Wisconsin Mathematics Performance Level Descriptors—Number & Quantity

The following descriptors represent the range of student performances associated with the respective performance level.*

Proficient — Number & Quantity

Students convert positive rational exponents to radical form, classify numbers expressed in terms of square roots of positive integers as rational or irrational, and describe properties of rational numbers and irrational numbers. Students recognize that x^{-2} is $\frac{1}{x^2}$ and not $\frac{1}{\sqrt{x}}$ and rewrite cube roots so there are no perfect cubes remaining inside. Students rewrite fractions using the division property of exponents even when the exponents are variable expressions. Students perform operations with radicals, simplify radicals, and rewrite fractions with an irrational denominator such as $\frac{4}{3-\sqrt{7}}$ in a form with a rational denominator. Students convert units in rate problems.

Basic

Students express data in "millions" in scientific notation, rewrite monomials with negative integer exponents in fraction form with positive exponents, and raise perfect squares to the 0.5 power, perhaps using a calculator. Students add binomials in the form $a + bi$ for integers a and b, which may represent complex numbers, and add 2×2 matrices. Students use units involving rates when solving problems.

Below Basic

Students understand the meaning of whole number exponents and multiply like terms such as $2b^3 \cdot 5b^5$. They should be able to identify reasonable units in a measurement context.

Advanced

Students express radical expressions using rational exponents in order to rewrite expressions and functions, including recognizing that an expression like $(2 + x)^{-10}$ can be written as $\frac{1}{(2+x)^{10}}$ but not as $\frac{1}{2^{10}} + \frac{1}{x^{10}}$. Students find inverse functions for power functions and radical functions and solve expressions for a given variable when exponents and radicals are involved. Students perform matrix subtraction, find the determinant of a 2×2 matrix, multiply a vector by a matrix, and can tell if a matrix product exists by knowing the dimensions of the matrices. Students represent complex numbers algebraically and graphically, recognize quadratic equations/functions that have complex solutions/zeros, and use complex numbers. Given a complex number, they identify a quadratic equation/function with that complex number as a solution/zero.

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Wisconsin Mathematics Performance Level Descriptors—Algebra

The following descriptors represent the range of student performances associated with the respective performance level.*

Proficient — Algebra

Students solve linear equations and inequalities in 1 variable that may require multiple applications of the distributive property, with values that can be fractions and decimals. They write equations to express relationships between quantities including linear, quadratic, linear systems, and simple cubic equations. They identify the graphs of linear equations, inequalities, and systems from equations or contexts. Students find a solution to equations of the form $ax^3 = c$, perhaps arising from the volume of a prism. Students apply the quadratic formula. They factor by recognizing the difference of squares. They should be able to factor trinomials of the form $\pm x^2 + bx + c$ that factor over the integers. Students solve problems that require several of these skills.

Basic

Students solve simple real-world problems that can be modeled by linear equations in 1 variable with rational coefficients. Students capture generalized arithmetic in terms of operations and letters representing numbers. They write equivalent linear equations and find solutions by applying the distributive property, collecting like terms, and isolating the variable. They do the same for linear inequalities when any multiplication and division steps are by whole numbers. Students graph solutions to linear equations and inequalities on a number line and graph a range expressed as a compound inequality such as $-3 \leq x < 7$, attending to the inclusion or exclusion of endpoints. Students write or identify linear equations in 2 variables from a context or table of values and verify whether a point is on its graph. They find the slope from various forms of linear equations or from 2 points, and they recognize that 1 point is not sufficient to find slope. They can find slope of a line from a graph of a linear system and verify a solution through substitution when coefficients and variables are integers. They can also isolate a particular unknown in a linear equation with 2 variables and solve a system of 2 linear equations in 2 variables by isolating a variable in the first and substituting that expression into the second. Students identify a solution for a system of linear inequalities with 2 or 3 linear constraints when values are integers. They identify a solution to a quadratic equation when the solution and coefficients are integers. They find the positive solution to equations of the form $x^2 = c$ and solve simple square-root equations by squaring both sides. Students add and subtract polynomials, multiply and divide monomials, and multiply binomials. They evaluate polynomials and rational functions at points in the domain. They use structure to suggest actions to take with expressions and equations.

