does your experience teaching boys to read and write concur with the research? What ideas do you have to address the achievement gaps related to gender?
- What are ways you can uncover, acknowledge, and use students' backgrounds and strengths to enhance learning?
- What are some strategies for valuing and taking advantage of the unique experiences that each student brings to the classroom?

Resources

A good resource still valid today is *Making Assessment Work for Everyone: How to Build on Student Strengths*. See the SEDL website to download this resource: http://www.sedl.org/pubs/tl05/.

A short, easy-to-digest article from Carnegie Mellon University is titled *Theory and Research-Based Principles of Learning*. The article and full bibliography are at http://www.cmu.edu/teaching/principles/learning.html.

References

Bell, S. (2010). Project-based learning for the 21st century: Skills for the future. *The Clearing House*, 83(2), 39–43. Retrieved June 3, 2011, from http://teacherscollegesj.org/resources/publications/PBL%20for%20the%2021%20Century.pdf

Blum, D. (1997). *Sex on the brain: The biological differences between men and women*. New York: Viking.

Bransford, J. D., Brown, A. L. & Cocking, R. R. (Eds.). (2000). How people learn: *Brain, mind, experience, and school* (Expanded ed.). Washington, DC: National Academy Press.

Cochran-Smith, M. (2004). Walking the road: *Race, diversity, and social justice in teacher education*. New York: Teachers College Press.

Epstein, J. (2005). Attainable goals? The spirit and letter of the No Child Left Behind Act on parental involvement. *Sociology of Education*, 78(2), 179–182.

Ferguson, A. A. (2001). *Bad boys: Public schools in the making of black masculinity*. Ann Arbor: University of Michigan Press.

Jensen, E. P. (2008). A fresh look at brain-based education. *Phi Delta Kappan*, 89(6), 408–417.

Sadik, A. (2008). Digital storytelling: A meaningful technology-integrated approach for engaged student learning. *Educational Technology Research and Development*, 56(4), 487–506.

Sax, L. (2005). *Why gender matters: What parents and teachers need to know about the emerging science of sex differences*. New York: Doubleday.

Sax, L. (2007). *Boys adrift: The five factors driving the growing epidemic of unmotivated boys and underachieving young men*. New York: Basic Books.

Sternberg, R. J. (1988). *The triarchic mind: A new theory of human intelligence*. New York: Viking.

Sternberg, R. J. (2000). Wisdom as a form of giftedness. *Gifted Child Quarterly*, 44(4), 252–259.

Sternberg, R. J., & Grigorenko, E. L. (2000). *Teaching for successful intelligence*. Arlington Heights, IL: Skylight Training.

Sternberg, R. J., Grigorenko, E. L., Jarvin, L., Clinkenbeard, P., Ferrari, M., & Torff, B. (2000, Spring). The effectiveness of triarchic teaching and assessment. *NRC/GT Newsletter,* 3–8. Retrieved June, 3, 2011, from http://www.gifted.uconn.edu/nrcgt/newsletter/spring00/spring00.pdf

Sternberg, R. J., & O'Hara, L. A. (2000). Intelligence and creativity. In R. J. Sternberg (Ed.), *Handbook of intelligence* (pp. 611–628). New York: Cambridge University Press.

Vygotsky, L. S. (1980). *Mind in society: The development of higher psychological processes*. Cambridge, MA: Harvard University Press.

Guiding Principle 6:
Responsive environments engage learners.

Meaningful learning happens in environments where creativity, awareness, inquiry, and critical thinking are part of instruction. Responsive learning environments adapt to the individual needs of each student and encourage learning by promoting collaboration rather than isolation of learners. Learning environments, whether classrooms, schools, or other systems, should be structured to promote engaged teaching and learning.

Research Summary

To be effective for all students, classroom learning environments must be responsive to a broad range of needs among a diverse student population. These diverse needs include cultural and linguistic differences as well as developmental levels, academic readiness, and learning styles. A responsive learning environment engages all students by providing a respectful climate where instruction and curriculum are designed to respond to the backgrounds and needs of every student.

