Wisconsin Student Assessment System

Criterion-Referenced Test Framework

For the fall Wisconsin Knowledge and Concepts Examinations Statewide Assessment

Assessment Framework for

Mathematics

In Grades 3 through 8 and 10

A QUALITY EDUCATION FOR EVERY CHILD

New Wisconsin Promise

Elizabeth Burmaster, State Superintendent
Wisconsin Department of Public Instruction

CTB/McGraw-Hill
Test Development Contractor

January, 2005

This document provides an indication of the range of coverage on the math portion of the Wisconsin Knowledge and Concepts Examination that will be administered statewide in Wisconsin each November beginning in 2005 in grades 3 through 8 and 10. It is intended to foster discussion among educators and others across grades and across subject areas. It should be used in conjunction with the Wisconsin Model Academic Standards for Mathematics and your local curriculum.
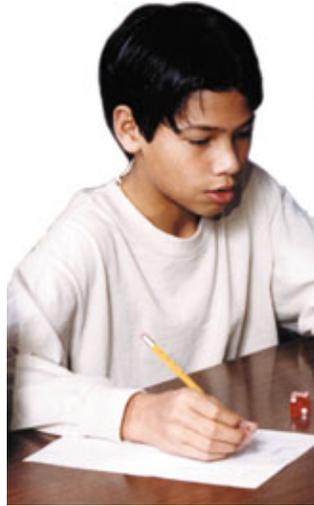
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Overview

Beginning in the 2005–2006 school year, the federal No Child Left Behind Act requires all states to test all students in reading and mathematics in grades 3 through 8 and once in high school (grade 10 under Wisconsin law s. 118.30). These tests are referred to as the Wisconsin Knowledge and Concepts Examination (WKCE) and will replace WKCE reading and mathematics tests beginning in fall 2005. Student performance on these tests is reported in proficiency categories and used to determine the adequate yearly progress of students at the school, district and state levels.

The Wisconsin Department of Public Instruction published a request for proposals to support the development of customized criterion-referenced reading and mathematics tests to be vertically scaled over grades 3 through 8 and grade 10. CTB/McGraw-Hill was awarded the contract. Panels of Wisconsin teachers began meeting during the 2003–2004 school year to select reading passages, establish grade-level descriptors for reading and mathematics and review (accept/reject/edit) all customized items developed by the contractor.

CTB/McGraw-Hill conducted item pilot testing in May 2004 and forms calibration in December 2004 based on a stratified sampling design drawing from all public schools in the state. The Wisconsin Department of Public Instruction also contracted with three national experts to evaluate the work of CTB/McGraw-Hill as well as to advise the department on issues of validity and reliability of the new WKCE test design for reading and mathematics. This Technical Advisory Committee (TAC) met initially in February 2004 and will meet twice annually in the future to assure the continued technical validity of the tests.

General Test Specifications

All test items developed for the new WKCE tests in reading and mathematics are either selected-response (multiple-choice) or constructed-response format. The test reporting categories and items assigned to measure each reporting category are aligned to the Wisconsin Model Academic Standards in reading and mathematics with grade-level appropriate descriptors supporting learning expectations for tests administered in the fall semester. The test design draws approximately 80% of the total score points from selected-response items and 20% of the score points from student-generated constructed-response items.

All students in grades 3 through 8 and 10 will be tested in reading and mathematics using these new customized WKCE tests beginning in fall 2005. Students with disabilities will be allowed accommodations during these tests unless an alternate assessment is required based on an IEP process. Students whose English language proficiency as tested on state-approved language proficiency examinations is level three or higher will take the WKCE tests with allowable accommodations. English language learners with language proficiency scores less than three will take an alternate assessment. All alternate assessments are aligned to state standards.

Students in grades 4, 8 and 10 will continue to be assessed in language arts, science and social studies as required by s. 118.30 Wisconsin Statutes. These assessments will be a shelf-test provided under the terms of the department's contract with CTB/McGraw-Hill.

Table 1. Mathematics Assessment Framework Objectives and Subskills.

WKCE MATHEMATICS

REPORTING CATEGORIES
Objectives and Sub-skills

Objective: A. MATHEMATICAL PROCESS
- **Reasoning**
- **Communication**
- **Connections**
- **Representation**
- **Problem Solving**

Objective: B. NUMBER OPERATIONS AND RELATIONSHIPS
Sub-skills:

- **B.a. Number Concepts**
- **B.b. Number Computation**

Objective: C. GEOMETRY
Sub-skills:

- **C.a. Describing Figures**
- **C.b. Spatial Relationships and Transformations**
- **C.c. Coordinate Systems**

Objective: D. MEASUREMENT
Sub-skills:

- **D.a. Measurable Attributes**
- **D.b. Direct Measurement**
- **D.c. Indirect Measurement**

Objective: E. STATISTICS AND PROBABILITY
Sub-skills:

- **E.a. Data Analysis and Statistics**
- **E.b. Probability**

Objective: F. ALGEBRAIC RELATIONSHIPS
Sub-skills:

- **F.a. Patterns, Relations and Functions**
- **F.b. Expressions, Equations and Inequalities**
- **F.c. Properties**

Table 2. Mathematics Assessment Framework Objectives and Subskills.

WKCE MATHEMATICS
REPORTING CATEGORIES
(80% of score points: selected response/20% constructed-response)

OBJECTIVES AND SUB-SKILLS	**Estimated Percentage of Score Points per Grade**						
	Gr3	**Gr4**	**Gr5**	**Gr6**	**Gr7**	**Gr8**	**Gr10**
A. MATHEMATICAL PROCESS Reasoning Communication Connections Representation Problem Solving	**15**	**18**	**18**	**19**	**19**	**22**	**18**
B. NUMBER OPERATIONS AND RELATIONSHIPS Number Concepts Number Computation	**21**	**19**	**18**	**19**	**20**	**14**	**10**
C. GEOMETRY Describe Figures Spatial Relationships and Transformations Coordinate Systems	**19**	**16**	**16**	**16**	**17**	**14**	**16**
D. MEASUREMENT Measurable Attributes Direct Measurement Indirect Measurement	**15**	**16**	**16**	**15**	**14**	**16**	**16**
E. STATISTICS AND PROBABILITY Data Analysis and Statistics Probability	**15**	**15**	**16**	**15**	**14**	**14**	**20**
F. ALGEBRAIC RELATIONSHIPS Patterns, Relations and Functions Expressions, Equations and Inequalities Properties	**15**	**16**	**16**	**16**	**16**	**20**	**20**

Figure 1. Distribution of WKCE mathematics score points by grade. Bands indicate percent of score points at grade level related to each objective

WKCE-CRT Mathematics Assessment Blueprint
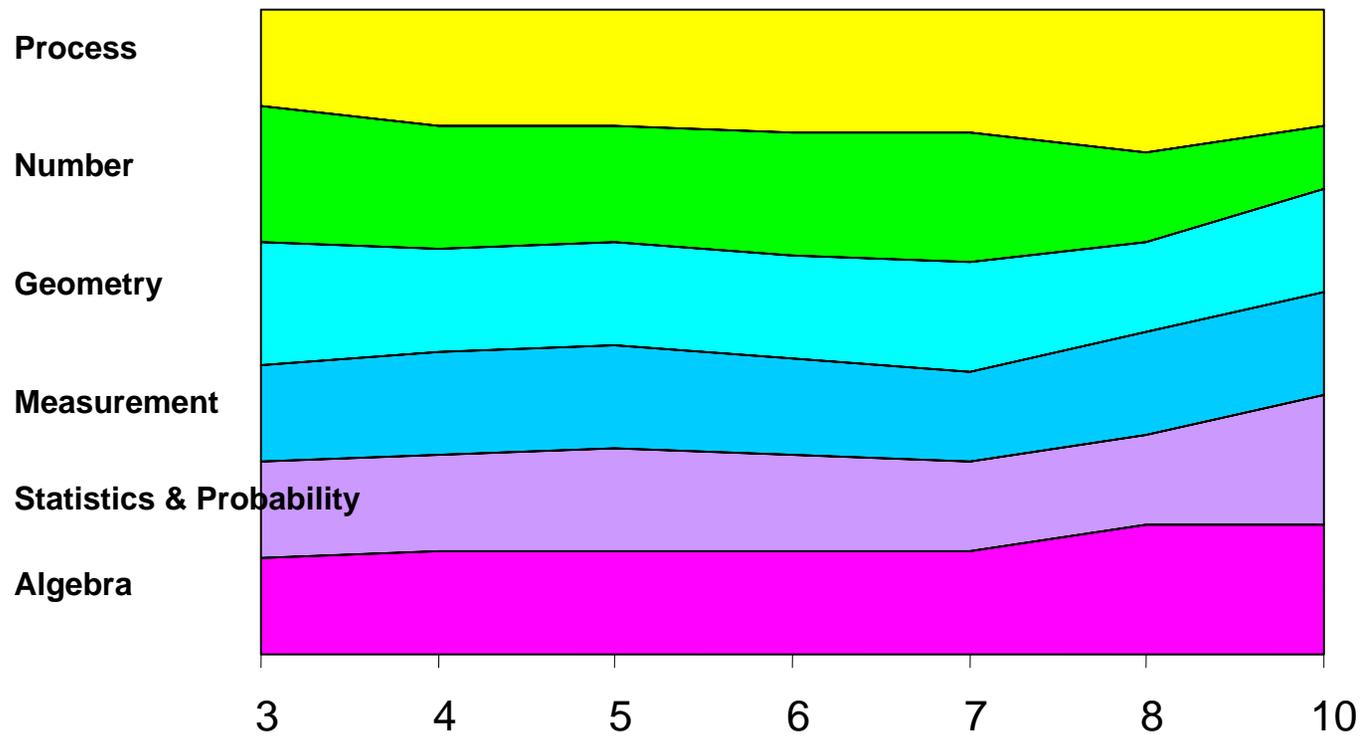
Frequently Asked Questions

What is the framework?

In order to develop a customized test aligned with Wisconsin standards, a diverse group of educators from around the state, content-area specialists from the Department of Public Instruction (DPI), and test developers from the test contractor met and developed a detailed plan for the test, called the test blueprint. The framework is derived from this test blueprint and was developed by the DPI to provide information about the range and coverage of the WKCE at each grade. Figure 2 indicates that the test blueprint is based on the academic standards, and that the WKCE and the framework are based on the test blueprint. As we gain experience with the WKCE, we may gain insight that leads us to revise our test blueprint.

Figure 2. Relationships among the Wisconsin Model Academic Standards, test blueprint, test, and framework.
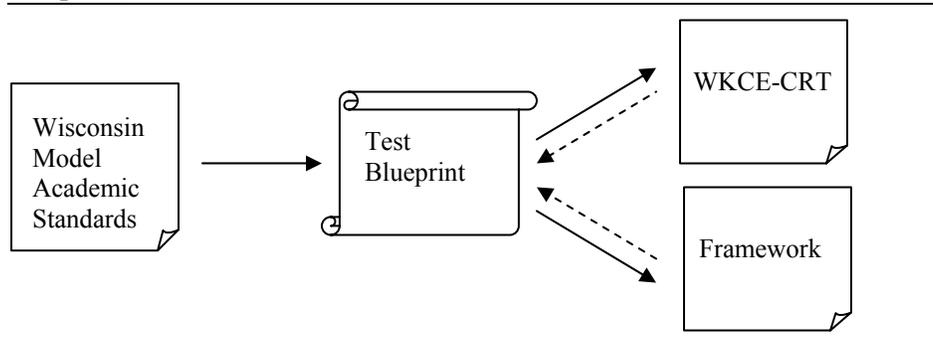
How can I use the framework?

One way you can use the framework is to ensure that your local curriculum includes the knowledge and skills described in the framework. However, since the mathematics assessment framework is only an indication of the knowledge and skills that will be assessed on the November WKCE, *this information does not replace your local curriculum.*

Another way to use the framework is as a basis for teachers to engage in multi-grade-level discussions. Since the test is administered in the fall, students should have an opportunity to acquire the knowledge and skills that will be assessed prior to the tested grade. Similarly, teachers will want to examine test results from the next-higher grade level for feedback on what is happening at their own grade level, as illustrated in the example in Figure 3.

Figure 3. Knowledge and skills assessed at one grade must be part of the curriculum prior to that grade.
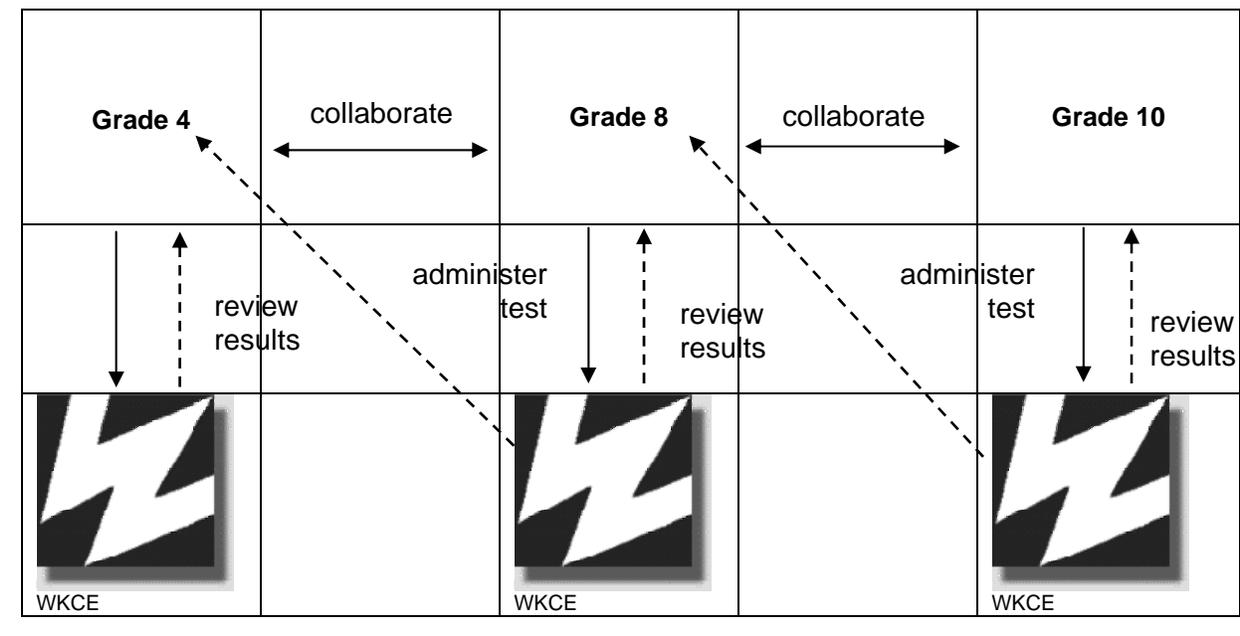
Do I need to teach something if it isn't assessed at my grade level?

Yes. You may want to ensure that you introduce students to the knowledge and skills that will be assessed at least one year (or more) before they are assessed. On the other hand, even if something is no longer assessed you may need to teach it if students haven't mastered it because it is assumed to be known and because it may be prerequisite for something that is assessed.

Does the mathematics assessment framework replace our local curriculum or the Wisconsin Model Academic Standards for Mathematics?

The mathematics assessment framework is an indication of the knowledge and skills that will be assessed on the November WKCE. *This information does not replace your local curriculum or the Model Academic Standards*. You may wish to ensure that your local curriculum includes – *but is not limited to* – the knowledge and skills described in the framework.

Table 4 on the following page is intended to help foster discussion among educators about local curriculum, state standards, and the framework.

To assist in curriculum planning, the Wisconsin Department of Public Instruction has prepared a comprehensive guide "*Planning Curriculum in Mathematics*."

How does the framework relate to our local curriculum?

If your local curriculum is linked to the Wisconsin Model Academic Standards then it is also linked to the framework, because the mathematics framework is based on the Wisconsin Model Academic Standards. However, since the framework provides additional information about what may be assessed at each grade level, you may wish to ensure that your local curriculum includes the knowledge and skills described in the framework. *Note, however, that the framework does not replace your local curriculum.* Figure 4 shows the relationships among the framework, local curriculum, and state standards.

Figure 4. Suggested relationships among the Wisconsin Model Academic Standards, mathematics assessment framework, and local curriculum, instruction, and assessment. Solid arrows indicate direct influence, and dotted arrows indicate indirect or recommended influence.
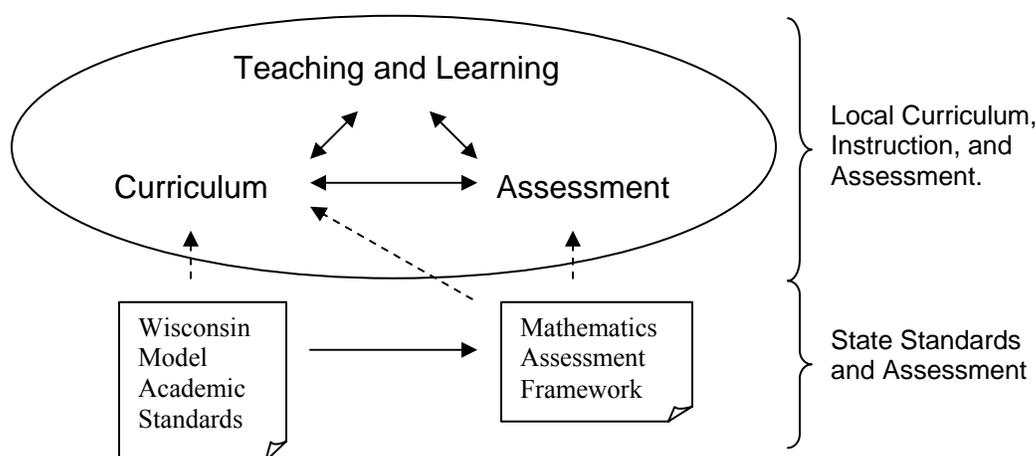
What is a criterion-referenced test?