Below Basic

Students solve or verify the solution to a linear equation in 1 variable where the variable is in a single term and the solution is an integer. They find the value of a simple algebraic expression for whole number values of the variable(s), and they identify appropriate values of the variables from a simple context. Students build an equation of the form $y = ax + b$ from a simple real-world context where the equation is a close parallel to the description. Students find products of monomials. They know that a solution for an equation means that the solution value makes the equation true, and they can use that to find the value of a parameter in a simple equation family. Students perform multiple operations to solve problems in real-world contexts and write expressions in 1 variable to represent a quantity.

Advanced

Students interpret structure to identify equations and systems of equations with no solutions and to identify equivalent expressions. They solve nonroutine problems using basic algebraic tools, treating parameters as numbers in performing algebraic operations and interpreting the results. Students capture geometric reasoning through algebraic relationships and interpret domain with respect to the context. They solve quadratic equations with complex solutions and write a quadratic equation with real coefficients that has a given complex number as a solution. Students find the maximum or minimum value of a linear expression in 2 variables subject to linear constraints and model a context with a quadratic equation in order to maximize or minimize a quantity. They should be able to find factorizations of trinomials over the integers. Students connect a suitable factorization of a higher-order polynomial with its graph.

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Wisconsin Mathematics Performance Level Descriptors—Functions

The following descriptors represent the range of student performances associated with the respective performance level.*

Proficient — Functions

Students identify graphs of linear functions from rate information and identify step functions from information about endpoints. They evaluate exponential functions and evaluate logarithmic functions where the argument may be expressed as a rational power of the base, perhaps using a calculator. Students find a vertical asymptote for a rational function with a linear denominator. For a functions expressed with a linear denominator, a quadratic denominator, or a denominator that is the square root of a linear factor, students identify the domain or where the function is undefined. Students find the change in a function's value due to a change in the independent variable's value. Students identify a reflection over the *y*-axis graphically. They compose functions and identify an equation of a function translated vertically or, for an absolute-value function, translated horizontally and vertically. For a proportional relationship, students use a pair of related values to find the constant of proportionality and then use that to find the value of the function at a different point. They recognize that unit rate is related to slope of a linear function, including in the context of a regression line. Students find the next few terms for an arithmetic or geometric sequence where the common difference or common ratio is a fraction or negative integer. They find the next few terms of a function using a recursive definition that involves subscripts. For a situation that can be modeled by an arithmetic sequence, students find a total for several terms or find how many terms it takes to make up a given total. Students use trigonometric relationships in a right triangle to find side lengths, angle measures, and trigonometric ratios. They apply the law of sines and can find $\cos^2(x)$ given $\sin^2(x)$ or vice versa. They create linear equations from different representations.

Basic

Students evaluate polynomial and absolute-value functions, perhaps given in function notation, with whole number computations. They identify linear functions that model a context. Given a linear context relating 2 variables, students find the value of one variable given the value of the other variable. Given 2 linear functions defined by equations or described in a context as a fixed part plus a variable part, students find where the functions are equal. Students find the next few terms in an arithmetic or geometric sequence where the subsequent terms are found by adding, subtracting, multiplying, or dividing by a whole number. Students identify a linear function when given a data table showing terms with a whole-number first difference. Students find values that are related by a proportional relationship described in a context. They find information about a function from a graph or table, such as change, average rate of change, vertical shift, and range.

Below Basic

Students use closely parallel structure to accomplish tasks such as identifying the values of variables from a context and substituting into an expression when the context sets the values out in the same order as in the expression. Students identify parts of an equation that are the same and that are different, for example, in $3^{a+1} = 3^4$, and recognize that making the "different" parts be equal will provide a solution to the equation. Students identify real zeros from a graph and find the value of a function, perhaps using function notation, when the computations are simple.