Culturally Responsive Teaching

Research on culturally responsive teaching emphasizes the importance of teachers' understanding the cultural characteristics and contributions of various ethnic groups (Smith, 1998) and showing respect toward these students and their culture (Ladson-Billings, 1995; Pewewardy & Cahape, 2003). Culturally responsive teaching is defined by Gay (2002) as "using the cultural characteristics, experiences, and perspectives of ethnically diverse students as conduits for teaching them more effectively" (p. 106).

Research on culturally responsive teaching has found that students both are more engaged in learning and learn more effectively when the knowledge and skills taught are presented within a context of their experience and cultural frames of references (Au & Kawakami, 1994; Gay, 2000; Ladson-Billings, 1995). Areas considered part of creating a culturally responsive learning environments are (1) understanding the cultural lifestyles of their students, such as which ethnic groups give priority to communal living and problem solving; (2) knowing differences in the modes of interaction between children and adults in different ethnic groups; and (3) becoming aware of cultural implications of gender role socialization among different groups (Banks & Banks, 2001). To provide a culturally responsive learning environment teachers need to:

- Communicate high expectations for all students (Gay, 2000; Hollins & Oliver, 1999; Ladson-Billings, 1994, Nieto, 1999).
- Use active teaching methods and act as learning facilitators (Banks & Banks, 2001; Gay, 2000).
- Maintain positive perspectives on families of diverse students (Delgado-Gaitin & Trueba, 1991).
- Gain knowledge of cultures of the students in their classrooms (Banks & Banks, 2001; Nieto, 1999).
- Reshape the curriculum to include culturally diverse topics (Banks & Banks, 2001; Gay, 2000; Hilliard, 1991).
- Use culturally sensitive instruction that includes student-controlled discussion and small-group work (Banks & Banks, 2001; Nieto, 1999).

Further research asserts that culturally responsive teachers help students understand that knowledge is not absolute and neutral but has moral and political elements. This knowledge can help students from diverse groups view learning as empowering (Ladson-Billings, 1995; Tharp & Gallimore, 1988).

Strategies for designing curriculum and instruction for culturally diverse students are similar to the strategies for differentiating curriculum and instruction. In fact, Mulroy and Eddinger (2003) point out that the research on differentiation emerged, in part, because of the demand on schools to serve an increasingly diverse student population. Heacox (2002) asserts that classrooms are diverse in cognitive abilities, learning styles, socioeconomic factors, readiness, learning pace, and gender and cultural influences.

Differentiation

Research on differentiation includes meeting the learning needs of all students through modifying instruction and curriculum to consider developmental level, academic readiness, and socioeconomic backgrounds, as well as cultural and linguistic differences. Tomlinson (2005) defines differentiated instruction as a philosophy of teaching based on the premise that students learn best when their teachers accommodate the difference in their readiness levels, interests, and learning profiles. In a differentiated learning environment, each student is valued for his or her unique strengths while being offered opportunities to learn and demonstrate learning through a variety of strategies (Mulroy & Eddinger, 2003). Hall (2002) states, "To differentiate instruction is to recognize students' varying backgrounds, readiness, language, learning preferences, and interests and to react responsively" (p. 1).

According to Tomlinson (2005), who has written extensively on differentiation, three elements guide differentiated instruction: content, process, and product. *Content* means that all students are given access to the same content but are allowed to master it in different ways. Process refers to the ways in which the content is taught. *Product* refers to how students demonstrate understanding. Corley (2005) provides three questions that drive differentiation: (1) What do you want the student to know? (2) How can each student best learn this? and (3) How can each student most effectively demonstrate learning? Maker (1986) offers a framework through which differentiation can occur in the classroom:

- Create an encouraging and engaging learning environment through student-centered activities, encouraging independent learning, accepting student contributions, using a rich variety of resources, and providing mobility and flexibility in grouping.
- Modify the content according to abstractness and complexity. Provide a variety of content and particularly content focused on people.
- Modify the learning process through use of inquiry, higher-order thinking activities, group interactions, variable pacing, creativity and student risk-taking, and freedom of choice in learning activities.
- Modify the product through facilitating different ways for students to demonstrate learning, such as the use of authentic assessments.