This term refers to the way test results are interpreted. A criterion-referenced interpretation of an assessment relates a student's performance to specific performance criteria, rather than to the performance of other students (which would be a norm-referenced interpretation). Wisconsin has defined five proficiency categories: pre-requisite skill, minimal performance, basic, proficient, and advanced. A combination of professional judgment (involving educators from around Wisconsin) and statistical analysis is used to link assessment scores with proficiency levels.

What kinds of questions are on the test?

There are both selected-response (multiple-choice) and constructed-response (short answer) items on the WKCE. Approximately 80% of a student's score points will come from selected-response items, and 20% from constructed-response items.

How can I help my students prepare for the test?

The best test preparation involves a rich, engaging curriculum coupled with ongoing assessment that is integrated into instruction. Students should be familiar with the kinds of items they will see on the test and with general test-taking strategies, but this should not be a major focus of instruction.

Characteristics of Mathematics Assessment

Constructed response—three score points: one point reports to assigned content category and two points report to the mathematical process category.

Mathematics Tools (vary by grade)

    Manipulatives (provided by test contractor)**:** pattern blocks, tangrams, pentomino
    Measuring Tools (provided by test contractor)**:** ruler, protractor

Grade Level	Tools	Tool Features
3	ruler (U.S. customary and metric) pattern blocks	ruler interval: 1/2 inch, centimeter
4	ruler (U.S. customary and metric) pattern blocks pentomino (one asymmetrical shape used for transformational geometry)	ruler interval: 1/4 inch, centimeter
5	ruler (U.S. customary and metric) pattern blocks	ruler interval: 1/8 inch, millimeter
6	ruler (U.S. customary and metric) protractor tangrams	ruler interval: 1/16 inch, millimeter
7	ruler (U.S. customary and metric) protractor	ruler interval: 1/16 inch, millimeter
8	ruler (U.S. customary and metric) protractor	ruler interval: 1/16 inch, millimeter
10	ruler (U.S. customary and metric) protractor	ruler interval: 1/16 inch, millimeter

    Calculators (vary by grade, provided by school district)

- **GRADES 3 and 4**
  Calculator use is **PROHIBITED** for all sessions of the test.

- **GRADES 5, 6, 7 and 8**
  Four-function calculator availability is **REQUIRED** for most sessions of the test.

  - All students may use a scientific calculator.
  - Districts may permit use of a graphing calculator.

    1. Graphing calculator memory must be cleared.
    2. Calculators with QWERTY keyboards, infrared capabilities and those that perform symbolic manipulation are not permitted.

- **GRADE 10**
  Scientific or graphing calculator availability is **<u>REQUIRED</u>** for most sessions of the test.

  - Districts may permit use of a graphing calculator.

    1. Graphing calculator memory must be cleared.
    2. Calculators with QWERTY keyboards, infrared capabilities, and those that perform symbolic manipulation are not permitted.

<u>Suggestions for managing calculator use and test security</u>.

- borrow classroom sets from grades or academic areas not being tested

- encourage teachers to have students bring their own calculator on test day

- provide teachers with extra calculators to use if needed

  1. borrow calculators from other grades or classes

  2. purchase additional calculators

- at grades 5-8, avoid graphing calculator use during pilot testing in May and December 2004

  1. if teachers are unfamiliar with clearing memory

  2. if students do not regularly use them in class

Graphing calculator use is a <u>district decision</u> to be made before the Fall 2005 WKCE testing window. Use of graphing calculators should be based on the district's mathematics curriculum as well as customary classroom practice. Graphing calculators offer no advantage to test takers, but students in some grades and in some classrooms may be more familiar with such calculators where they are routinely used. The memory function of all graphing calculators permitted at a testing site must be cleared both <u>before</u> and <u>after</u> each testing session.

The information on the following pages provides a framework to describe what skills in mathematics are being tested at each grade. The descriptions of the reporting categories by objectives and sub-skills provide information about the range and coverage of the mathematics test at each grade. The descriptors represent the item content specifications used to develop customized items for each test form at each grade. The descriptors further offer teachers insight into the test content in order to develop instructional strategies to prepare students for successful performance on the test.

Wisconsin Knowledge and Concepts Examinations
Criterion Referenced Test
Mathematics Rubrics

Grades 3-8 Rubric for Brief Constructed Response Questions

2 points   The student demonstrates a thorough understanding of the mathematical concepts and/or procedures represented in the problem. The student uses appropriate mathematical procedures and/or concepts to explain or justify the response to Step A, and provides clear and complete explanations and interpretations containing words, calculations, or symbols, unless otherwise specified in the item stem.

The response may contain minor flaws that do <u>not</u> detract from the demonstration of a thorough understanding of the problem.

1 point   The student demonstrates only a partial understanding of the mathematical concepts and/or procedures represented in the problem. The response lacks an essential understanding of the underlying mathematical concepts used to provide the response to Step A.

The response contains errors related to the misinterpretation of important aspects of the problem, misuse of mathematical procedures and/or concepts, or misinterpretation of results.

0 points   The student provides a completely incorrect explanation or justification, or one that cannot be interpreted.

At grades 3-8, the brief constructed response questions are worth a total of 3 points which are assigned as follows:

- 1 point: Mathematical Content
- 2 points: Mathematical Process

The item bank contains constructed response items from all mathematics objectives:

- Mathematical Process
- Number Operations and Relationships
- Geometry
- Measurement
- Statistics and Probability
- Algebraic Relationships

Wisconsin Knowledge and Concepts Examinations

Criterion Referenced Test

Mathematics Rubrics

Grade 10 Rubric for Brief Constructed Response Questions

2 points The student demonstrates a thorough understanding of the mathematical concepts and/or procedures represented in the problem. The student responds correctly to the problem, uses mathematical procedures and/or concepts, and provides clear and complete explanations and interpretations containing words, diagrams, or calculations unless otherwise specified. The response may contain minor flaws that do not detract from the demonstration of a thorough understanding of the problem.

1 point The student provides a response that is only partially correct. The student provides a correct solution, but may demonstrate a misunderstanding of the underlying mathematical concepts and/or procedures. The student provides a correct solution, but in place of showing his/her work writes, "I used my calculator." The student provides a thorough demonstration of understanding the problem, but states an incorrect solution or conclusion.

0 points The student provides a completely incorrect solution, a response that cannot be interpreted, or no response at all.

Grade 10 Rubric for Extended Constructed Response Questions

4 points The student demonstrates a thorough understanding of the mathematical concepts and/or procedures represented in the problem. The student responds correctly to the problem, uses mathematical procedures and/or concepts, and provides clear and complete explanations and interpretations containing words, diagrams, or calculations unless otherwise specified.

The response may contain minor flaws that do <u>not</u> detract from the demonstration of a thorough understanding of the problem.

3 points The student demonstrates an understanding of the mathematical concepts and/or procedures represented in the problem. The student's response to the problem is essentially correct. The mathematical procedures and/or concepts used and the explanations and interpretations provided demonstrate an essential but less than thorough understanding of the problem.

3 points   (continued)

The response may contain minor errors that reflect inattentive execution of mathematical procedures and/or concepts, or minor errors indicating of some misunderstanding of the underlying mathematical concepts and/or procedures.

2 points   The student demonstrates only a partial understanding of the mathematical concepts and/or procedures represented in the problem. Although the student may have used the correct approach to obtain a solution or may have provided a correct solution, the response lacks an essential understanding of the underlying mathematical concepts.

The response contains errors related to the misinterpretation of important aspects of the problem, misuse of mathematical procedures and/or concepts, or misinterpretation of results.

1 point   The student demonstrates a very limited understanding of the mathematical concepts and/or procedures represented in the problem. The response is incomplete and exhibits many flaws. Although the response may have addressed some of the conditions of the problem, the conclusion is inadequate and/or includes reasoning that was faulty or incomplete.

The response exhibits many errors or may be incomplete.

0 points   The student provides a completely incorrect solution, a response that cannot be interpreted, or no response at all.

* The student will not receive points for writing, "I used my calculator" on any part of the problem in place of showing his/her work.

The item bank for brief constructed response and extended constructed response questions contains items from all mathematics objectives:

- Mathematical Process

- Number Operations and Relationships

- Geometry

- Measurement

- Statistics and Probability

- Algebraic Relationships

Whom can I contact with questions or suggestions?

We welcome your questions and suggestions! Please contact:

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            Mailing address:

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            Madison, WI 53707-7841

There is a questionnaire at the end of this document with some questions that are of interest to us for evaluating the mathematics framework. Please take a moment to complete the survey and return it to us.

Grade Level Framework

Beginning of Grade 3

How to use the Framework

The mathematics assessment framework is an indication of the knowledge and skills that will be assessed on the November WKCE. ***This information does not replace your local curriculum.*** However, you may wish to ensure that your local curriculum includes the knowledge and skills described in the framework.

This section of the framework describes the types of content that students may encounter on the WKCE

The knowledge and skills to be assessed are organized into objectives, subskills, and descriptors as shown below. WKCE results will be reported by objectives and subskill.

A. **Objective:** A group of cognitively related skills.
    A.a. **Subskill:** A group of related knowledge and skills that ***may include, but is not limited to,*** the descriptors that follow.
    - **Descriptor:** an example of a specific knowledge or skill that may be assessed.

Objectives, Subskills, and Descriptors

Objective A:

Mathematical Processes

Students will effectively use mathematical knowledge, skills and strategies related to reasoning, communication, connections, representation and problem solving.

Descriptors, such as but not limited to

- Use reasoning and logic to:
    - Perceive patterns
    - Identify relationships
    - Formulate questions
    - Pose problems
    - Make conjectures
    - Justify strategies
    - Test reasonableness of results

- Communicate mathematical ideas and reasoning using the vocabulary of mathematics in a variety of ways e.g., using words, numbers, symbols, pictures, charts, tables, diagrams, graphs, and models.

- Connect mathematics to the real world, as well as within mathematics.

- Create and use representations to organize, record, and communicate mathematical ideas.

- Solve and analyze routine and non-routine problems.

Objective B:	**Number Operations and Relationships**

Subskill B.a.:	**Concepts**

Descriptors, such as but not limited to

- Recognize and apply place-value concepts to whole numbers less than 1,000
- Read, write, and represent numbers using words, numerals, pictures (e.g. base-ten blocks), number lines, , arrays, expanded forms (24=20+4) and symbolic renaming e.g., 24=30-6.
- Compare and order whole numbers less than 1,000
- Count by 2s, 3s, 5s, 10s, 25s and 100s
- Count, compare and make change using a collection of coins (up to one dollar) and one-dollar bills.
- Identify a fractional part of a collection/set.

  Read, write and represent fractional parts of a whole e.g., 1/4, 1/2.

Subskill B.b.:	**Computation**

Descriptors, such as but not limited to

- Use addition and subtraction in everyday situations and solve one-step word problems.
- Solve single and double-digit addition and subtraction problems with regrouping including horizontal format in problems with and without context.
- Demonstrate the concept of multiplication as grouping or repeated addition in context with products up to 50.
- Demonstrate understanding of the concept of division as repeated subtraction, partitioning/sharing or measuring (dividend up to 30 and divisors up to 5).
- Use fractions to represent quantities when solving problems involving equal sharing or partitioning.
- Represent with shaded circles, rods, squares, pictorial representations of a whole.
- Estimate sums to tens and hundreds and differences to ten.
- Determine reasonableness of answers.

Objective C:	**Geometry**

Subskill C.a.:	**Describing figures**

Descriptors, such as but not limited to

- Identify, describe, and compare properties of 2 and 3 dimensional figures such as squares, triangles, rectangles, circles, pattern block shapes, cubes, pyramids, rectangular prisms, cylinders, and spheres (e.g., comparing sides, faces, corners, and edges).

Subskill C.b.:	**Spatial relationships and transformations**

Descriptors, such as but not limited to

- Identify 2-dimensional geometric shapes created by combining or decomposing other shapes e.g., square/triangles; trapezoid/rhombus, triangle; hexagon/triangles, rhombus, trapezoid.
- Apply concepts of single-motion geometry (e.g., slides, flips and turns) to match two identical shapes.

Subskill C.c.:	**Coordinate systems**

Descriptors, such as but not limited to

- Use simple 2-dimensional coordinate systems to find locations on maps and to represent points and simple figures with coordinates of letters and numbers, (e.g., (E, 3)).

Objective D: **Measurement**

Subskill D.a.: **Measurable attributes**

Descriptors, such as but not limited to

- Describe attributes of length, time and temperature and identify appropriate units to measure them. Units include: inches, feet, yards, centimeters, meters, seconds, minutes, hours, days, months, years and degrees Fahrenheit/Celsius.

- Compare attributes of length and weight by observation or when given actual measurements.

Subskill D.b.: **Direct measurement**

Descriptors, such as but not limited to

- Read and interpret measuring instruments to determine the measurement of objects with non-standard and standard units to the nearest centimeter or 1/2-inch.

- Read thermometers to the nearest 5 degrees F/C.

- Tell time to the nearest minute using analog and digital clocks; translate time from analog to digital clocks and vice versa.

- Investigate measurements of area.

Subskill D.c.: **Indirect measurement**

Descriptors, such as but not limited to

- Apply estimation techniques using non-standard units.

Objective E: **Statistics and Probability**

Subskill E.a.: **Data analysis and statistics**

Descriptors, such as but not limited to

- Answer and pose questions about collecting, organizing and displaying data. Work with data in the context of real-world situations by determining what data to collect and when and how to collect it to answer questions.

- Collect, organize and display data in simple bar graphs and charts including translating data from one form to the other.

- Draw reasonable conclusions based on simple interpretations of data.

- Read, use information and draw reasonable conclusions from data in graphs, tables, charts and Venn diagrams.

Subskill E.b.: **Probability**

Descriptors, such as but not limited to

- Determine if the occurrence of future events are more, less or equally likely to occur.

- Choose a fair and an unfair spinner.

Objective F :	**Algebraic Relationships**

Subskill F.a.:	**Patterns, relations and functions**

Descriptors, such as but not limited to

- Recognize, extend, describe, create and replicate a variety of patterns including attribute, number and geometric patterns.
  Such as:
    - Picture patterns
    - Patterns in tables and charts
    - "What's-my–rule?" patterns
    - Patterns using addition and subtraction rules.

  Focusing on relationships within patterns as well as extending patterns e.g., patterns and relationships represented with pictures, tables and charts, and "what's-my–rule?" patterns using addition and subtraction rules.
- Determine odd or even with a total set of 20 or less.

Subskill F.b.:	**Expressions, equations and inequalities**

Descriptors, such as but not limited to

- Demonstrate an understanding that the "=" sign means "the same as" by solving open or true/false number sentences.
- Use notation to represent mathematical thinking: letter or box (variable); operation symbols (+, -, =).

Subskill F.c.:	**Properties**

Descriptors, such as but not limited to

- Use properties and or relationships of arithmetical thinking to determine and to reason about what number goes in a "box" to make a number sentence true,
    - identity property of e.g., zero Ex: property $12 + 0 =$ "box"
    adding 1 to any number, commutative property for addition of single-digits
- Use simple equations in a variety of ways to demonstrate the properties above.

Grade Level Framework

Beginning of Grade 4

How to use the Framework

The mathematics assessment framework is an indication of the knowledge and skills that will be assessed on the November WKCE. *This information does not replace your local curriculum.* However, you may wish to ensure that your local curriculum includes the knowledge and skills described in the framework.

This section of the framework describes the types of content that students may encounter on the WKCE

The knowledge and skills to be assessed are organized into objectives, subskills, and descriptors as shown below. WKCE results will be reported by objectives and subskill.

- A. **Objective:** A group of cognitively related skills.
   A.a. **Subskill:** A group of related knowledge and skills that *may include, but is not limited to,* the descriptors that follow.
   - **Descriptor:** an example of a specific knowledge or skill that may be assessed.

Objectives, Subskills, and Descriptors

Objective A:	**Mathematical Processes**

Students will effectively use mathematical knowledge, skills and strategies related to reasoning, communication, connections, representation and problem solving.

Descriptors, such as but not limited to

- Use reasoning and logic to:
  - Perceive patterns
  - Identify relationships
  - Formulate questions
  - Pose problems
  - Make conjectures
  - Justify strategies
  - Test reasonableness of results

- Communicate mathematical ideas and reasoning using the vocabulary of mathematics in a variety of ways e.g., using words, numbers, symbols, pictures, charts, tables, diagrams, graphs, and models.

- Connect mathematics to the real world, as well as within mathematics.

- Create and use representations to organize, record, and communicate mathematical ideas.

- Solve and analyze routine and non-routine problems.

Objective B:	**Number Operations and Relationships**
Subskill B.a.:	**Concepts**

Descriptors, such as but not limited to

- Recognize and apply place-value concepts to whole numbers less than 10,000.

- Read, write, and represent numbers using words, numerals, pictures (e.g. base ten blocks), number lines, , arrays, expanded forms (243=200+40+3) and symbolic renaming e.g., 243=250-7.