Advanced

Students find the equations of functions that have been translated horizontally or reflected across the *x*-axis, and given a graph of $f(x)$, they identify the graph of $|f(x)|$. Students determine the nature of the zeros of a quadratic function and find the domain of rational functions, radical functions, logarithmic functions, and composition functions. They determine asymptotes and end behavior of rational functions. Students build functions with specified properties using the relationship between slopes of parallel and perpendicular lines. Students identify the graph of a piecewise-defined function from contextual information about rates. They connect a suitable factorization of higher-order polynomial functions with a graph. Students understand that if a system of 2 linear equations in 2 variables has multiple solutions, then the equations describe the same line. Students find inverse functions. They use logarithmic functions to solve exponential equations and vice versa. Students build quadratic functions to model a quantity and predict the minimum or maximum value for the quantity. They choose a function family that is a logical fit to data trends. Given 2 terms in an arithmetic sequence, not necessarily sequential, students build a function for the sequence and calculate other terms. For situations that can be modeled with a geometric sequence, they find a total for several terms or find how many terms make up a given total. Students apply the law of cosines, identify characteristics of a trigonometric function from a graph or equation, and rewrite trigonometric expressions to be in terms of a different trigonometric ratio. They find the area of a triangle when given the lengths of 2 sides and the measure of the included angle.

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Wisconsin Mathematics Performance Level Descriptors—Geometry

The following descriptors represent the range of student performances associated with the respective performance level.*

Proficient — Geometry

Students reflect points across an axis in the coordinate plane, and they translate points left, right, up, and down. Students recognize similarity of triangles determined by 2 transversals that intersect between 2 parallel lines. They should be able to identify a sequence of basic transformations that shows congruence. They recognize the correct similarity correspondence even when the chain of transformation requires a rotation. Students find an endpoint of a line segment when given the midpoint and the other endpoint. They find the lengths of the other 3 sides of a parallelogram when given the length of 1 side and perimeter. They find the measure of an interior angle in a polygon when given the other interior angle measures. Students convert between degrees and radians. Students identify general conclusions about angles in a standard configuration such as crossing lines or a triangle. Given the lengths of a pair of corresponding sides of similar triangles, students find another side length from the length of its corresponding side. Students find the length of a side in a right triangle given the lengths of the other two sides, and they use that to find perimeter or area. Students find distance in the coordinate plane. Students are attuned to a triangle needing to be a right triangle before $a^2 + b^2 = c^2$ applies. They find the perimeter or area of a rectangle by using the Pythagorean theorem. Students determine the area, height, length of base 1, or length of base 2 in a trapezoid when given the other three quantities. They determine the volume, length, width, or depth of a prism when given relationships that determine the other three quantities. Students find the surface area of prisms and, when given a suitable net, of other figures. They find a quantity that is proportional to area when given unit rate (for example, cost per square foot) in a composite figure made from rectangles. They find a quantity that is proportional to volume when given a unit rate (for example, pounds per cubic foot) for a prism. Students find the circumference of a circle when given the diameter. Students find sine, cosine, and tangent of an acute angle in a right triangle with 2 given numeric side lengths. They use trigonometric ratios in a right triangle to find side lengths, angle measures, and other trigonometric ratios. They apply the law of sines. Students identify counterexamples.

Basic

Students translate points on a graph. They translate points using coordinates when left/right translation is given before up/down translation, which aligns with the structure of coordinate pairs. Students apply standard congruence relationships based on translation, rotation, and reflection such as base angles of isosceles triangles, alternate interior angles for parallel lines with a transversal, and vertical angles. Students find an endpoint of a line segment when given the midpoint and the other endpoint when all coordinates are whole numbers. They find angle measures using the relationship between interior angles in a triangle, the relationship between adjacent angles that make a straight angle, and the relationship between exterior angles and opposite interior angles in a triangle. Students apply scale factors for lengths and distances, such as between similar figures or using a map or scale drawing. Students use the proportional relationship between lengths along a pair of transversals cut by a set of parallel lines. They recognize that the line segment connecting the midpoints of 2 sides of a triangle is half as long as the third side. Students add and subtract areas to find the area of a composite figure. Students determine the length, width, depth, or volume of a prism when given the other three quantities. They determine the length, width, or perimeter of a rectangle when given the other two quantities. Students find a quantity that is proportional to perimeter (for example, cost of fence) when given unit rate and the length and width of the rectangle. They find the area of a right triangle when given the lengths of the legs and find the area of a parallelogram when the height can be determined without the Pythagorean theorem. Students identify characteristics of quadrilaterals.