In addition, researchers have found that the use of flexible grouping and tiered instruction for differentiation increases student achievement (Corley, 2005; Tomlinson & Eidson, 2003). Heacox (2002) describes differentiation as follows:

The focus is not on the adjustment of the students, but rather the adjustment of teaching and instructional strategies making it about learning, not teaching. The teacher is the facilitator who…puts students at the center of teaching and learning and lets his or her students' learning needs direct instructional planning (p. 1).

Several studies conducted in elementary and middle school classroom have found that student achievement is increased in differentiated classrooms (Connor, Morrison, & Katch 2004; McAdamis, 2001). Tomlinson and Eidson (2003) emphasize the need to include the components of student readiness, student interest, and student learning profile in differentiating instruction. Students' interests and learning profiles are often tied to their learning styles.

Learning Styles

The body of research on learning styles has coalesced around the work of Howard Gardner, who introduced the theory of multiple intelligences in 1983. Gardner's work suggests that the concept of a pure intelligence that can be measured by a single I.Q. score is flawed, and he has identified nine intelligences that people possess to various degrees. His theory asserts that a person's type of intelligence determines how he or she learns best (Gardner, 1999).

Learning style refers to how a student learns, and the concept takes into account cultural background and social and economic factors as well as multiple intelligences. Beishuizen and Stoutjesdjik (1999) define *learning style* as a consistent mode of acquiring knowledge through study, or experience. Research has shown that the quality of learning at all levels of education (primary, secondary, and higher education) is enhanced when instruction and curriculum take into account individual learning styles (Dunn, Griggs, Olsen, Beasley & Gorman, 1995). Another study found that student learning improved when the learning environment was modified to allow students to construct personally relevant knowledge and to engage in the materials at different levels and from different points of view (Dearing, 1997).

A responsive classroom environment considers the individual learning needs of all students. These learning needs include a variety of factors that influence how students learn: culture, language, developmental level, readiness, social and economic background, and learning style.

Creativity

Creativity is an essential component for creating an engaging and accessible classroom environment. The Wisconsin Task Force on Arts and Creativity in Education (2009) defines *creativity* as a process that combines "imagination, creativity, and innovation to produce something novel that has value" (p. 14). Sir Ken Robinson (2011) and Daniel Pink (2006) both support the need for schools to focus on creating classroom that foster this type of creativity in students. According to Robinson (2011), classrooms that foster creativity and allow students to question assumptions, look at content through various lenses, and create new understandings can help students be more successful in postsecondary education and the workplace.

Probing Questions

- Describe two or three ways you might differentiate the instruction in your classroom. How might you share this with a new teacher?
- How might you implement a simple strategy for assessing your students' learning styles?

Resources

ASCD offers a number of resources on differentiated instruction, including work by Carol Ann Tomlinson, at http://www.ascd.org.

For resources on culturally responsive teaching, the Center for Culturally Responsive Teaching and Learning can be accessed at http://www.culturallyresponsive.org/.

The website of the National Center for Culturally Responsive Education Systems (NCCRESt) can be accessed at http://www.nccrest.org.

For learning styles and resources on multiple intelligences, Thomas Armstrong hosts a website with information on Gardner's Theory of Multiple Intelligences and related teaching resources at http://www.thomasarmstrong.com/multiple_intelligences.php.

Creativity: Its Place in Education is a report that offers suggestions for creative classrooms and teaching. This report can be found at http://www.jpb.com/creative/Creativity_in_Education.pdf.

The report of the Wisconsin Task Force on Arts and Creativity in Education offers recommendations for policy and practice. This report can be found at ftp://doaftp04.doa.state.wi.us/doadocs/taskforce_report_final2009pdf.

References

Au, K. H., & Kawakami, A. J. (1994). Cultural congruence in instruction. In E. R. Hollins, J. E. King, & W. C. Hayman (Eds.), *Teaching diverse populations: Formulating a knowledge base* (p. 5–23). Albany: State University of New York Press.

Banks, J. A., & Banks, C. A. (2001). *Multicultural education: Issues and perspectives* (4th ed.). New York: Wiley.