- Compare and order whole numbers less than 10,000

- Count by 2s, 3s, 5s, 10s, 25s and 100s starting with any multiple and 100s starting with any number.
  - Identify and name counting patterns
- Count, compare and make change up to $10.00 using a collection of coins and one-dollar bills..

- Identify a fractional part of a collection/set or parts of a whole.

Read, write, order and represent unit fractions (e.g.,1/2, 1/3, 1/4) and part(s) of a set.

Subskill B.b.:   **Computation**

Descriptors, such as but not limited to

- Use addition and subtraction in everyday situations and solve one-and two-step word problems

- Solve double-and triple-digit addition and subtraction problems with regrouping in horizontal and vertical format in problems with and without context.

- Demonstrate understanding of multiplication as grouping or repeated addition or arrays in problems with and without context (without context up to 5 x 9; in context products up to 100).

- Demonstrate understanding of the concept of division as repeated subtraction, partitioning/sharing or measuring (dividend up to 45 and divisors up to 5).

- Use fractions to represent quantities when solving problems involving equal sharing or partitioning including fractions less than one as well as mixed numbers.

Represent with shaded circles, rods, squares or pictorial representations of objects (for a set).
- Estimate sums to tens, hundreds and thousands and differences of ten and hundreds.

- Determine reasonableness of answers.

Objective C: **Geometry**

Subskill C.a.:   **Describe figures**

Descriptors, such as but not limited to;
- Identify, describe, and compare properties of 2 and 3 dimensional figures such as squares, triangles, rectangles, pentagon, hexagon, octagon, pattern block shapes, circles, cubes, pyramids, rectangular prisms, tetrahedrons, cylinders, and spheres (e.g., comparing sides, faces, corners, and edges).

Subskill C.b.:   **Spatial relationships and transformations**

Descriptors, such as but not limited to
- Create and identify 2-dimensional geometric shapes by combining or decomposing other shapes.

- Identify cubes and square pyramid shapes from their nets (flat patterns).

- Apply concepts of single-motion geometry (e.g., slides, flips and turns) to match two identical shapes.

Subskill C.c.:   **Coordinate Systems**

Descriptors, such as but not limited to
- Use simple 2-dimensional coordinate systems to find locations on maps and to represent points and simple figures with coordinates using letters and numbers, (e.g., (E, 3)).

- Identify and use relationships among figures (e.g., location, position and intersection).

Objective D:	**Measurement**

Subskill D.a.:	**Measurable attributes**

Descriptors, such as but not limited to

- Describe attributes of length, time, temperature, liquid capacity, weight/mass, volume and identify appropriate units to measure them. Units include: inches, feet, yards, miles, meters, centimeters, millimeters, cups quarts, gallons, liters, seconds, minutes, hours, days, months, years, ounces, pounds, grams and degrees Fahrenheit/Celsius.

- Compare attributes of length, volume and weight by observation or when given actual measurements.

- Make measurement conversions within a system (e.g., yards to feet; feet to inches; hours to minutes; days to hours; years to months; gallons to quarts).

Subskill D.b.:	**Direct measurement**

Descriptors, such as but not limited to

- Read and interpret and use measuring instruments to determine the measurement of objects with non-standard and standard units to the nearest centimeter, 1/4-inch.

- Read thermometers to the nearest 5 degrees F/C.

- Tell time to the nearest minute and translate time from analog to digital clocks and vice versa.

- Determine and compare elapsed time in multiples of 15 minutes in problem-solving situations.

- Investigate measurements of area and perimeter.

Subskill D.c.:	**Indirect measurement**

Descriptors, such as but not limited to

- Apply estimation techniques using non-standard units.

Objective E:	**Statistics and Probability**

Subskill E.a.:	**Data analysis and statistics**

Descriptors, such as but not limited to

- Answer and pose questions about collecting, organizing and displaying data. Work with data in the context of real-world situations by formulating questions that lead to data collection and analysis and determining what data to collect and when and how to collect the data.

- Collect, organize and display data in simple bar graphs and charts, including translating data from one form to the other.

- Draw reasonable conclusions based on simple interpretations of data.

- Read, use information and draw reasonable conclusions from data in graphs, tables, charts and Venn diagrams.

Subskill E.b.:	**Probability**

Descriptors, such as but not limited to

- Determine if the occurrence of future events are more, less or equally likely to occur.

- Design a fair and an unfair spinner.

- Predict the outcomes of a simple event using words to describe probability.
  Ex: Flipping a coin has a 1 out of 2 chance of getting a head.

- Describe and determine the number of combinations for choosing 2 out of 3 items.
  Ex: Red hat, blue jacket and green jacket. What are the combinations of wearing a hat and a jacket?

Objective F:	**Algebraic Relationships**
Subskill F.a.:	**Patterns, relations and functions**

Descriptors, such as but not limited to

- Recognize, extend, describe, create and replicate a variety of patterns including attribute, number and geometric patterns.
  Such as:
    - Picture patterns
    - Patterns in tables and charts
    - "What's-my–rule?" patterns
    - Patterns using addition and subtraction rules.

  Focusing on relationships within patterns as well as extending patterns e.g., patterns and relationships represented with pictures, tables and charts; "what's-my–rule?" patterns using addition and subtraction rules.

- Determine odd or even.

Subskill F.b.:	**Expressions, equations and inequalities**

Descriptors, such as but not limited to

- Demonstrate an understanding that the " =" sign means "the same as" by solving open or true/false number sentences.
- Use notation to represent mathematical thinking: letter or box (variable); operation symbols (+, - , =).
- Demonstrate a basic understanding of equality and inequality using symbols ($<, >, =$) with simple addition and subtraction.

Subskill F.c.:	**Properties**

Descriptors, such as but not limited to

- Use properties and relationships of arithmetic to determine what number goes in a "box" to make a number sentence true,
    - Identity property of zero Ex: $12 + 0 =$ "box"
    - Identity property of one Ex: $5 \times 1 =$ "box"
    - Commutative property for addition of single-digits
    - Associative property

- Use simple equations in a variety of ways to demonstrate the properties above.

Grade Level Framework

Beginning of Grade 5

How to use the Framework

The mathematics assessment framework is an indication of the knowledge and skills that will be assessed on the November WKCE. *This information does not replace your local curriculum*. However, you may wish to ensure that your local curriculum includes the knowledge and skills described in the framework.

This section of the framework describes the types of content that students may encounter on the WKCE

The knowledge and skills to be assessed are organized into objectives, subskills, and descriptors as shown below. WKCE results will be reported by objectives and subskill.

- A. **Objective:** A group of cognitively related skills.
    - A.a. **Subskill:** A group of related knowledge and skills that *may include, but is not limited to,* the descriptors that follow.
        - **Descriptor:** an example of a specific knowledge or skill that may be assessed.

Objectives, Subskills, and Descriptors

Objective A:

Mathematical Processes

Students will effectively use mathematical knowledge, skills and strategies related to reasoning, communication, connections, representation and problem solving.

Descriptors, such as but not limited to

- Use reasoning and logic to:
    - Perceive patterns
    - Identify relationships
    - Formulate questions
    - Pose problems
    - Make conjectures
    - Justify strategies
    - Test reasonableness of results
- Communicate mathematical ideas and reasoning using the vocabulary of mathematics in a variety of ways e.g., using words, numbers, symbols, pictures, charts, tables, diagrams, graphs, and models.
- Connect mathematics to the real world, as well as within mathematics.
- Create and use representations to organize, record, and communicate mathematical ideas.
- Solve and analyze routine and non-routine problems.

Objective B:

Number Operations and Relationships

Subskill B.a.:

Concepts

Descriptors, such as but not limited to

- Recognize and apply place-value concepts to whole numbers less than 1,000,000
- Read, write, and represent numbers using words, numerals, pictures (e.g.,. base ten blocks), number lines, , arrays, expanded forms (243=200+40+3) and symbolic renaming e.g., 243=250-7.
- Compare and order numbers less than 10,000 represented in numbers, arrays, symbols (<, > , =) and words.

- Use basic facts to determine the first ten multiples of 2-10 and determine factors for numbers up to 100. Recognize the divisibility potential of numbers (divisors of 2, 5, 10, 25) Count using whole numbers less than 10,000 and by any number 1-12 and 'friendly numbers' through 100. (ex. 20, 25, etc.)

- Read, write, represent, count, compare and order, and make change using a collection of coins and bills equal to and less than $20.00.

- Read, write and identify, equivalent fractions (1/4s, 1/2s, 1/8s, 1/10s, 1/16s)

  Represent fractions (1/4s, 1/2s, 1/8s, 1/10s, 1/16s) using numbers, pictures (e.g. drawings or base ten blocks), and number lines.

  Order and compare fractions (1/4s, 1/2s, 1/8s, 1/10s, 1/16s)represented numerically or as models (including parts of a set and parts of a whole)

  Rename improper fractions to mixed numbers.

Subskill B.b.:	**Computation**

Descriptors, such as but not limited to

- Use all operations in everyday situations to solve single or multi-step word problems.

- Solve three-and four-digit addition and subtraction with regrouping; multiplication of two-digit by one-digit numbers; division with single-digit divisors and two-digit dividends and with two-step or mixed operation problems with single-digit numbers.

  Add and subtract decimals in the context of money.
- Solve problems using basic multiplication and division facts.

- Add and subtract fractions with like denominators.

- Estimate: multiplication of two-digit by one-digit problems, addition and subtraction of decimals using money, and division in context
- Determine reasonableness of answers.

Objective C:	**Geometry**
Subskill C.a.:	**Describe figures**

Descriptors, such as but not limited to

- Identify, describe and compare properties of 2-and 3-dimensional figures, comparing sides, faces, vertices and edges of regular figures including parallel and perpendicular lines and line segments.

- Determine the number of faces, edges and vertices given an illustration of a 3-dimensional figure.

Subskill C.b.:	**Spatial relationships and transformations**

Descriptors, such as but not limited to

- Use pattern blocks and dot paper (geoboards) to describe, model and construct plane figures.

- Identify cubes, rectangular and triangular prisms and rectangular and triangular pyramids from simple nets (flat patterns).

- Use slides, flips and turns on figures. Identify congruent shapes using figures that have been manipulated by one or two motions (slides, flips and turns).

- Discern a shape with one line of symmetry.

- Identify and describe 3-dimensional figures from multiple perspectives.

Subskill C.c.:	**Coordinate systems**

Descriptors, such as but not limited to

- Use simple 2-dimensional coordinate systems to identify or plot locations on maps and to represent points and simple figures with coordinates using letters and numbers, (e.g., (E, 3)).
- Identify and use relationships among figures (e.g., location, position and intersection).

Objective D:	**Measurement**

Subskill D.a.:	**Measurable attributes**

Descriptors, such as but not limited to

- Identify appropriate units to measure length, liquid capacity, volume, weight/mass, time, temperature. Units include: inches, feet, yards, miles, millimeters, centimeters, meters, kilometers, ounces, cups quarts, gallons, liters, seconds, minutes, hours, days, months, years, ounces, pounds, grams, kilograms and degrees Fahrenheit/Celsius.
- Compare attributes of length and weight by direct observation or when given actual measurements.
- Make measurement conversions within a system between units (e.g., feet and yards; inches and feet; quarts and gallons; meters and centimeters; minutes and hours; hours and days; months and years).

Subskill D.b.:	**Direct measurement**

Descriptors, such as but not limited to

- Read, interpret and use measuring instruments to determine the measurement of objects with non- standard and standard units to the nearest ¼- inch or centimeter.
- Read thermometers to the nearest five degrees F/C and read a scale to the nearest ounce or five grams.
- Translate time on an analog clock to a digital clock and vice versa.
- Determine and compare elapsed time in problem-solving situations.

Subskill D.c.:	**Indirect Measurement**

Descriptors, such as but not limited to

- Estimate measurement using U.S customary and metric measurements.
- Determine perimeter and area of regular shapes and the area of plane rectangular shapes.
  Determine perimeter and area of irregular shapes when given a reference tool such as a grid.

Objective E:	**Statistics and Probability**

Subskill E.a.:	**Data analysis and statistics**

Descriptors, such as but not limited to

- Formulate questions to collect, organize and display data.
- Collect, organize and display data in appropriate graphs or charts.
- Draw reasonable conclusions based on contextual data.
- Use data to predict outcomes or trends from graph or table.
- Read and interpret information from single bar graphs, line plots, picture graphs and Venn diagrams.
- Describe a given set of data of seven items/numbers or fewer using the terms range, mode and median in problems with and without context.

Subskill E.b.:	**Probability**

Descriptors, such as but not limited to

- Determine if future events are more, less or equally likely, impossible or certain to occur.
- Choose or design an event that is fair or unfair.
- Predict the outcomes of a simple event using words to describe probability and test predictions using data from a variety of sources.
- Describe and determine the number of combinations for choosing 2 out of 4 items
  Ex: What are the possible combinations when selecting 2 items from a menu of 4 items (chips, cookie, pizza, banana, etc.)?

Objective F:	**Algebraic Relationships**
Subskill F.a.:	**Patterns, relations and functions**

Descriptors, such as but not limited to
- Recognize, extend, describe, create and replicate a variety of patterns including attribute, numeric and geometric patterns.
- Represent patterns and relationships with pictures, tables and charts.
- Describe a rule that explains a functional relationship or pattern using addition, subtraction or multiplication rules.
- Determine a future event in a pattern up to the eighth item when given the first five.

Subskill F.b.:	**Expressions, equations and inequalities**

Descriptors, such as but not limited to
- Solve simple one-step open sentences involving all operations in context.
- Demonstrate a basic understanding of equality and inequality using symbols ($<$, $>$, $=$) with all operations.
- Solve simple one-step open sentences including missing factor in problems with and without context e.g., "box" or letter variable and whole number coefficients.
- Represent problem situations with one-step equations involving multiplication and division with simple open sentences.
- Represent problem situations with one-step equations or expressions using one of the four operations.

Subskill F.c.:	**Properties**

Descriptors, such as but not limited to
- Use the commutative property of multiplication with positive single digits.
- Use the inverse relationship of division and multiplication with single digit, whole numbers.
- Demonstrate understanding of order of operations by solving two-step open sentences involving all operations.

Grade Level Framework

Beginning of Grade 6

How to use the Framework

The mathematics assessment framework is an indication of the knowledge and skills that will be assessed on the November WKCE. ***This information does not replace your local curriculum.*** However, you may wish to ensure that your local curriculum includes the knowledge and skills described in the framework.

This section of the framework describes the types of content that students may encounter on the WKCE

The knowledge and skills to be assessed are organized into objectives, subskills, and descriptors as shown below. WKCE results will be reported by objectives and subskill.

- **A. Objective:** A group of cognitively related skills.
  - A.a. **Subskill:** A group of related knowledge and skills that *may include, but is not limited to,* the descriptors that follow.
    - **Descriptor:** an example of a specific knowledge or skill that may be assessed.

Objectives, Subskills, and Descriptors

Objective A:	**Mathematical Processes**

Students will effectively use mathematical knowledge, skills and strategies related to reasoning, communication, connections, representation and problem solving.

Descriptors, such as but not limited to

- Use reasoning and logic to:
  - Perceive patterns
  - Identify relationships
  - Formulate questions
  - Pose problems
  - Make conjectures
  - Justify strategies
  - Test reasonableness of results
- Communicate mathematical ideas and logical reasoning using the vocabulary of mathematics in a variety of ways e.g., using words, numbers, symbols, pictures, charts, tables, diagrams, graphs, and models.
- Connect mathematics to the real world, as well as within mathematics.
- Create and use representations to organize, record, and communicate mathematical ideas.
- Solve and analyze routine and non-routine problems.

Objective B:	**Number Operations and Relationships**
Subskill B.a.:	**Concepts**

Descriptors, such as but not limited to

- Recognize and apply place-value concepts to whole numbers less than 10,000,000.

- Read, write and represent numbers using words, numerals, pictures (base-ten blocks), number lines, , arrays, expanded forms (12,436=10,000+2,000+400+30+6) and symbolic renaming e.g., 12,436=12,450-14.

- Compare and order numbers less than 100,000 represented in numbers, arrays, symbols (<, >, =) and words.

- Identify and use number theory concepts:
  - prime and composite numbers
  - divisibility potential of numbers (divisors of 1-10, 25).
  - least common multiples through 24

  greatest common factors through 50

- Read, write and identify monetary amounts represented with visual models.

  Compare and order monetary amounts.

  Equate a monetary value with its benchmark fraction and percent. (Eg. $.25=1/4=25%)
- Demonstrate basic understanding of proportionality in proportional contexts.

- Read, write, identify, order, compare and mixed fractions.

  Represent fractions using numbers, pictures, and number lines.

  Rename improper fractions to mixed numbers in lowest terms.

  Identify and represent equivalence between fractions, percents, and decimals.

Subskill B.b.:	**Computation**

Descriptors, such as but not limited to

- Use all operations in everyday situations to solve single or multi-step word problems.

- Solve three and four-digit addition and subtraction with regrouping, multiplication of three-digit by two-digit numbers, division with single-digit divisors and four-digit dividends with two-step or mixed operation problems.

  Compute with decimals in the context of money and make change.
- Solve problems using basic multiplication and division facts.
- Rename improper fractions.

  Add and subtract fractions with unlike denominators (halves, thirds, fourths, fifths, and tenths) with sums or differences between 0 and 1.
- Estimate using basic whole number operations, benchmark fractions and benchmark decimals.

- Determine reasonableness of answers.

Objective C:	**Geometry**
Subskill C.a.:	**Describing figures**

Descriptors, such as but not limited to

- Recognize and name polygons with 3, 4, 5, 6 or 8 sides.