Below Basic

Students determine length, width, or area of a rectangle when given the other two quantities. They use geometric formulas to find area or volume when given all needed quantities. Students find the perimeter of a rectangle. They identify corresponding sides of congruent triangles using the structure of congruence notation.

Advanced

Students recognize and use congruences that appear through the use of auxiliary lines. When a problem involves multiple copies of figures that are congruent via translation, rotation, or reflection (such as when a hexagon can be seen as divided into multiple triangles), students can find lengths or relative lengths, areas or relative areas. Students strategically employ the Pythagorean theorem to find lengths when solving problems. They use the converse of the Pythagorean theorem to know whether angle measures are less than 90°, equal to 90°, or more than 90°. Students recognize and apply similarity in conjunction with other relationships between lengths. They find the area or volume of similar figures or of the same figure with a different unit of measure. Students recognize similarity between a right triangle and the two triangles formed by adding the altitude to the hypotenuse. They recognize conditions leading to a regular hexagon inscribed in a circle, or equivalently, to the 6 equilateral triangles that would make up this hexagon. Students add and subtract volumes to find the volume of a composite shape. They use the relationship between the area of a sector, the central angle measure, and the radius to find any one given the other two quantities. Students find the area of an equilateral triangle in terms of side length or altitude. They use chord-radius relationships in a circle. Students recognize properties of diagonals in quadrilaterals.

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Wisconsin Mathematics Performance Level Descriptors—Statistics & Probability

The following descriptors represent the range of student performances associated with the respective performance level.*

Proficient — Statistics & Probability

Students find the probability of a compound event defined with "NOT" when given a frequency table. They find a ratio from a frequency table and calculate frequencies from a cumulative frequency table. Students determine a geometric probability based on rectangular area. Students find a median of a list of data, including the odd and even cases. They recognize that the median of an odd number of data values is one of the data values. Students recognize differences between empirical and theoretical probability. They should be able to recognize a random sample and bias. Students interpret the slope of the line of best fit when it is provided on a scatterplot.

Basic

Students calculate average/mean from a list of data, and they find weighted average/mean of a few whole-number data values with small frequencies where the students can easily make a list of all the individual data points. They find a missing data value given the average/mean and all the data values but 1. They divide a total to find average/mean when given the number of data values or the length of the interval. Students identify a bar graph by matching frequency counts. Students use sample counts to determine expected population counts. Students count combinations of 3 objects, 1 from each of 3 categories. Students find the probability of a compound event defined with "or" when given a frequency table. They find conditional probability when they can see it as finding the probability of a simple event in a smaller sample space.

Below Basic

Students find the probability of a simple event happening or not happening when the sample space is clear. Students find a median when given an odd number of data points. They interpret a value from 1 region of a Venn diagram.

Advanced

Students find a median from a frequency table or stem-and-leaf graph. They calculate weighted average/mean. They find an average/mean from a frequency table or graph. Students find the probability of a compound event when the simple events are independent or when selecting twice at random, either with or without replacement. Students find the number of possibilities when cases overlap and it is easy to miss or to double-count if the counting method is not organized. They count using ordered selection with replacement (fundamental counting principle), ordered selection without replacement (permutations), and unordered selection without replacement (combinations). Students recognize that greater standard deviation represents more spread away from the mean of the data. They calculate expected value. Students should be able to create a random sample. Students model data with nonlinear functions.

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