Beishuizen, J. J., & Stoutjesdjik, E. T. (1999). Study strategies in a computer assisted study environment. *Learning and Instruction*, 9(3), 281–301.

Connor, C. M., Morrison, F. J., & Katch, L. E. (2004). Beyond the reading wars: Exploring the effect of child-instruction interactions on growth in early reading. *Scientific Studies of Reading*, 8(2), 305–336.

Corley, M. (2005). Differentiated instruction: Adjusting to the needs of all learners. *Focus on Basics: Connecting Research and Practice*, 7(C), 13–16.

Dearing, R. (1997). *Higher education in the learning society: Report of the National Committee*. London: HMSO.

Delgado-Gaitan, C., & Trueba, H. (1991). *Crossing cultural borders: Education for immigrant families in America*. London: Falmer.

Dunn, R., Griggs, S., Olsen, J., Beasley, M., & Gorman, B. (1995). A meta-analytic validation of the Dunn and Dunn model of learning-style preferences. *Journal of Educational Research*, 88(6), 353–362.

Gardner, H. (1999). *Intelligence reframed: Multiple intelligences for the 21st century*. New York: Basic Books.

Gay, G. (2000). *Culturally responsive teaching: Theory, research, and practice*. New York: Teachers College Press.

Gay, G. (2002). Preparing for culturally responsive teaching. *Journal of Teacher Education*, 53(2) 106–116.

Heacox, D. (2002). *Differentiating instruction in the regular classroom: How to reach and teach all learners, Grades 3–12*. Minneapolis, MN: Free Spirit.

Hilliard, A. G., III. (1991). Why we must pluralize the curriculum. *Educational Leadership*, 49(4), 12–16.

Hollins, E. R., & Oliver, E. I. (1999). *Pathways to success in school: Culturally responsive teaching*. Mahwah, NJ: Erlbaum.

Ladson-Billings, G. (1994). *The dreamkeepers: Successful teachers of African American children*. San Francisco: Jossey-Bass.

Ladson-Billings, G. (1995). Toward a theory of culturally relevant pedagogy. American *Educational Research Journal*, 32(3), 465–491.

Maker, C. J. (1986). *Critical issues in gifted education: Defensible programs for the gifted.* Rockville, MD: Aspen.

McAdamis, S. (2001). Teachers tailor their instruction to meet a variety of student needs. *Journal of Staff Development*, 22(2), 1–5.

Mulroy, H., & Eddinger, K. (2003, June). *Differentiation and literacy*. Paper presented at the Institute on Inclusive Education, Nazareth College of Rochester, Rochester, NY.

Nieto, S. (1999). *The light in their eyes: Creating multicultural learning opportunities*. New York: Teachers College Press.

Pewewardy, C. H., & Cahape, P. (2003). Culturally responsive teaching for American Indian students. *ERIC Digest*. Retrieved June 3, 2011, from http://www.ericdigests.org/2005-1/teaching.htm

Pink, D. H. (2006). *A whole new mind: Why right-brainers will rule the future*. New York: Riverhead.

Robinson, Ken. (2011). *Out of our minds: Learning to be creative*. West Sussex, United Kingdom: Capstone.

Smith, G. P. (1998). *Common sense about common knowledge*: The knowledge bases for diversity. Washington, DC: American Association of Colleges for Teacher Education.

Tharp, R. G., & Gallimore, R. (1988). *Rousing minds to life: Teaching, learning, and schooling in social context*. Cambridge: England: Cambridge University Press.

Tomlinson, C. A. (2005). Grading and differentiation: Paradox or good practice? *Theory Into Practice*, 44(3) 262–269.

Tomlinson, C. A., & Eidson, C. C. (2003). *Differentiation in practice: A resource guide for differentiating curriculum. Grades 5–9*. Alexandria, VA: Association for Supervision and Curriculum Development.

Wisconsin Task Force on Arts and Creativity in Education. (2009). *A plan for action*. Madison: Wisconsin Department of Public Instruction. Retrieved June 3, 2011, from ftp://doaftp04.doa.state.wi.us/doadocs/taskforce_report_final2009.pdf