- Identify lines and line segments in a plane figure.

- Classify plane figures by characteristics of angles (acute, obtuse and right) and describe rays found in open-angle situations.

Subskill C.b.:	**Spatial relationships and transformations**

Descriptors, such as but not limited to

- Use tangrams to describe, model, and construct plane figures.

- Identify figures that are congruent and/or similar.

- Describe and compare cubes, rectangular and triangular prisms and rectangular and triangular pyramids from nets (flat patterns).

- Use slides, flips and turns on figures. Identify congruent shapes using figures that have been manipulated by one or two motions (slides, flips and turns).

- Identify lines of symmetry and the number of lines of symmetry in figures and design shapes that have at least one line of symmetry.

- Identify and describe 3-dimensional figures from multiple perspectives.

| **Subskill C.c.:** | **Coordinate systems** |

- Identify and plot the coordinates of locations or objects on simple one quadrant grids using numbers only for coordinates, (e.g., (3, 2)).
- Locate the fourth coordinate pair when given three vertices of a rectangle or parallelogram on a coordinate grid.

| **Objective D:** | **Measurement** |

| **Subskill D.a.:** | **Measurable attributes** |

Descriptors, such as but not limited to

- Identify appropriate units to measure length, liquid capacity, volume, time, weight/mass, temperature, including mixed measures. Units include: inches, feet, yards,(i.e. 1 foot 3 inches) miles, centimeters, millimeters, meters, kilometers, ounces, cups quarts, gallons, liters, hours, minutes, seconds (i.e. 1 hour 15 minutes) , days, months, years, ounces, pounds, grams, kilograms and degrees Fahrenheit/Celsius.
- Compare attributes of length, volume and weight by observation or when given actual measurements.
- Make measurement conversions within a system between units (e.g., feet and yards; inches and yards; quarts and gallons; meters and centimeters; seconds and hours).

| **Subskill D.b.:** | **Direct measurement** |

Descriptors, such as but not limited to

- Measure down to the nearest -1/8-inch, centimeter or millimeter.
  Determine angle measurement to nearest five degrees using a protractor.
- Read and interpret measuring instruments to determine the measurement of objects with standard units (U.S. customary).
- Determine and compare elapsed time in problem-solving situations.

| **Subskill D.c.:** | **Indirect measurement** |

Descriptors, such as but not limited to

- Estimate measurements using U.S. customary and metric measurement.
- Determine the area of regular shapes including right triangles.
- Determine distance between points using a scale.

| **Objective E:** | **Statistics and Probability** |

| **Subskill E.a** | **Data analysis and statistics** |

Descriptors, such as but not limited to

- Formulate questions to collect, organize and display data.
- Collect, organize and display data in appropriate graphs or charts.
- Draw reasonable conclusions based on contextual data.
- Use data to predict outcomes or trends from graphs and tables.
- Extract, interpret and analyze data from single bar graphs, tables and charts, line plots, context, circle graphs and Venn diagrams.
- Describe a given set of data of ten or fewer items/numbers using the terms mean, median, mode and range to extract information from organized charts, tables, graphs and Venn diagrams in problems with and without context.

Subskill E.b.:	**Probability**

Descriptors, such as but not limited to

- Determine the likelihood of future events, predict outcomes of future events and test predictions using data from a variety of sources.
- Choose or design an event that is fair or unfair.
- Determine the probability of events in context using words, percents or fractions.
- Describe and determine the number of combinations of selecting 3 items from 4 or more items.

Objective F:	**Algebraic Relationships**

Subskill F.a.:	**Patterns, relations and functions**

Descriptors, such as but not limited to

- Recognize, extend, describe, create and replicate a variety of patterns including attribute, numeric and geometric patterns.
- Represent patterns and relationships with pictures, table and charts.
- Describe a rule that explains a functional relationship or pattern using addition, subtraction or multiplication rules.
- Determine a future event in a pattern up to the tenth item when given the first five.
- Solve simple two-step, two operation patterns.
  Ex: 5, 8, 6, 9, 7, 10, 8……..
  (Pattern: +3-2….)Represent patterns and relationships with pictures, table and charts.

Subskill F.b:	**Expressions, equations and inequalities**

Descriptors, such as but not limited to

- Demonstrate basic understanding of equality and inequality using symbols ($<$, $>$, $=$) with multi-step, mixed operations.
- Solve one-step equations with "box" variable and whole number coefficients in problems with and without context using whole number coefficients.
- Solve two-step multi-operation equations with "box" or letter variable and whole number coefficients with and without context.
  Ex: 3 * "box" +1 = 7
- Represent problem situations with one or two-step equations or expressions. Solve simple two-step, two operation patterns.
- Solve two-step open sentences involving all operations.
- Solve equations involving any two operations.
  Ex: 3 * 4 -2=?
  Ex: 12/3 +1="box"
  Ex: 5 * 2 – 1 = a

Subskill F.c.:	**Properties**

Descriptors, such as but not limited to

- Use the commutative property of multiplication with positive single digits.
- Use the inverse relationship of division and multiplication with single whole digits.
- Simplify (evaluate) two-step numerical expressions using correct order of operations.
- Demonstrate understanding of distributive property.
- Demonstrate understanding of order of operations by solving two-step open sentences involving all operations.

Grade Level Framework

Beginning of Grade 7

How to use the Framework

The mathematics assessment framework is an indication of the knowledge and skills that will be assessed on the November WKCE. ***This information does not replace your local curriculum***. However, you may wish to ensure that your local curriculum includes the knowledge and skills described in the framework.

This section of the framework describes the types of content that students may encounter on the WKCE

The knowledge and skills to be assessed are organized into objectives, subskills, and descriptors as shown below. WKCE results will be reported by objectives and subskill.

> **A. Objective:** A group of cognitively related skills.
> > A.a. **Subskill:** A group of related knowledge and skills that ***may include, but is not limited to,*** the descriptors that follow.
> > - **Descriptor:** an example of a specific knowledge or skill that may be assessed.

Objectives, Subskills, and Descriptors

Objective A:	**Mathematical Processes**

Students will effectively use mathematical knowledge, skills and strategies related to reasoning, communication, connections, representation and problem solving.

Descriptors, such as but not limited to

- Use reasoning and logic to:
  - Perceive patterns
  - Identify relationships
  - Formulate questions
  - Pose problems
  - Make conjectures
  - Justify strategies
  - Test reasonableness of results

- Communicate mathematical ideas and logical reasoning using the vocabulary of mathematics in a variety of ways e.g., using words, numbers, symbols, pictures, charts, tables, diagrams, graphs, and models.

- Connect mathematics to the real world, as well as within mathematics.

- Create and use representations to organize, record, and communicate mathematical ideas.

- Solve and analyze routine and non-routine problems.

Objective B:	**Number Operations and Relationships**
Sub-skill B.a.:	**Concepts**

Descriptors, such as but not limited to

- Recognize and apply place-value concepts to numbers less than 10,000,000 with decimals to the thousandths place.

- Read, write and represent numbers using words, numerals, number lines, arrays, and expanded form (12.09=10+2+.09) and symbolic renaming (12.09= 13-.91).

- Compare and order a set of fractions or decimals (to the hundredths place)
  and use symbols ($<, >, =, \neq, \leq, \geq$).

- Identify and use number theory concepts:
  - prime and composite numbers
  - divisibility potential of numbers (divisors of 1-10, 25, and multiples of 10).
  - least common multiples
  - greatest common factor of two numbers

- Demonstrate understanding of fractions and benchmark percents in problems with context.
  e.g., Joe got six questions correct and two were wrong, what percent did he get correct?.

- Apply proportional reasoning to a variety of problem situations. (E.g., comparisons and/or rates).

- Identify equivalent forms of fractions, decimals and percents.

Sub-skill B.b.: **Computation**

- ▪ Use all operations in everyday situations (including monetary contexts) to solve single or multi-step word problems.
  ▪ Solve problems involving percents with and without context.
  ▪ Add and subtract decimals including thousandths with and without context.
  ▪ Multiply decimals including hundredths with and without context.
  ▪ Divide decimals including hundredths by single-digit divisors in problems with and without context.

- Demonstrate understanding of the concept of division of fractions in a contextual setting.

- Add, subtract, and multiply mixed numbers and fractions with like and unlike denominators.

- Estimate the sum, difference and product of whole numbers, common fractions, mixed numbers and decimals to thousandths and estimate benchmark fractions.

- Determine reasonableness of answers.

Objective C: **Geometry**

Sub-skill C.a.: **Describing figures**

Descriptors, such as but not limited to

- Name regular and irregular polygons up to eight sides and identify and justify by characteristics whether a shape is a polygon.

- Determine the number of faces, edges and vertices given an illustration of a 3-dimensional figure.

- Classify shapes according to characteristics such as parallel and perpendicular lines; identify right, acute and obtuse angles with varied orientations.

- Find the measure of the third angle of a triangle when given the measures of two interior angles.

- Decompose convex polygons into triangles using diagonals from a single vertex.

Sub-skill C.b.: **Spatial relationships and transformations**

Descriptors, such as but not limited to

- Draw and/or describe a similar figure when given a polygon drawn on graph paper with vertices at lattice points.

- Identify figures that are congruent and/or similar.

- Demonstrate understanding of similarity by finding the relationship between the sides of two figures.
- Draw or identify the image of a figure based on one or more transformations (reflection, rotation and/or translation).
- Design symmetrical shapes.
  Draw or identify lines of symmetry.
- Identify and describe 3-dimensional figures from multiple perspectives.

Sub-skill C.c.: **Coordinate systems**

Descriptors, such as but not limited to

- Identify, locate, plot coordinates in the four quadrants and transformations of points across the x- or y-axis.
- Locate or plot coordinates in the four quadrants using a geometric figure (e.g., transformations).

Objective D:	**Measurement**
Sub-skill D.a.:	**Measurable attributes**

Descriptors, such as but not limited to

- Select the appropriate unit of measure to estimate the length, liquid capacity, volume, weight/mass of everyday objects using U.S. customary and metric.

- Convert units within a system e.g., feet to yards; ounces to pounds; inches to feet; pints to quarts. Approximate conversions of units between metric and U.S. customary systems using a model or in context (quart/liter; yard/meter).

Sub-skill D.b.:	**Direct measurement**

Descriptors, such as but not limited to

- Apply appropriate tools and techniques to measure down to the nearest 1/4-, 1/8- or 1/16-inch or nearest centimeter or millimeter.
- Determine and compare elapsed time in problem-solving situations.

- Measure and/or draw angles up to 180 degrees.

Sub-skill D.c.:	**Indirect measurement**

Descriptors, such as but not limited to
- Estimate area given a reference.

- Determine perimeter/circumference and area of squares, rectangles, triangles, parallelograms and circles in real-world context.
- Determine the distance between points using a scale.

Objective E:	**Statistics and Probability**
Sub-skill E.a.:	**Data analysis and statistics**

Descriptors, such as but not limited to

- Summarize data sets in tables, charts and diagrams with and or without context.

- Evaluate a set of data to generate or confirm/deny hypotheses.

- Extract, interpret and analyze data from tables, simple stem-and-leaf plots, simple bar graphs, line plots, line graphs, simple circle graphs, charts and diagrams.

- Create graph with one-variable data sets using simple stem-and-leaf plots, bar graphs, circle graphs, line plots and line graphs; discuss appropriateness of graphs selected.

- Find mean, median (with odd set of data), mode and range of a set of data with and without context.

- Evaluate sources of data in context and multiple representations of a given data set.

Sub-skill E.b.:	**Probability**

Descriptors, such as but not limited to
- Determine the likelihood of an event and probability based on one independent event, e.g., spinning the arrow on a spinner.
- Use probabilities to estimate outcomes and evaluate fair and unfair simple events.

- Use data from simulations provided in charts/tables to solve and interpret probability problems.

- Describe and determine the number of combinations of selecting 3 items from 4 or more items.

- Solve problems involving sample spaces or diagrams.

- Analyze outcomes based on an understanding of theoretical and experimental probability.

Objective F:	**Algebraic Relationships**

Sub-skill F.a.: **Patterns, relations and functions**

Descriptors, such as but not limited to

- Use two concurrent numeric patterns to describe and analyze functional relationships between two variables in two concurrent numeric patterns using addition and subtraction.

- Extend a given arithmetic sequence of pictures or numbers.

- Describe and interpret linear patterns in tables and graphs.

- Identify the rule to complete or extend a function table or any combination of the two using one operation (+, -, x, ÷) and numbers (0 through 100) in the function table.

- Describe real-world phenomena represented by a graph. Describe real-world phenomena that a given graph might represent.

Sub-skill F.b.: **Expressions, equations and inequalities**

Descriptors, such as but not limited to

- Demonstrate understanding of equality and inequality and solve single-variable equations using symbols ($<$, $>$, $=+$).
- Solve single-variable one-step equations and algebraic expressions with one variable and one operation and whole number coefficients with and without context.
- Describe in words the generalization for a given one-operation pattern.
- Solve two-step multi-operation equations with letter variables and whole number coefficients with and without context.   Ex: $3x + 1 = 7$
- Represent problem situations with one or two-step equations or expressions.
- Describe in words the generalization for a given one-operation pattern.
- Evaluate formulas with and without context by solving for a specified variable.

Sub-skill F.c.: **Properties**

Descriptors, such as but not limited to

- Identify a pair of equivalent numerical expressions where the commutative property of either addition or multiplication has been used.
- Demonstrate understanding of up to three-step order of operations expression with and without context using parentheses and exponents.
- Demonstrate understanding of distributive property.

Grade Level Framework

Beginning of Grade 8

How to use the Framework

The mathematics assessment framework is an indication of the knowledge and skills that will be assessed on the November WKCE. *This information does not replace your local curriculum.* However, you may wish to ensure that your local curriculum includes the knowledge and skills described in the framework.

This section of the framework describes the types of content that students may encounter on the WKCE

The knowledge and skills to be assessed are organized into objectives, subskills, and descriptors as shown below. WKCE results will be reported by objectives and subskill.

A. Objective: A group of cognitively related skills.
   A.a. **Subskill:** A group of related knowledge and skills that *may include, but is not limited to,* the descriptors that follow.
   - **Descriptor:** an example of a specific knowledge or skill that may be assessed.

Objectives, Subskills, and Descriptors

Objective A:	**Mathematical Processes**

Students will effectively use mathematical knowledge, skills and strategies related to reasoning, communication, connections, representation and problem solving.

Descriptors, such as but not limited to

- Use reasoning and logic to:
  - Perceive patterns
  - Identify relationships
  - Formulate questions
  - Pose problems
  - Make conjectures
  - Justify strategies
  - Test reasonableness of results
- Communicate mathematical ideas and logical reasoning using the vocabulary of mathematics in a variety of ways e.g., using words, numbers, symbols, pictures, charts, tables, diagrams, graphs, and models.
- Connect mathematics to the real world, as well as within mathematics.
- Create and use representations to organize, record, and communicate mathematical ideas.
- Solve and analyze routine and non-routine problems.

Objective B:	**Number Operations and Relationships**
Subskill B.a.:	Concepts

Descriptors, such as but not limited to

- Recognize and apply place-value concepts to numbers less than 100,000,000 with decimals to the thousandths place.
- Read, write and represent numbers using words, numerals, number lines, arrays, and expanded form (12.09=10+2+.09) and symbolic renaming (12.09= 13-.91).
- Compare and order a set of fractions or decimals (to the hundredths place) and use symbols ($<, >, =, \neq$).
- Identify and use number theory concepts:
  - prime and composite numbers
  - divisibility potential of numbers (divisors of 1-10, 25, and multiples of 10).
  - least common multiples
  - greatest common factor of two numbers

- Demonstrate understanding of fractions and percents with and without contexts (e.g.,sales tax and discounts, 40 is 25 percent of what number?; What number is 25 percent of 160?)

- Apply proportional reasoning to a variety of problem situations. (E.g. comparisons, rates, and similarities).

- Identify equivalent forms of fractions, decimals and percents.

Subskill B.b.:   **Computation**

Descriptors, such as but not limited to

- Use all operations in everyday situations to solve single or multi-step word problems.

   Solve problems involving percents with and without context.

- ▪ Add and subtract decimals including thousandths with and without text.
  - ▪ Multiply decimals and integers (-100 to 100) including thousandths with and without context. (Ex. interest rates)
  - ▪ Divide decimals and integers in problems with and without context.
- Demonstrate understanding of the concept of division of fractions in a contextual setting.

- Add and subtract mixed numbers and fractions with unlike denominators, multiply mixed numbers.

- Estimate the sum, difference and product of whole numbers, common fractions, mixed numbers and decimals to thousandths.
- Determine reasonableness of answers.

Objective C: **Geometry**

Subskill C.a.:   **Describing figures**

Descriptors, such as but not limited to

- Name 3-dimensional figures (e.g., rectangular prisms, square pyramids, cones, cylinders and spheres.)

- Find the measure of the third angle of a triangle when given the measures of two interior or exterior angles.

- Determine the sum of the angles of a polygon using diagonals drawn from one vertex.

- Determine the measure of an angle in a drawing of an adjacent and supplementary or adjacent and complementary pair of angles when given the measure of the other angle.

Subskill C.b.:   **Spatial relationships and transformations**

Descriptors, such as but not limited to

- Draw and/or describe a similar figure when given a polygon drawn on graph paper with vertices at lattice points.

- Identify figures that are congruent and/or similar.

- Demonstrate understanding of similarity by finding the relationship between the sides of two figures.

- Draw or identify the image of a figure based on one or more transformations (reflection, rotation and/or translation).
- Design symmetrical shapes.
  Draw or identify lines of symmetry.
- Classify figures possessing line symmetry only; line and rotation symmetry; rotational symmetry only; no symmetry.

- Identify and describe 3-dimensional figures from multiple perspectives.

Subskill C.c.:   **Coordinate systems**

Descriptors, such as but not limited to

- Identify, locate, plot coordinates in all four quadrants; draw or identify the reflection of a point across the x- or y-axis or the translation of a point at integer coordinates in any of the four quadrants.

- Locate or plot coordinates in any of the four quadrants using a geometric figure (e.g., transformations).

Objective D:	**Measurement**
Subskill D.a.:	**Measurable attributes**

Descriptors, such as but not limited to
- Select the appropriate unit of measure (U.S. customary and metric) to estimate the length, liquid capacity, volume, time, and weight/mass of everyday objects.
- Convert units within a system e.g., feet to yards; ounces to pounds; inches to feet; pints to quarts.

Approximate conversions of units between metric and U.S. customary systems using a model or in context (quart/liter; yard/meter).

Subskill D.b.:	**Direct measurement**

Descriptors, such as but not limited to
- Apply appropriate tools techniques to measure down to the nearest 1/4-, 1/8- or 1/16-inch or nearest centimeter or millimeter.
- Determine and compare elapsed time in problem-solving situations.
- Measure and/or draw angles up to 360 degrees.

Subskill D.c.:	**Indirect measurement**

Descriptors, such as but not limited to
- Estimate area given a reference.
- Determine perimeter/circumference and area of polygons and circles with and without context.
- Determine the distance between points using a scale.
- Determine volume and surface area of cylinders, rectangular prisms and pyramids with base shapes of triangle, square, regular pentagon and regular hexagon in real-world context.
- Draw similar figures in any shape using a scale factor (e.g., enlarge/shrink).
- Use ratio and proportion in context.
- Use $d = r*t$ formula in simple contexts.

Objective E:	**Statistics and Probability**
Subskill E.a.:	**Data analysis and statistics**

Descriptors, such as but not limited to
- Compare two sets of data to generate or confirm/deny hypotheses.
- Extract, interpret and analyze data including multiple representations of the same data from tables, double back-to-back stem-and-leaf plots, double bar graphs, simple circle graphs, line plots, line graphs, charts and diagrams with and without context.
- Create graph with one-variable data sets using back-to-back stem-and-leaf plots, double bar graphs, circle graphs, line plots and line graphs; discuss appropriateness of graph selected.
- Find mean, median (with odd or even number of data), mode and range of a set of data with and without context.
- Evaluate sources of data in context and multiple representations of a given data set.
- Compare two sets of data to generate or confirm/deny hypotheses.

Subskill E.b.:	**Probability**

Descriptors, such as but not limited to
- Determine the likelihood of an event and probability based on one or two dependent or independent events.
- Use probabilities to estimate outcomes and evaluate fair and unfair simple events.
- Use data from simulations provided in charts/tables to solve and interpret probability problems.

- Determine the number of arrangements from a set of 5 or less.
  Ex: How many different ways could 5 students stand in line?

- Solve problems involving sample spaces or diagrams.

- Analyze outcomes based on an understanding of theoretical and experimental probability.

Objective F:	**Algebraic Relationships**
Subskill F.a.:	**Patterns, relations and functions**

Descriptors, such as but not limited to

- Use two concurrent numeric patterns to describe and analyze functional relationships between two variables.
  Describe and analyze in words functional relationships in two concurrent numeric patterns s using multiplication and exponents and describe the relationship in words.
- Extend an increasing or decreasing arithmetic or geometric pattern.

- Describe and interpret linear patterns in tables and graphs.

- Identify the rule to complete or extend a function table or any combination of the two using one or two operations (+, -, x, ÷) and numbers (-100 through 100) in the function table..

- Describe real-world phenomena represented by a graph. Describe real-world phenomena that a given graph might represent.

- Justify the accuracy of the chosen item in a sequence.

Subskill F.b.:	**Expressions, equations and inequalities**

Descriptors, such as but not limited to

- Solve single-variable inequalities using symbols.

- Solve single-variable one- and two-step equations with whole number, whole number integer, or rational, coefficients with and without context.

- Find values of expressions with one variable and up to two operations including basic operations and exponents.

- Solve two-step multi-operation equations with letter variables and whole number or integer coefficients with and without context.
  Ex: $-3x + 1 =$.

- Write an algebraic expression (with one or two operations) which generalizes a linear pattern.

- Create a corresponding algebraic expression when given an arithmetic operation/relationship expressed in words.

- Evaluate formulas with and without context by solving for a specified variable.

Subskill F.c.:	**Properties**

Descriptors, such as but not limited to

- Identify a pair of equivalent numerical or one-variable expressions when using commutative or associative properties with addition and multiplication.
- Demonstrate understanding of up to four-step order of operations expression using parentheses, exponents and fraction symbol.
- Demonstrate understanding of distributive property without variables.
- Solve order of operations problems with one variable to demonstrate understanding of commutativity and associativity.

Grade Level Framework

Beginning of Grade 10

How to use the Framework

The mathematics assessment framework is an indication of the knowledge and skills that will be assessed on the November WKCE. ***This information does not replace your local curriculum***. However, you may wish to ensure that your local curriculum includes the knowledge and skills described in the framework.

This section of the framework describes the types of content that students may encounter on the WKCE

The knowledge and skills to be assessed are organized into objectives, subskills, and descriptors as shown below. WKCE results will be reported by objectives and subskill.

    A. Objective: A group of cognitively related skills.
        A.a. **Subskill:** A group of related knowledge and skills that ***may include, but is not limited to,*** the descriptors that follow.
          • **Descriptor:** an example of a specific knowledge or skill that may be assessed.

Objectives, Subskills, and Descriptors

Objective A:	**Mathematical Processes**

Students will effectively use mathematical knowledge, skills and strategies related to reasoning, communication, connections, representation and problem solving.

Descriptors, such as but not limited to

- Use reasoning and logic to:
    - Perceive patterns
    - Identify relationships
    - Formulate questions
    - Pose problems
    - Make conjectures
    - Justify strategies
    - Test reasonableness of results

- Communicate mathematical ideas and logical reasoning using the vocabulary of mathematics in a variety of ways e.g., using words, numbers, notation, symbols, pictures, charts, tables, diagrams, graphs, and models.

- Connect mathematics to the real world, as well as within mathematics.

- Create and use representations to organize, record, and communicate mathematical ideas.

- Solve and analyze routine and non-routine problems.

Objective B:	**Number Operations and Relationships**
Subskill B.a.:	**Concepts**

Descriptors, such as but not limited to

- Compare and order real numbers.

- Analyze and solve problems using percents.

- Apply proportional reasoning and ratios in mathematical and real-world contexts.

Subskill B.b.:	**Computation**

Descriptors, such as but not limited to

- Compare, perform and explain operations on real numbers with and without context e.g., transitivity, rate of change, exponential functions, scientific notation, roots, powers, reciprocals, absolute value, ratios, proportions, percents.

- Select and use appropriate properties, computational procedures, and modes of representation with and without context e.g., simple and compound interest, commission, percents, proportions.
- Determine reasonableness of answers.

Objective C:	**Geometry**

Subskill C.a.:	**Describing figures**

Descriptors, such as but not limited to

- Identify, describe and analyze properties of 2 and 3 dimensional figures, relationships among figures and relationships among their parts (e.g., parallel, perpendicular and congruent sides, diagonals, various types of angles and triangles, complementary and supplementary angles, sum of angles in a triangle).
- Present convincing geometric arguments by means of informal proof, counter-examples or other logical means.
- Model problems using the Pythagorean Theorem and right triangle trigonometry.

Subskill C.b.:	**Spatial relationships and transformations**

Descriptors, such as but not limited to

- Use proportional reasoning to solve congruence and similarity problems (e.g., scale drawings and similar geometric figures).
- Use transformations and symmetry to solve problems.
- Visualize 3-dimensional figures in problem-solving situations.

Subskill C.c.:	**Coordinate systems**

Descriptors, such as but not limited to

- Use the two-dimensional rectangular coordinate system to describe and characterize properties of geometric figures. Identify and apply symmetry about an axis.
- Use the two-dimensional rectangular coordinate system and algebraic procedures to describe and characterize geometric properties and relationships (e.g., slope, intercepts, parallelism, and perpendicularity, Pythagorean Theorem, distance formula).

Objective D:	**Measurement**

Subskill D.a.:	**Measurable attributes**

Descriptors, such as but not limited to

- Identify, describe and use derived attributes to represent and solve problems (e.g., speed, acceleration, density, money conversion.)

Subskill D.b.:	**Direct measurement**

Descriptors, such as but not limited to

- Select and use tools with appropriate degree of precision to determine measurements directly.

Subskill D.c.:	**Indirect measurement**

Descriptors, such as but not limited to

- Determine the perimeter/area of two-dimensional figures.
- Determine the surface area/volume of three-dimensional figures.
- Solve for angles, and segments in similar polygons and arcs in circles.
- Use right-triangle trig functions and the Pythagorean Theorem to solve right-triangle problems.
- Use formulas in applications (e.g., Distance Formula, simple and compound interest).

Objective E:	**Statistics and Probability**

Subskill E.a:	**Data analysis and statistics**

Descriptors, such as but not limited to

- Organize, display, compare and interpret data in a variety of ways in mathematical and real-world contexts e.g., histograms, line graphs, stem-and-leaf plots, scatter plots, box-and whiskers, bar charts, Venn diagrams, tables, circle graphs.

- Interpret, analyze and make predictions from organized and displayed data. e.g., measures of central tendency such as mean, median, mode, and, measures of variation such as standard deviation, mean, median, mode, range, dispersion, outliers, line of best fit, percentiles.

- Analyze, evaluate and critique methods and conclusions of statistical experiments, e.g., randomness, sampling, techniques, surveys.

Subskill E.b.:	**Probability**

Descriptors, such as but not limited to

- Determine the likelihood of occurrence of simple and complex events
Ex: Combinations and permutations, fundamental counting principle, experimental versus theoretical probability and independent, dependent and conditional probability.

Objective F:	**Algebraic Relationships**

Subskill F.a.:	**Patterns, relations and functions**

Descriptors, such as but not limited to

- Describe, recognize, interpret and translate graphical representations of mathematical and real-world phenomena on coordinate grids, e.g., slope, intercepts, rate of change, linear and non-linear functions, and quadratic, exponential and constant functions.

- Analyze, generalize and represent patterns of change, e.g., direct and inverse variations, including numerical sequences, patterns to a given term, algebraic expressions and equations.

Subskill F.b:	**Expressions, equations and inequalities**

Descriptors, such as but not limited to

- Solve linear and quadratic equations, linear inequalities and systems of linear equations and inequalities.

- Model and solve a variety of mathematical and real-world problems by using algebraic expressions, equations and inequalities, e.g., linear, exponential, quadratic.

- Translate between different representations and describe the relationship among variable quantities in a problem, e.g., tables, graphs, functional notations, formulas.

Subskill F.c.:	**Properties**

Descriptors, such as but not limited to

- Demonstrate understanding of properties by evaluating and simplifying expressions.

- Demonstrate understanding of properties by solving linear and quadratic equations, linear inequalities, and systems of linear equations and inequalities with one or two variables.

	Grade 3	Grade 4	Grade 5	Grade 6	Grade 7	Grade 8	Grade 10
A. Mathematical Processes — Students will effectively use mathematical knowledge, skills and strategies related to reasoning, communication, connections, representation and problem solving.	Use reasoning and logic to: ▪ perceive patterns ▪ identify relationships ▪ formulate questions ▪ pose problems ▪ make conjectures ▪ justify strategies ▪ test reasonableness of results	Use reasoning and logic to: ▪ perceive patterns ▪ identify relationships ▪ formulate questions ▪ pose problems ▪ make conjectures ▪ justify strategies ▪ test reasonableness of results	Use reasoning and logic to: ▪ perceive patterns ▪ identify relationships ▪ formulate questions ▪ pose problems ▪ make conjectures ▪ justify strategies ▪ test reasonableness of results	Use reasoning and logic to: ▪ perceive patterns ▪ identify relationships ▪ formulate questions ▪ pose problems ▪ make conjectures ▪ justify strategies ▪ test reasonableness of results	Use reasoning and logic to: ▪ perceive patterns ▪ identify relationships ▪ formulate questions ▪ pose problems ▪ make conjectures ▪ justify strategies ▪ test reasonableness of results	Use reasoning and logic to: ▪ perceive patterns ▪ identify relationships ▪ formulate questions ▪ pose problems ▪ make conjectures ▪ justify strategies ▪ test reasonableness of results	Use reasoning and logic to: ▪ perceive patterns ▪ identify relationships ▪ formulate questions ▪ pose problems ▪ make conjectures ▪ justify strategies ▪ test reasonableness of results
	Communicate mathematical ideas and reasoning using the vocabulary of mathematics in a variety of ways e.g., using words, numbers, symbols, pictures, charts, tables, diagrams, graphs, and models.	Communicate mathematical ideas and reasoning using the vocabulary of mathematics in a variety of ways e.g., using words, numbers, symbols, pictures, charts, tables, diagrams, graphs, and models.	Communicate mathematical ideas and reasoning using the vocabulary of mathematics in a variety of ways e.g., using words, numbers, symbols, pictures, charts, tables, diagrams, graphs, and models.	Communicate mathematical ideas and reasoning using the vocabulary of mathematics in a variety of ways e.g., using words, numbers, symbols, pictures, charts, tables, diagrams, graphs, and models.	Communicate mathematical ideas and reasoning using the vocabulary of mathematics in a variety of ways e.g., using words, numbers, symbols, pictures, charts, tables, diagrams, graphs, and models.	Communicate mathematical ideas and reasoning using the vocabulary of mathematics in a variety of ways e.g., using words, numbers, symbols, pictures, charts, tables, diagrams, graphs, and models.	Communicate mathematical ideas and reasoning using the vocabulary of mathematics in a variety of ways e.g., using words, numbers, symbols, pictures, charts, tables, diagrams, graphs, and models.
	Connect mathematics to the real world, as well as within mathematics.	Connect mathematics to the real world, as well as within mathematics.	Connect mathematics to the real world, as well as within mathematics.	Connect mathematics to the real world, as well as within mathematics.	Connect mathematics to the real world, as well as within mathematics.	Connect mathematics to the real world, as well as within mathematics.	Connect mathematics to the real world, as well as within mathematics.
	Create and use representations to organize, record, and communicate mathematical ideas.	Create and use representations to organize, record, and communicate mathematical ideas.	Create and use representations to organize, record, and communicate mathematical ideas.	Create and use representations to organize, record, and communicate mathematical ideas.	Create and use representations to organize, record, and communicate mathematical ideas.	Create and use representations to organize, record, and communicate mathematical ideas.	Create and use representations to organize, record, and communicate mathematical ideas.
	Solve and analyze routine and non-routine problems.	Solve and analyze routine and non-routine problems.	Solve and analyze routine and non-routine problems.	Solve and analyze routine and non-routine problems.	Solve and analyze routine and non-routine problems.	Solve and analyze routine and non-routine problems.	Solve and analyze routine and non-routine problems.

	Grade 3	Grade 4	Grade 5	Grade 6	Grade 7	Grade 8	Grade 10
B. Number Operations & Relationships Sub Skill B.a.: Concepts	Recognize and apply place-value concepts to whole numbers less than 1,000.	Recognize and apply place-value concepts to whole numbers less than 10,000.	Recognize and apply place-value concepts to whole numbers less than 1,000,000.	Recognize and apply place-value concepts to whole numbers less than 10,000,000.	Recognize and apply place-value concepts to numbers less than 10,000,000 with decimals to the thousandths place.	Recognize and apply place-value concepts to numbers less than 100,000,000 with decimals to the thousandths place.	
	Read, write, and represent numbers using words, numerals, pictures (e.g., base-ten blocks), number lines, arrays, expanded forms (24=20+4) and symbolic renaming, e.g., 24=30-6.	Read, write, and represent numbers using words, numerals, pictures (e.g., base ten blocks), number lines, arrays, expanded forms (243=200+40+3) and symbolic renaming, e.g., 243=250-7.	Read, write, and represent numbers using words, numerals, pictures (e.g., base ten blocks), number lines, arrays, expanded forms (243=200+40+3) and symbolic renaming, e.g., 243=250-7.	Read, write and represent numbers using words, numerals, pictures (base-ten blocks), number lines,, arrays, expanded forms (12,436=10,000+2,000+ 400+30+6) and symbolic renaming, e.g., 12,436=12,450-14.	Read, write and represent numbers using words, numerals, number lines, arrays, and expanded form (12.09=10+2+.09) and symbolic renaming (12.09= 13-.91).	Read, write and represent numbers using words, numerals, number lines, arrays, and expanded form (12.097=10+2+.09+.007) and symbolic renaming (12.097= 13-.003).	
	Compare and order whole numbers less than 1,000.	Compare and order whole numbers less than 10,000.	Compare and order numbers less than 10,000 represented in numbers, arrays, symbols (<, >, =) and words.	Compare and order numbers less than 100,000 represented in numbers, arrays, symbols (<, >, =) and words.	Compare and order a set of fractions or decimals (to the hundredths place) and use symbols (<, >, =, ≠, ≤, ≥).	Compare and order a set of fractions, decimals (to the thousandths) and use symbols (<, >, =, ≠, ≤, ≥).	Compare and order real numbers.
	Count by 2's, 3's, 5's, 10's, 25's and 100's.	Count by 2's, 3's, 5's, 10's, 25's and 100's starting with any multiple and 100's starting with any number. Identify and name counting patterns.	Use basic facts to determine the first ten multiples of 2-10 and determine factors for numbers up to 100. Recognize the divisibility potential of numbers (divisors of 2, 5, 10, 25). Count using whole numbers less than 10,000 and by any number 1-12 and 'friendly numbers' through 100. (ex. 20, 25, etc.)	Identify and use number theory concepts: ▪ prime and composite numbers ▪ divisibility potential of numbers (divisors of 1-10, 25) ▪ least common multiples through 24 ▪ greatest common factors through 50	Identify and use number theory concepts: ▪ prime and composite numbers ▪ divisibility potential of numbers (divisors of 1-10, 25) ▪ least common multiples ▪ greatest common factors of two numbers	Identify and use number theory concepts: ▪ prime and composite numbers ▪ divisibility potential of numbers (divisors of 1-10, 25) ▪ least common multiples ▪ greatest common factors of two or three numbers	

	Grade 3	Grade 4	Grade 5	Grade 6	Grade 7	Grade 8	Grade 10
B. Number Operations & Relationships - Continued Sub Skill B.a.: Concepts Continued	Count, compare and make change using a collection of coins (up to one dollar) and one-dollar bills.	Count, compare and make change up to $10.00 using a collection of coins and one-dollar bills.	Read, write, represent, count, compare and order, and make change using a collection of coins and bills equal to and less than $20.00.	Read, write and identify monetary amounts represented with visual models. Compare and order monetary amounts. Equate a monetary value with its benchmark fraction and percent. (Eg. $.25=1/4=25%)	Demonstrate understanding of fractions and benchmark percents in problems with context. E.g., Joe got six questions correct and two were wrong, what percent did he get correct?	Demonstrate understanding of fractions and percents with and without contexts (e.g.,sales tax and discounts, 40 is 25 percent of what number?; What number is 25 percent of 160?)	Analyze and solve problems using percents.
				Demonstrate basic understanding of proportionality in proportional contexts.	Apply proportional reasoning to a variety of problem situations. (E.g. comparisons and rates).	Apply proportional reasoning to a variety of problem situations. (E.g. comparisons, rates, and similarities).	Apply proportional reasoning and ratios in mathematical and real-world contexts.
	Identify a fractional part of a collection/set. Read, write and represent fractional parts of a whole e.g., 1/4, 1/2.	Identify a fractional part of a collection/set or parts of a whole. Read, write, order and represent unit fractions (e.g., 1/2, 1/3, 1/4) and part(s) of a set.	Read, write and identify, equivalent fractions (1/4s, 1/2s, 1/8s, 1/10s, 1/16s) Represent fractions (1/4s, 1/2s, 1/8s, 1/10s, 1/16s) using numbers, pictures (E.g. drawings or base ten blocks), and number lines. Order and compare fractions (1/4s, 1/2s, 1/8s, 1/10s, 1/16s)represented numerically or as models (including parts of a set and parts of a whole) Rename improper fractions to mixed numbers.	Read, write, identify, order, compare and mixed fractions. Represent fractions using numbers, pictures, and number lines. Rename improper fractions to mixed numbers in lowest terms. Identify and represent equivalence between fractions, percents, and decimals.	Identify equivalent forms of fractions, decimals and percents.	Identify equivalent forms of fractions, decimals and percents.	

	Grade 3	Grade 4	Grade 5	Grade 6	Grade 7	Grade 8	Grade 10
B. Number Operations & Relationships Sub Skill B.b.: Computation	Use addition and subtraction in everyday situations and solve one-step word problems.	Use addition and subtraction in everyday situations and solve one-and two-step word problems.	Use all operations in everyday situations to solve single or multi-step word problems.	Use all operations in everyday situations to solve single or multi-step word problems.	Use all operations in everyday situations (including monetary contexts) to solve single or multi-step word problems. Solve problems involving percents with and without context.	Use all operations in everyday situations to solve single or multi-step word problems. Solve problems involving percents with and without context.	Compare, perform and explain operations on real numbers with and without context e.g., transitivity, rate of change, exponential functions, scientific notation, roots, powers, reciprocals, absolute value, ratios, proportions, percents.
	Solve single and double-digit addition and subtraction problems with regrouping including horizontal format in problems with and without context.	Solve double-and triple-digit addition and subtraction problems with regrouping in horizontal and vertical format in problems with and without context.	Solve three-and four-digit addition and subtraction with regrouping; multiplication of two-digit by one-digit numbers; division with single-digit divisors and two-digit dividends and with two-step or mixed operation problems with single-digit numbers. Add and subtract decimals in the context of money.	Solve three and four-digit addition and subtraction with regrouping, multiplication of three-digit by two-digit numbers, division with single-digit divisors and four-digit dividends with two-step or mixed operation problems. Compute with decimals in the context of money and make change.	Add and subtract decimals including thousandths with and without context. Multiply decimals including hundredths with and without context. Divide decimals including hundredths by single-digit divisors in problems with and without context.	Add and subtract decimals including thousandths with and without text. Multiply decimals and integers (-100 to 100) including thousandths with and without context. (Ex. interest rates) Divide decimals and integers in problems with and without context.	
	Demonstrate the concept of multiplication as grouping or repeated addition in context with products up to 50.	Demonstrate understanding of multiplication as grouping or repeated addition or arrays in problems with and without context (without context up to 5 x 9; in context products up to 100).	Solve problems using basic multiplication and division facts.	Solve problems using basic multiplication and division facts.	Demonstrate understanding of the concept of division of fractions in a contextual setting.	Demonstrate understanding of the concept of division of fractions in a contextual setting.	
	Demonstrate understanding of the concept of division as repeated subtraction, partitioning/sharing or measuring (dividend up to 30 and divisors up to 5).	Demonstrate understanding of the concept of division as repeated subtraction, partitioning/sharing or measuring (dividend up to 45 and divisors up to 5).					

	Grade 3	Grade 4	Grade 5	Grade 6	Grade 7	Grade 8	Grade 10
B. Number Operations & Relationships Sub Skill B.b.: Computation Continued	Use fractions to represent quantities when solving problems involving equal sharing or partitioning. Represent with shaded circles, rods, squares, pictorial representations of a whole.	Use fractions to represent quantities when solving problems involving equal sharing or partitioning including fractions less than one as well as mixed numbers. Represent with shaded circles, rods, squares or pictorial representations of objects (for a set).	Add and subtract fractions with like denominators.	Rename improper fractions. Add and subtract fractions with unlike denominators (halves, thirds, fourths, fifths, and tenths) with sums or differences between 0 and 1.	Add, subtract, and multiply mixed numbers and fractions with like and unlike denominators.	Add and subtract mixed numbers and fractions with unlike denominators, multiply mixed numbers.	Select and use appropriate properties, computational procedures, and modes of representation with and without context e.g., simple and compound interest, commission, percents, proportions.
	Estimate sums to tens and hundreds and differences to ten.	Estimate sums to tens, hundreds and thousands and differences of ten and hundreds.	Estimate: multiplication of two-digit by one-digit problems, addition and subtraction of decimals using money, and division in context.	Estimate using basic whole number operations, benchmark fractions and benchmark decimals.	Estimate the sum, difference and product of whole numbers, common fractions, mixed numbers and decimals to thousandths and estimate benchmark fractions.	Estimate the sum, difference and product of whole numbers, common fractions, mixed numbers and decimals to thousandths.	
	Determine reasonableness of answers.	Determine reasonableness of answers.	Determine reasonableness of answers.	Determine reasonableness of answers.	Determine reasonableness of answers.	Determine reasonableness of answers.	Determine reasonableness of answers.

	Grade 3	Grade 4	Grade 5	Grade 6	Grade 7	Grade 8	Grade 10
C. Geometry **Sub-Skill C.a.: Describing Figures**	Identify, describe, and compare properties of 2 and 3 dimensional figures such as squares, triangles, rectangles, circles, pattern block shapes, cubes, pyramids, rectangular prisms, cylinders, and spheres (e.g., comparing sides, faces, corners, and edges).	Identify, describe, and compare properties of 2 and 3 dimensional figures such as squares, triangles, rectangles, pentagon, hexagon, octagon, pattern block shapes, circles, cubes, pyramids, rectangular prisms, tetrahedrons, cylinders, and spheres (e.g., comparing sides, faces, corners, and edges).	Identify, describe and compare properties of 2- and 3-dimensional figures, comparing sides, faces, vertices and edges of regular figures including parallel and perpendicular lines and line segments.	Recognize and name polygons with 3, 4, 5, 6 or 8 sides.	Name regular and irregular polygons up to eight sides and identify and justify by characteristics whether a shape is a polygon.	Name 3-dimensional figures (e.g., rectangular prisms, square pyramids, cones, cylinders and spheres.)	Identify, describe and analyze properties of 2 and 3 dimensional figures, relationships among figures and relationships among their parts (e.g., parallel, perpendicular and congruent sides, diagonals, various types of angles and triangles, complementary and supplementary angles, sum of angles in a triangle).
			Determine the number of faces, edges and vertices given an illustration of a 3-dimensional figure.	Identify lines and line segments in a plane figure.	Determine the number of faces, edges and vertices given an illustration of a 3-dimensional figure.		
				Classify plane figures by characteristics of angles (acute, obtuse and right) and describe rays found in open-angle situations.	Classify shapes according to characteristics such as parallel and perpendicular lines; identify right, acute and obtuse angles with varied orientations.		
					Find the measure of the third angle of a triangle when given the measures of two interior angles.	Find the measure of the third angle of a triangle when given the measures of two interior or exterior angles.	Present convincing geometric arguments by means of informal proof, counter-examples or other logical means.
					Decompose convex polygons into triangles using diagonals from a single vertex.	Determine the sum of the angles of a polygon using diagonals drawn from one vertex.	Model problems using the Pythagorean Theorem and right triangle trigonometry.
						Determine the measure of an angle in a drawing of an adjacent and supplementary or adjacent and complementary pair of angles when given the measure of the other angle.	

C. Geometry
Sub-Skill C.b: Spatial Relationships and Transformations

	Grade 3	Grade 4	Grade 5	Grade 6	Grade 7	Grade 8	Grade 10
	Identify 2-dimensional geometric shapes created by combining or decomposing other shapes e.g., square/triangles; trapezoid/rhombus, triangle; hexagon/triangles, rhombus, trapezoid.	Create and identify 2-dimensional geometric shapes by combining or decomposing other shapes.	Use pattern blocks and dot paper (geoboards) to describe, model and construct plane figures.	Use tangrams to describe, model, and construct plane figures.	Draw and/or describe a similar figure when given a polygon drawn on graph paper with vertices at lattice points.	Draw and/or describe a similar figure when given a polygon drawn on graph paper with vertices at lattice points.	Use proportional reasoning to solve congruence and similarity problems (e.g., scale drawings and similar geometric figures).
				Identify figures that are congruent and/or similar.	Identify figures that are congruent and/or similar.	Identify figures that are congruent and/or similar.	
					Demonstrate understanding of similarity by finding the relationship between the sides of two figures.	Demonstrate understanding of similarity by finding the relationship between the sides of two figures.	
		Identify cubes and square pyramid shapes from their nets (flat patterns).	Identify cubes, rectangular and triangular prisms and rectangular and triangular pyramids from simple nets (flat patterns).	Describe and compare cubes, rectangular and triangular prisms and rectangular and triangular pyramids from nets (flat patterns).			
	Apply concepts of single-motion geometry (e.g., slides, flips and turns) to match two identical shapes.	Apply concepts of single-motion geometry (e.g., slides, flips and turns) to match two identical shapes.	Use slides, flips and turns on figures. Identify congruent shapes using figures that have been manipulated by one or two motions (slides, flips and turns).	Use slides, flips and turns on figures. Identify congruent shapes using figures that have been manipulated by one or two motions (slides, flips and turns).	Draw or identify the image of a figure based on one or more transformations (reflection, rotation and/or translation).	Draw or identify the image of a figure based on one or more transformations (reflection, rotation and/or translation).	Use transformations and symmetry to solve problems.
			Discern a shape with one line of symmetry.	Identify lines of symmetry and the number of lines of symmetry in figures and design shapes that have at least one line of symmetry.	Design symmetrical shapes. Draw or identify lines of symmetry.	Design symmetrical shapes. Draw or identify lines of symmetry.	

	Grade 3	Grade 4	Grade 5	Grade 6	Grade 7	Grade 8	Grade 10
C. Geometry - Continued — Sub-Skill C.b: Spatial Relationships and Transformations						Classify figures possessing line symmetry only; line and rotation symmetry; rotational symmetry only; no symmetry	
			Identify and describe 3-dimensional figures from multiple perspectives.	Identify and describe 3-dimensional figures from multiple perspectives.	Identify and describe 3-dimensional figures from multiple perspectives.	Identify and describe 3-dimensional figures from multiple perspectives.	Visualize 3-dimensional figures in problem-solving situations.

	Grade 3	Grade 4	Grade 5	Grade 6	Grade 7	Grade 8	Grade 10
C. Geometry Sub-Skill C.c.: Coordinate Systems	Use simple 2-dimensional coordinate systems to find locations on maps and to represent points and simple figures with coordinates of letters and numbers, (e.g., (E, 3)).	Use simple 2-dimensional coordinate systems to find locations on maps and to represent points and simple figures with coordinates using letters and numbers, (e.g., (E, 3)).	Use simple 2-dimensional coordinate systems to identify or plot locations on maps and to represent points and simple figures with coordinates using letters and numbers, (e.g., (E, 3)).	Identify and plot the coordinates of locations or objects on simple one quadrant grids using numbers only for coordinates, (e.g., (3, 2)).	Identify, locate, plot coordinates in the four quadrants and transformations of points across the x- or y-axis.	Identify, locate, plot coordinates in all four quadrants; draw or identify the reflection of a point across the x- or y-axis or the translation of a point at integer coordinates in any of the four quadrants.	Use the two-dimensional rectangular coordinate system to describe and characterize properties of geometric figures. Identify and apply symmetry about an axis.
		Identify and use relationships among figures (e.g., location, position and intersection).	Identify and use relationships among figures (e.g., location, position and intersection).	Locate the fourth coordinate pair when given three vertices of a rectangle or parallelogram on a coordinate grid.	Locate or plot coordinates in the four quadrants using a geometric figure (e.g., transformations).	Locate or plot coordinates in any of the four quadrants using a geometric figure (e.g., transformations).	Use the two-dimensional rectangular coordinate system and algebraic procedures to describe and characterize geometric properties and relationships (e.g., slope, intercepts, parallelism, and perpendicularity, Pythagorean Theorem, distance formula).

	Grade 3	Grade 4	Grade 5	Grade 6	Grade 7	Grade 8	Grade 10
D. Measurement Sub-Skill D.a.: Measurable Attributes	Describe attributes of length, time and temperature and identify appropriate units to measure them. Units include: inches, feet, yards, centimeters, meters, seconds, minutes, hours, days, months, years and degrees Fahrenheit/Celsius.	Describe attributes of length, time, temperature, liquid capacity, weight/mass, volume and identify appropriate units to measure them. Units include: inches, feet, yards, miles, meters, centimeters, millimeters, cups quarts, gallons, liters, seconds, minutes, hours, days, months, years, ounces, pounds, grams and degrees Fahrenheit/Celsius.	Identify appropriate units to measure length, liquid capacity, volume, weight/mass, time, temperature. Units include: inches, feet, yards, miles, millimeters, centimeters, meters, kilometers, ounces, cups quarts, gallons, liters, seconds, minutes, hours, days, months, years, ounces, pounds, grams, kilograms and degrees Fahrenheit/Celsius.	Identify appropriate units to measure length, liquid capacity, volume, time, weight/mass, temperature, including mixed measures. Units include: inches, feet, yards,(i.e. 1 foot 3 inches) miles, centimeters, millimeters, meters, kilometers, ounces, cups quarts, gallons, liters, hours, minutes, seconds, (i.e. 1 hour 15 minutes) , days, months, years, ounces, pounds, grams, kilograms and degrees Fahrenheit/Celsius.	Select the appropriate unit of measure to estimate the length, liquid capacity, volume, weight/mass of everyday objects using U.S. customary and metric.	Select the appropriate unit of measure (U.S. customary and metric) to estimate the length, liquid capacity, volume, time, and weight/mass of everyday objects.	
	Compare attributes of length and weight by observation or when given actual measurements.	Compare attributes of length, volume and weight by observation or when given actual measurements.	Compare attributes of length and weight by direct observation or when given actual measurements.	Compare attributes of length, volume and weight by observation or when given actual measurements.			
		Make measurement conversions within a system (e.g., yards to feet; feet to inches; hours to minutes; days to hours; years to months; gallons to quarts).	Make measurement conversions within a system between units (e.g., feet and yards; inches and feet; quarts and gallons; meters and centimeters; minutes and hours; hours and days; months and years).	Make measurement conversions within a system between units (e.g., feet and yards; inches and yards; quarts and gallons; meters and centimeters; seconds and hours).	Convert units within a system e.g., feet to yards; ounces to pounds; inches to feet; pints to quarts. Approximate conversions of units between metric and U.S. customary systems using a model or in context (quart/liter; yard/meter).	Convert units within a system e.g., feet to yards; ounces to pounds; inches to feet; pints to quarts. Approximate conversions of units between metric and U.S. customary systems using a model or in context (quart/liter; yard/meter).	
							Identify, describe and use derived attributes to represent and solve problems (e.g., speed, acceleration, density, money conversion.)

	Grade 3	Grade 4	Grade 5	Grade 6	Grade 7	Grade 8	Grade 10
D. Measurement Sub-Skill D.b.: Direct Measurement	Read and interpret measuring instruments to determine the measurement of objects with non-standard and standard units to the nearest centimeter or 1/2-inch.	Read and interpret and use measuring instruments to determine the measurement of objects with non-standard and standard units to the nearest centimeter, or 1/4-inch.	Read, interpret and use measuring instruments to determine the measurement of objects with non- standard and standard units to the nearest ¼- inch or centimeter.	Measure down to the nearest -1/8-inch, centimeter or millimeter Determine angle measurement to nearest five degrees using a protractor.	Apply appropriate tools and techniques to measure down to the nearest 1/4-, 1/8- or 1/16- inch or nearest centimeter or millimeter.	Apply appropriate tools techniques to measure down to the nearest 1/4-, 1/8- or 1/16-inch or nearest centimeter or millimeter.	Select and use tools with appropriate degree of precision to determine measurements directly.
	Read thermometers to the nearest 5 degrees F/C.	Read thermometers to the nearest 5 degrees F/C.	Read thermometers to the nearest five degrees F/C and read a scale to the nearest ounce or five grams.	Read and interpret measuring instruments to determine the measurement of objects with standard units (U.S. customary).			
	Tell time to the nearest minute using analog and digital clocks; translate time from analog to digital clocks and vice versa.	Tell time to the nearest minute and translate time from analog to digital clocks and vice versa.	Translate time on an analog clock to a digital clock and vice versa.				
		Determine and compare elapsed time in multiples of 15 minutes in problem-solving situations.	Determine and compare elapsed time in problem-solving situations.	Determine and compare elapsed time in problem-solving situations.	Determine and compare elapsed time in problem-solving situations.	Determine and compare elapsed time in problem-solving situations.	
	Investigate measurements of area.	Investigate measurements of area and perimeter.			Measure and/or draw angles up to 180 degrees.	Measure and/or draw angles up to 360 degrees.	

	Grade 3	Grade 4	Grade 5	Grade 6	Grade 7	Grade 8	Grade 10
D. Measurement Sub-Skill D.c.: Indirect Measurement	Apply estimation techniques using non-standard units.	Apply estimation techniques using non-standard units.	Estimate measurement using U.S customary and metric measurements.	Estimate measurements using U.S. customary and metric measurement.	Estimate area given a reference.	Estimate area given a reference.	
			Determine perimeter and area of regular shapes and the area of plane rectangular shapes. Determine perimeter and area of irregular shapes when given a reference tool such as a grid.	Determine the area of regular shapes including right triangles.	Determine perimeter/circumference and area of squares, rectangles, triangles, parallelograms and circles in real-world context.	Determine perimeter/circumference and area of polygons and circles with and without context.	Determine the perimeter/area of two-dimensional figures.
				Determine distance between points using a scale.	Determine distance between points using a scale.	Determine distance between points using a scale.	
						Determine volume and surface area of cylinders, rectangular prisms and pyramids with base shapes of triangle, square, regular pentagon and regular hexagon in real-world context.	Determine the surface area/volume of three-dimensional figures.
						Draw similar figures in any shape using a scale factor (e.g., enlarge/shrink).	Solve for angles, and segments in similar polygons and arcs in circles.
						Use ratio and proportion in context.	Use right-triangle trig functions and the Pythagorean Theorem to solve right-triangle problems.
						Use $d = r*t$ formula in simple contexts.	Use formulas in applications (e.g., Distance Formula, simple and compound interest).

	Grade 3	Grade 4	Grade 5	Grade 6	Grade 7	Grade 8	Grade 10
E. Statistics and Probability Sub-Skill E.a.: Data analysis and statistics	Answer and pose questions about collecting, organizing and displaying data. Work with data in the context of real-world situations by determining what data to collect and when and how to collect it to answer questions.	Answer and pose questions about collecting, organizing and displaying data. Work with data in the context of real-world situations by formulating questions that lead to data collection and analysis and determining what data to collect and when and how to collect the data.	Formulate questions to collect, organize and display data.	Formulate questions to collect, organize and display data.			Organize, display, compare and interpret data in a variety of ways in mathematical and real-world contexts e.g., histograms, line graphs, stem-and-leaf plots, scatter plots, box-and whiskers, bar charts, Venn diagrams, tables, circle graphs.
	Collect, organize and display data in simple bar graphs and charts including translating data from one form to the other.	Collect, organize and display data in simple bar graphs and charts including translating data from one form to the other.	Collect, organize and display data in appropriate graphs or charts.	Collect, organize and display data in appropriate graphs or charts.			
	Draw reasonable conclusions based on simple interpretations of data.	Draw reasonable conclusions based on simple interpretations of data.	Draw reasonable conclusions based on contextual data.	Draw reasonable conclusions based on contextual data.	Summarize data sets in tables, charts and diagrams with and or without context.		
			Use data to predict outcomes or trends from graph or table.	Use data to predict outcomes or trends from graphs and tables.	Evaluate a set of data to generate or confirm/deny hypotheses.	Compare two sets of data to generate or confirm/deny hypotheses.	

	Grade 3	Grade 4	Grade 5	Grade 6	Grade 7	Grade 8	Grade 10
E. Statistics and Probability Sub-Skill E.a.: Data analysis and statistics Continued	Read, use information and draw reasonable conclusions from data in graphs, tables, charts and Venn diagrams.	Read, use information and draw reasonable conclusions from data in graphs, tables, charts and Venn diagrams.	Read and interpret information from single bar graphs, line plots, picture graphs and Venn diagrams.	Extract, interpret and analyze data from single bar graphs, tables and charts, line plots, context, circle graphs and Venn diagrams.	Extract, interpret and analyze data from tables, simple stem-and-leaf plots, simple bar graphs, line plots, line graphs, simple circle graphs, charts and diagrams.	Extract, interpret and analyze data including multiple representations of the same data from tables, double back-to-back stem-and-leaf plots, double bar graphs, simple circle graphs, line plots, line graphs, charts and diagrams with and without context.	Interpret, analyze and make predictions from organized and displayed data. e.g., measures of central tendency such as mean, median, mode, and, measures of variation such as standard deviation, mean, median, mode, range, dispersion, outliers, line of best fit, percentiles.
			Describe a given set of data of seven items/numbers or fewer using the terms range, mode and median in problems with and without context.	Describe a given set of data of ten or fewer items/numbers using the terms mean, median, mode and range to extract information from organized charts, tables, graphs and Venn diagrams in problems with and without context.	Create graph with one-variable data sets using simple stem-and-leaf plots, bar graphs, circle graphs, line plots and line graphs; discuss appropriateness of graphs selected.	Create graph with one-variable data sets using back-to-back stem-and-leaf plots, double bar graphs, circle graphs, line plots and line graphs; discuss appropriateness of graph selected.	
					Find mean, median (with odd set of data), mode and range of a set of data with and without context.	Find mean, median (with odd or even number of data), mode and range of a set of data with and without context.	
					Evaluate sources of data in context and multiple representations of a given data set.	Evaluate sources of data in context and multiple representations of a given data set.	Analyze, evaluate and critique methods and conclusions of statistical experiments, e.g., randomness, sampling, techniques, surveys.

	Grade 3	Grade 4	Grade 5	Grade 6	Grade 7	Grade 8	Grade 10
E. Statistics and Probability Sub-Skill E.b.: Probability	Determine if the occurrence of future events are more, less or equally likely to occur.	Determine if the occurrence of future events are more, less or equally likely to occur.	Determine if future events are more, less or equally likely, impossible or certain to occur.	Determine the likelihood of future events, predict outcomes of future events and test predictions using data from a variety of sources.	Determine the likelihood of an event and probability based on one independent event, e.g., spinning the arrow on a spinner.	Determine the likelihood of an event and probability based on one or two dependent or independent events.	Determine the likelihood of occurrence of simple and complex events Ex: Combinations and permutations, fundamental counting principle, experimental versus theoretical probability and independent, dependent and conditional probability.
	Choose a fair and an unfair spinner.	Design a fair and an unfair spinner.	Choose or design an event that is fair or unfair.	Choose or design an event that is fair or unfair.	Use probabilities to estimate outcomes and evaluate fair and unfair simple events.	Use probabilities to estimate outcomes and evaluate fair and unfair simple events.	
		Predict the outcomes of a simple event using words to describe probability. Ex: Flipping a coin has a 1 out of 2 chance of getting a head.	Predict the outcomes of a simple event using words to describe probability and test predictions using data from a variety of sources.	Determine the probability of events in context using words, percents or fractions.	Use data from simulations provided in charts/tables to solve and interpret probability problems.	Use data from simulations provided in charts/tables to solve and interpret probability problems.	
		Describe and determine the number of combinations for choosing 2 out of 3 items. Ex: Red hat, blue jacket and green jacket. What are the combinations of wearing a hat and a jacket?	Describe and determine the number of combinations for choosing 2 out of 4 items Ex: What are the possible combinations when selecting 2 items from a menu of 4 items (chips, cookie, pizza, banana, etc.)?	Describe and determine the number of combinations of selecting 3 items from 4 or more items.	Describe and determine the number of combinations of selecting 3 items from 4 or more items.	Determine the number of arrangements from a set of 5 or less. Ex: How many different ways could 5 students stand in line?	
					Solve problems involving sample spaces or diagrams.	Solve problems involving sample spaces or diagrams.	
					Analyze outcomes based on an understanding of theoretical and experimental probability.	Analyze outcomes based on an understanding of theoretical and experimental probability.	

	Grade 3	Grade 4	Grade 5	Grade 6	Grade 7	Grade 8	Grade 10
F. Algebraic Relationships Sub-Skill F.a: Patterns, Relations and Functions	Recognize, extend, describe, create and replicate a variety of patterns including attribute, number and geometric patterns. Such as: • Picture patterns • Patterns in tables and charts • "What's–my–rule?" patterns • Patterns using addition and subtraction rules. Focusing on relationships within patterns as well as extending patterns e.g., patterns and relationships represented with pictures, tables and charts, and "what's-my–rule?" patterns using addition and subtraction rules.	Recognize, extend, describe, create and replicate a variety of patterns including attribute, number and geometric patterns. Such as: • Picture patterns • Patterns in tables and charts • "What's–my–rule?" patterns Patterns using addition and subtraction rules. Focusing on relationships within patterns as well as extending patterns e.g., patterns and relationships represented with pictures, tables and charts; "what's–my–rule?" patterns using addition and subtraction rules.	Recognize, extend, describe, create and replicate a variety of patterns including attribute, numeric and geometric patterns.	Recognize, extend, describe, create and replicate a variety of patterns including attribute, numeric and geometric patterns.	Use two concurrent numeric patterns to describe and analyze functional relationships between two variables in two concurrent numeric patterns using addition and subtraction.	Use two concurrent numeric patterns to describe and analyze functional relationships between two variables. Describe and analyze in words functional relationships in two concurrent numeric patterns s using multiplication and exponents and describe the relationship in words.	Describe, recognize, interpret and translate graphical representations of mathematical and real-world phenomena on coordinate grids, e.g., slope, intercepts, rate of change, linear and non-linear functions, and quadratic, exponential and constant functions.
			Represent patterns and relationships with pictures, tables and charts.	Represent patterns and relationships with pictures, table and charts.	Extend a given arithmetic sequence of pictures or numbers.	Extend an increasing or decreasing arithmetic or geometric pattern.	
	Determine odd or even with a total set of 20 or less.	Determine odd or even.	Describe a rule that explains a functional relationship or pattern using addition, subtraction or multiplication rules.	Describe a rule that explains a functional relationship or pattern using addition, subtraction or multiplication rules.	Describe and interpret linear patterns in tables and graphs.	Describe and interpret linear patterns in tables and graphs.	

	Grade 3	Grade 4	Grade 5	Grade 6	Grade 7	Grade 8	Grade 10
F. Algebraic Relationships Continued Subskill F.a: Patterns, Relations and Functions Continued			Determine a future event in a pattern up to the eighth item when given the first five.	Determine a future event in a pattern up to the tenth item when given the first five.	Identify the rule to complete or extend a function table or any combination of the two using one operation (+, -, x, ÷) and numbers (0 through 100) in the function table.	Identify the rule to complete or extend a function table or any combination of the two using one or two operations (+, -, x, ÷) and numbers (-100 through 100) in the function table.	
				Solve simple two-step, two operation patterns. Ex: 5, 8, 6, 9, 7,10, 8…….. (Pattern: +3-2….) Represent patterns and relationships with pictures, table and charts.	.	.	Analyze, generalize and represent patterns of change, e.g., direct and inverse variations, including numerical sequences, patterns to a given term, algebraic expressions and equations.
					Describe real-world phenomena represented by a graph. Describe real-world phenomena that a given graph might represent.	Describe real-world phenomena represented by a graph. Describe real-world phenomena that a given graph might represent.	
						Justify the accuracy of the chosen item in a sequence.	

	Grade 3	Grade 4	Grade 5	Grade 6	Grade 7	Grade 8	Grade 10
F. Algebraic Relationships Sub-Skill F.b.: Expressions, Equations and Inequalities	Demonstrate an understanding that the "=" sign means "the same as" by solving open or true/false number sentences.	Demonstrate an understanding that the " =" sign means "the same as" by solving open or true/false number sentences.	Solve simple one-step open sentences involving all operations in context.				
			Demonstrate a basic understanding of equality and inequality using symbols (<, >, =) with all operations.	Demonstrate basic understanding of equality and inequality using symbols (<, >, =) with multi-step, mixed operations.	Demonstrate understanding of equality and inequality and solve single-variable equations using symbols (<, >, =+).	Solve single-variable inequalities using symbols.	Solve linear and quadratic equations, linear inequalities and systems of linear equations and inequalities.
	Use notation to represent mathematical thinking: letter or box (variable); operation symbols (+, -, =).	Use notation to represent mathematical thinking: letter or box (variable); operation symbols (+, - , =).	Solve simple one-step open sentences including missing factor in problems with and without context e.g., "box" or letter variable and whole number coefficients.	Solve one-step equations with "box" variable and whole number coefficients in problems with and without context using whole number coefficients.	Solve single-variable one-step equations and algebraic expressions with one variable and one operation and whole number coefficients with and without context.	Solve single-variable one- and two-step equations with whole number, whole number integer, or rational, coefficients with and without context.	
		Demonstrate a basic understanding of equality and inequality using symbols (<, >, =) with simple addition and subtraction.	Represent problem situations with one-step equations involving multiplication and division with simple open sentences.		Describe in words the generalization for a given one-operation pattern.	Find values of expressions with one variable and up to two operations including basic operations and exponents	Model and solve a variety of mathematical and real-world problems by using algebraic expressions, equations and inequalities, e.g., linear, exponential, quadratic.
				Solve two-step multi-operation equations with "box" or letter variable and whole number coefficients with and without context. Ex: 3 * "box" +1 = 7	Solve two-step multi-operation equations with letter variables and whole number coefficients with and without context. Ex: 3x +1 = 7	Solve two-step multi-operation equations with letter variables and whole number or integer coefficients with and without context. Ex: -3x +1 = 7	

	Grade 3	Grade 4	Grade 5	Grade 6	Grade 7	Grade 8	Grade 10
F. Algebraic Relationships Sub-Skill F.b.: Expressions, Equations and Inequalities Continued			Represent problem situations with one-step equations or expressions using one of the four operations.	Represent problem situations with one or two-step equations or expressions. Solve simple two-step, two operation patterns.	Represent problem situations with one or two-step equations or expressions.	Write an algebraic expression (with one or two operations) which generalizes a linear pattern.	Translate between different representations and describe the relationship among variable quantities in a problem, e.g., tables, graphs, functional notations, formulas.
				Solve two-step open sentences involving all operations.	Describe in words the generalization for a given one-operation pattern.	Create a corresponding algebraic expression when given an arithmetic operation/relationship expressed in words.	
				Solve equations involving any two operations. Ex: 3 * 4 -2=? Ex: 12/3 +1=”box” Ex: 5 * 2 – 1 = a	Evaluate formulas with and without context by solving for a specified variable.	Evaluate formulas with and without context by solving for a specified variable.	

	Grade 3	Grade 4	Grade 5	Grade 6	Grade 7	Grade 8	Grade 10
F. Algebraic Relationships Sub-Skill F.c.: Properties	Use properties and or relationships of arithmetical thinking to determine and to reason about what number goes in a "box" to make a number sentence true, • identity property of e.g., zero; Ex: 12 + 0 = "box" adding 1 to any number, commutative property for addition of single-digits	Use properties and relationships of arithmetic to determine what number goes in a "box" to make a number sentence true, • identity property of zero; ex: 2 + 0 = "box" • identity property of one; Ex: 5 x 1 = "box" • commutative property for addition of single-digits • Associative property	Use the commutative property of multiplication with positive single digits.	Use the commutative property of multiplication with positive single digits.	Identify a pair of equivalent numerical expressions where the commutative property of either addition or multiplication has been used.	Identify a pair of equivalent numerical or one-variable expressions when using commutative or associative properties with addition and multiplication.	Demonstrate understanding of properties by evaluating and simplifying expressions.
	Use simple equations in a variety of ways to demonstrate the properties above.	Use simple equations in a variety of ways to demonstrate the properties above.					
			Use the inverse relationship of division and multiplication with single digit, whole numbers.	Use the inverse relationship of division and multiplication with single whole digits.			
			Demonstrate understanding of order of operations by solving two-step open sentences involving all operations.	Simplify (evaluate) two-step numerical expressions using correct order of operations.	Demonstrate understanding of up to three-step order of operations expression with and without context using parentheses and exponents.	Demonstrate understanding of up to four-step order of operations expression using parentheses, exponents and fraction symbol.	Demonstrate understanding of properties by solving linear and quadratic equations, linear inequalities, and systems of linear equations and inequalities with one or two variables.
				Demonstrate understanding of distributive property.	Demonstrate understanding of distributive property.	Demonstrate understanding of distributive property without variables.	
				Demonstrate understanding of order of operations by solving two-step open sentences involving all operations.		Solve order of operations problems with one variable to demonstrate understanding of commutativity and associativity.	

Calculator Use for Statewide Assessment
WKCE – Beginning November 2005

Background
The newest ESEA reauthorization known as *No Child Left Behind* (NCLB) has impacted schools and districts nationwide. State education agencies across the United States have been redesigning statewide assessment systems and implementing new policies and procedures in order to comply. The NCLB federal legislation mandates yearly testing in both reading and mathematics at grades 3-8 and once in high school. Since the mid-1990's, Wisconsin law (WI stats.118.30) has required testing in grades 4, 8 and 10 in reading, language arts, mathematics, science and social studies. With the enactment of NCLB, reading and mathematics testing in Wisconsin will be expanded to cover grades 3, 4, 5, 6, 7, 8 and 10. Testing of language arts, science and social studies will continue at grades 4, 8 and 10.

WKCE Test Development
Development of the Wisconsin Knowledge and Concepts Examination-Criterion Referenced Test (WKCE) for reading and mathematics began in 2003. Experts from across the state were involved in all phases of test development. Field testing and forms calibrations of the new test items were conducted during May and December 2004. The first official administration of WKCE will be November 2005.

WKCE Calculator Use
During the development phase of the WKCE, educators, administrators, statewide committees and mathematics leaders, including the Wisconsin Mathematics Council Board of Directors and past-presidents, were consulted regarding the use of calculators on statewide assessment. The consensus of the various groups was that calculators are a tool of mathematics and are used to enhance learning. Calculators are not a replacement for student knowledge of basic computation and estimation skills. The groups recommended that during statewide assessments, students should be allowed to use the same calculator that is used for classroom instruction. It is important to note that at grades 5-8 and 10, the WKCE items have been designed so that scientific or graphing calculators do not give students an advantage over four-function calculators.

In consultation with state educators, administrators, and mathematics leaders, the Wisconsin Department of Public Instruction has established specifications for the use of calculators on statewide mathematics assessments beginning November 2005.

- ✓ Grades 3 & 4:   - No calculators permitted during any session
- ✓ Grades 5, 6, 7, 8, 10:   - No calculator sessions <u>and</u> calculator required sessions

In grades 5-8, all students must have either a four-function, scientific or graphing calculator during the calculator-required sessions. In grade 10, all students must have either a scientific or graphing calculator. Calculators with QWERTY keyboards, infrared capabilities, memory disks or those that perform symbolic manipulations are not permitted. Graphing calculators must have the memory cleared prior to the test. (These are the same specifications for the current 10th grade WKCE.) If the memory cannot be cleared prior to test administration, students are not permitted to use that calculator and another calculator must be provided. In grades 5-8 and 10, anything other than a four-function calculator offers no advantage to students during the test. Allowable

accommodations for students with IEP or Section 504 plans will be specified in the test directions.

The use of calculators on WKCE has implications for districts and schools to consider prior to the 2005 testing window. First, it is the district's responsibility to ensure that all students in grades 5-8 and 10 have access to a calculator during the calculator-required sessions of WKCE. Second, a policy regarding the type of calculator to be used at grades 5-8 and 10 needs to be established. The use of graphing calculators on the test is a district decision and should be aligned with the district's curriculum and instructional practices. Third, if the district has determined that graphing calculators will be allowed at any levels, then procedures need to be established for disabling the specified functions. Fourth, districts and schools need to consider how all students will have access to a calculator during the 'calculator required' sessions of the mathematics test, including having students bring calculators from home or sharing calculators across grade levels by scheduling the mathematics tests at different times or on different days. Finally, although calculators are not permitted on the tests at grades 3 and 4, there are curricular implications for all grade levels. It is particularly important to have discussions about the appropriate use of calculators at early elementary grades.

Future assessment updates will continue to be distributed to districts and schools.

Glossary of Terms Used in the Wisconsin Assessment Framework

A

Analyze. In Bloom's Taxonomy this refers to breaking down a text into its component parts in order to make the relationships between the ideas more explicit.

Associative property. When adding or multiplying three numbers, it doesn't matter if the first two or the last two numbers are added or multiplied first.

Attribute (measurable). An identifiable property of an object, set, or event that is subject to being measured. For example, some of the measurable attributes of a box are its length, weight, and capacity (how much it holds).

B

Bias. A preference or attitude that may prevent impartial judgment.

Box plot. A graphic method that shows the distribution of a set of data by using the median, quartiles, and the extremes of the data set. The box shows the middle 50% of the data; the longer the box, the greater the spread of the data.

C

Cause and effect. A way of organizing text that emphasizes the causal relationships between two or more events or situations.

Central tendencies. A number which in some way conveys the "center" or "middle" of a set of data. The most frequently used measures are the mean and the median.

Combinations. Subsets chosen from a larger set of objects in which the order of the items in the subset does not matter. For example, determining how many different committees of four persons could be chosen from a set of nine persons. (See also, Permutations)

Commutative property. Numbers can be added or multiplied in either order.
For example, 15 + 9 = 9 + 15; 3 x 8 = 8 x 3.

Congruence. The relationship between two objects that have exactly the same size and shape.

Correlation. The amount of positive or negative relationship existing between two measures. For example, if the height and weight of a set of individuals were measured, it could be said that there is a positive correlation between height and weight if the data showed that larger weights tended to be paired with larger heights and smaller weights tended to be paired with smaller heights. The stronger those tendencies, the larger the measure of correlation.

Constructed response. On the WKCE reading test, a type of item that requires a brief written response from a student.

Criterion-referenced. An interpretation of a test score relative to specified performance criteria.

D

Descriptor. In the WKCE assessment framework, an example of a specific knowledge or skill that may be assessed on the test.

Direct measurement. A process of obtaining the measurement of some entity by reading a measuring tool, such as a ruler for length, a scale for weight, or a protractor for angle size.

Dispersion. The scattering of the values of a frequency distribution (of data) from an average.

Distributive property. Property indicating a special way in which multiplication is applied to addition of two (or more) numbers. For example,
5 x 23 = 5 x (20 + 3) = 5 x 20 + 5 x 3 = 100 + 15 = 115.

E

Evaluate. In Bloom's Taxonomy this refers to making a judgment about the value of some idea, text, and so on for some purpose.

Expanded notation. Showing place value by multiplying each digit in a number by the appropriate power of 10. For example, $523 = 5 \times 100 + 2 \times 10 + 3 \times 1$ or $5 \times 10^2 + 2 \times 10^1 + 3 \times 10^0$.

Exponential function. A function that can be represented by an equation of the form $y = ab^x + c$, where a, b, and c are arbitrary, but fixed, numbers and $a \neq 0$ and $b > 0$ and $b \neq 1$.

Exponential notation (exponent). A symbolic way of showing how many times a number or variable is used as a factor. In the notation 5^3, the exponent 3 shows that 5 is a factor used three times; that is $5^3 = 5 \times 5 \times 5 = 125$.

Extend. To draw conclusions or make predictions that go beyond what is stated.

F

Framework. For the WKCE, a document developed by the Department of Public Instruction to help educators understand the range of coverage of the test.

Frequency distribution. An organized display of a set of data that shows how often each different piece of data occurs.

Function. A relationship between two sets of numbers or other mathematical objects where each member of the first set is paired with only one member of the second set. Functions can be used to understand how one quantity varies in relation to (is a function of) changes in the second quantity. For example, there is a functional relationship between the price per pound of a particular type of meat and the total amount paid for ten pounds of that type of meat.

G – I

Identity. For addition: The number 0; that is $N + 0 = N$ for any number N. For multiplication: The number 1; that is, $N \times 1 = N$ for any number N.

Indirect measurement. A process where the measurement of some entity is not obtained by the direct reading of a measuring tool, or by counting of units superimposed alongside or on that entity. For example if the length and width of a rectangle are multiplied to find the area of that rectangle, then the area is an indirect measurement.

Integers. The set of numbers: {..., -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6,...}

Intercept. The points where a line drawn on a rectangular-coordinate-system graph intersect the vertical and horizontal axes.

Inverse. For addition: For any number N, its inverse (also called opposite) is a number -N so that $N + (-N) = 0$ (e.g., the opposite of 5 is -5, the opposite of -3/4 is 3/4).

For multiplication: For any number N, its inverse (also called reciprocal) is a number N* so that $N \times (N^*) = 1$ (e.g., the reciprocal of 5 is 1/5; the reciprocal of -3/4 is -4/3.

J – L

Line of best fit. A straight line used as a best approximation of a summary of all the points in a scatter-plot* (See definition below). The position and slope of the line are determined by the amount of correlation* (See definition above) between the two paired variables involved in generating the scatter-plot. This line can be used to make predictions about the value of one of the paired variables if only the other value in the pair is known.

Line plot. A graphical display of a set of data where each separate piece of data is shown as a dot or mark above a number line.

Linear equation. An equation of the form $y = ax + b$, where a and b can be any real number. When the ordered pairs (x, y) that make the equation true for specific assigned values of a and b are graphed, the result is a straight line.

M

Matrix (pl.: matrices). A rectangular array of numbers, letters, or other entities arranged in rows and columns.

Maximum/minimum (of a graph). The highest/lowest point on a graph. A relative maximum/minimum is higher/lower than any other point in its immediate vicinity.

Mean. The arithmetic average of a set of numerical data.

Median. The middle value of an ordered set of numerical data. For example, the median value of the set {5, 8, 9, 10, 11, 11,13} is 10.

Mode. The most frequently occurring value in a set of data. For example, the mode of the set {13, 5, 9, 11, 11, 8, 10} is 11.

Model (mathematical). A [verb] and a noun. [Generate] a mathematical representation (e.g., number, graph, matrix, equation(s), geometric figure) for real world or mathematical objects, properties, actions, or relationships.

N

(Non)-Linear functional relationship. (See definition of Function above.) Many functions can be represented by pairs of numbers. When the graph of those pairs results in points lying on a straight line, a function is said to be linear. When not on a line, the function is nonlinear.

Norm-referenced. An interpretation of a test score relative to the scores of other test-takers.

O

Outlier. For a set of numerical data, any value that is markedly smaller or larger than other values. For example, in the data set {3, 5, 4, 4, 6, 2, 25, 5, 6, 2} the value of 25 is an outlier.

P

Patterns. Recognizable regularities in situations such as in nature, shapes, events, sets of numbers. For example, spirals on a pineapple, snowflakes, geometric designs on quilts or wallpaper, the number sequence {0, 4, 8, 12, 16,...}.

Percentile. A value on a scale that indicates the percent of a distribution that is equal to it or below it. For example, a score at the 95th percentile is equal to or better than 95 percent of the scores.

Permutations. Possible arrangements of a set of objects in which the order of the arrangement makes a difference. For example, determining all the different ways five books can be arranged in order on a shelf.

Prime number. A whole number greater than 1 that can be divided exactly (i.e., with no remainder) only by itself and 1. The first few primes are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37.

Pythagorean theorem (relationship). In a right triangle, $c^2 = a^2 + b^2$, where c represents the length of the hypotenuse (the longest side of the triangle which is opposite the right (angle), and a and b represent the lengths of the other two, shorter sides of the triangle.

Q

Quadratic function. A function that can be represented by an equation of the form $y = ax^2$ (or ax^2) + bx + c, where a, b, and c are arbitrary, but fixed, numbers and a 0. The graph of this function is a parabola.

Quartiles. The 25th, 50th and 75th percentile points. (See definition of Percentile.)

R

Range (of a set of data). The numerical difference between the largest and smallest values in a set of data.

Rational number. A number that can be expressed as the ratio, or quotient, of two integers, a/b, provided $b \neq 0$. Rational numbers can be expressed as common fractions or decimals, such as 3/5 or 0.6. Finite decimals, repeating decimals, mixed numbers and whole numbers are all rational numbers. Nonrepeating decimals cannot be expressed in this way, and are said to be irrational.

Real numbers. All the numbers which can be expressed as decimals.

Real-world problems. Quantitative and spatial problems that arise from a wide variety of human experiences, applications to careers. These do not have to be highly complex ones and can include such things as making change, figuring sale prices, or comparing payment plans.

Rectangular coordinate system. This system uses two (for a plane) or three (for space) mutually perpendicular lines (called coordinate axes) and their point of intersection (called the origin) as the frame of reference. Specific locations are described by ordered pairs or triples (called coordinates) that indicate distance from the origin along lines that are parallel to the coordinate axes.

Rubric. A scoring guide used to evaluate a student's performance.

S

Scaling (Scale drawing). The process of drawing a figure either enlarged or reduced in size from its original size. Usually the scale is given, as on a map 1 inch equals 10 miles.

Scatter plot. Also known as scattergram or scatter diagram. A two dimensional graph representing a set of bi-variate data. That is, for each element being graphed, there are two separate pieces of data. For example, the height and weight of a group of 10 teenagers would result in a scatter plot of 10 separate points on the graph.

Scientific notation. A short-hand way of writing very large or very small numbers. The notation consists of a decimal number between 1 and 10 multiplied by an integral power of 10. For example, $47{,}300 = 4.73 \times^{4}$; $0.000000021 = 2.1 \times 10^{-8}$

Selected-response. A kind of test item in which a student must choose the best response from among several choices. Also known as multiple-choice.

Similarity. The relationship between two objects that have exactly the same shape but not necessarily the same size.

Simulation. Carrying out extensive data collection with a simple, safe, inexpensive, easy-to-duplicate event that has essentially the same characteristics as another event which is of actual interest to an investigator. For example, suppose one wanted to gather data about the actual order of birth of boys and girls in families with five children. (e.g., BBGBG is one possibility) Rather than wait for five children to be born to a single family, or identifying families that already have five children, one could simulate births by repeatedly tossing a coin five times. Heads vs. tails has about the same chance of happening as a boy vs. a girl being born.

Slope. A measure of the steepness or incline of a straight line drawn on a rectangular-coordinate-system graph. The measure is obtained by the quotient "rise/run" (vertical change divided by horizontal change) between any two points on that line.

Stem-and-leaf plot. A way of showing the distribution of a set of data along a vertical axis. The plot at right shows the data 13, 19, 33, 26, 19, 22, 34, 16, 28, 34. The ten's digits of these data are the stems and the one's digits are the leaves.

1	3699	
2	268	
3	344	
Key: 1|5 means 15	

Summary statistics. A single number representation of the characteristics of a set of data. Usually given by measures of central tendency and measures of dispersion (spread).

Symmetry. A figure has symmetry if it has parts that correspond with each other in terms of size, form, and arrangement. For example, a figure with line (or mirror) symmetry has two halves which match each other perfectly if the figure is folded along its line of symmetry.

T

Technical Advisory Committee. For the Wisconsin Student Assessment System, a group of nationally-recognized experts that meets twice a year to advise the state on technical issues related to the assessments.

Transformation. A change in the size, shape, location or orientation of a figure.

Transitive property. For equality: If a=b and b=c, then a=c;
For inequality: If a>b and b>c, then a>c; or If a<b and b<c, then a<c.

Tree diagram. A schematic way of showing the number of ways a compound event may occur. For example, the tree diagram at the right shows the eight possible ways the tossing of three coins could happen.

U

Unit fraction. A fraction with a numerator of 1, such as 1/4 or 1/7.

V

Variable. A quantity that may assume any one of a set of values. Usually represented in algebraic notation by the use of a letter. In the equation $y = 2x + 7$, both x and y are variables.

Variance. The value of the standard deviation squared.

Vertical angles. The pair of angles that are directly across from each other when two straight lines intersect. Angles a and b at the right are an example of vertical angles.

W- Z

Whole Numbers. The numbers: 0, 1, 2, 3, 4, 5, ...

Wisconsin Mathematics Assessment Framework
Feedback Form

We welcome your comments and questions. To use this form to provide feedback please print a copy of this page, fill out the form, and mail or fax it to:

Viji Somasundaram WKCE Program Manager (608) 267-7268 Office (608) 266-8770 FAX   Wisconsin Department of Public Instruction P.O. Box 7841 Madison, WI 53707-7841

Name:

School: District:

Optional contact information:

Email: Phone:

1. Please describe your primary role in education (e.g., curriculum coordinator, director of instruction, district administrator, parent, principal, math specialist, teacher, etc.):

2. Please answer the following questions about the mathematics assessment framework:

Key: 1 = strongly disagree, 5 = strongly agree	1	2	3	4	5
1. The framework is easy to understand.					
2. The framework will be useful as we review our curriculum.					
3. The framework reflects major conceptual understandings, principles, and theories of mathematics.					
4. The framework reflects knowledge and skills that are appropriate at each grade level.					
5. The framework reflects knowledge and skills that are relevant and engaging to students.					

3. Please indicate your comments or questions: